ELEMENTARY APPLIED MECHANICS

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PREFACE

k planning this book I have had particularly in mind the very mited mathematical attainments of many beginners, and especially the earlier chapters much use is made of simple graphical lculation, which experience shows appeals strongly to young gineering students. On the other hand, the subject will generally rm part of a systematic course of technical instruction, and some aders will be well prepared in this respect, so that in the later rtions particularly, opportunities are given to the student to apply s elementary practical mathematics, and to turn it to useful count.

I am a firm believer in the value of numerical calculations as a eans of teaching elementary mechanics, and as symbols may be rmidable obstacles to some readers, simple numerical illustration nerally precedes or replaces algebraic formulæ.

While the volume is not a laboratory manual, mechanical boratory work is so valuable an agent of instruction that observans obtained from experiments on simple apparatus illustrated d described are frequently used; most students will make similar servations for themselves from more or less similar apparatus, ile readers, unable to avail themselves of such advantages, will t be precluded from understanding the conclusions drawn from ch simple experiments.

The ground covered is that indicated by the Board of Education Stage I of the subject, and care has been taken not to make the rk more difficult by going much outside those limits. Building dents and others requiring a partial course on mechanics, may ct Chapters I to X and XIV to XXI inclusive.

Mr. Inchley has written parts of the text, most of the worked-out

examples, and all of those to be solved by the reader, which form so important a feature of the book; he is also responsible for all of the diagrams, and any merit which the book may possess must be very largely attributed to the way in which he has carried out his share of the work.

I am indebted to my friend Prof. J. H. Smith, D.Sc., for suggestions on the plan of this book; figs. 158 and 193 represent apparatus of his design.

Our thanks are tendered to Messrs. Tangye, Ltd., of Birmingham, for the blocks of Figs. 237 and 238.

ARTHUR MORLEY.

NOTTINGHAM,
Jan., 1911.

PREFACE TO THIRD EDITION

SEVERAL numerical and other errors have been corrected and thanks are given to teachers and others who have kindly pointed out any such inaccuracies. Since the issue of the first edition a set of laboratory instruction sheets suitable for this stage of the work has been published to meet the requirements of those numerous teachers who have not time and opportunity to prepare their own.

WM. INCHLEY.

NORTH SHIELDS,
December, 1914.

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ELEMENTARY APPLIED MECHANICS

INTRODUCTION

SOME of the information given in the following pages of introduction will be known to all readers, while the whole of it may be already known to others. It is placed here for convenient reference, and will also provide suitable introductory exercises in laboratory work for a systematic course of mechanics. The beginner need not master the whole introduction before reading the book. Laboratory exercises should *follow* lecture work, and in many laboratories this is only possible by starting part of the class on the preliminary but instructive exercises on measurement.

Rules for the Calculation of Areas.

Square.—Multiply the length of side (in inches) by the length of side (in inches); the result will be the area expressed in square inches. *Or* if s = length of side in inches, then area $= s \times s = s^2$ square inches (Fig. 1).

Rectangle.—Multiply the length of one side by the length of its adjacent side. If a and b are the sides (Fig. 1), then area $= a \times b$.

Triangle.—The area is equal to one-half the product of the length of the base and the height. If b = base and h = height, then area $= \frac{1}{2}b \times h$.

Parallelogram.—Multiply the length of one side by the perpendicular distance from that side to the opposite side. In Fig. 1 area $= a \times d$.

Trapezoid.—Multiply one-half the sum of the parallel sides by the perpendicular distance between them, *or* area $= \dfrac{a+b}{2} \times d$ (Fig. 1).

Circle.—Multiply the square of the diameter by 3·1416 and

divide the product by 4; *or*, if $d =$ diameter, then area $= \frac{\pi d^2}{4}$, where $\pi = 3{\cdot}1416$.

This may also be written $0{\cdot}7354d^2$, or πr^2 where $r =$ radius of circle $= \frac{d}{2}$.

Ellipse.—Multiply the product of the two axes by $\frac{\pi}{4}$; *or*, if d_1 and d_2 be the two axes (Fig. 1), then area $= \frac{\pi}{4}d_1d_2$.

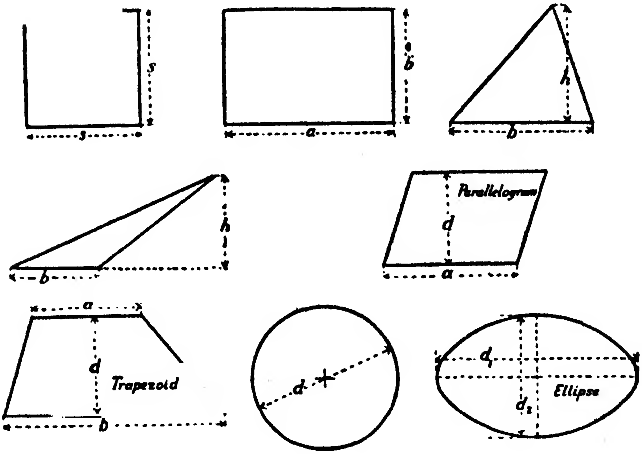


FIG. 1.

Irregular Area.—An irregular area, such as is shown in Fig. 2, may be measured by the method of mean ordinates. Divide the length of the figure into any number of equal parts as shown. Then measure the width of the figure at the middle of each part. The area is then taken as the sum of a number of rectangles of equal width and varying length, *i.e.*—

$$\begin{aligned}\text{Area} &= (d \times h_1) + (d \times h_2) + (d \times h_3) + (d \times h_4) + \text{etc.}\\ &= d(h_1 + h_2 + h_3 + h_4 + h_5 + h_6 + h_7 + h_8 + h_9 + h_{10}).\end{aligned}$$

NOTE.—If all dimensions are in inches, the area is in square inches.

Laboratory Exercise.

Trace the area of an irregular-shaped piece of metal sheet, and

find its area by the above method, using a steel rule graduated in tenths of an inch, and reading it to hundredths of an inch by estimating the second place of decimals. Check the area so found by weighing the irregular piece of plate and also a square piece cut from the same sheet. If a = area of square piece (which is known) and w is its weight, and W the weight of the irregular piece, then the area of the irregular piece $= \frac{a \times W}{w}$

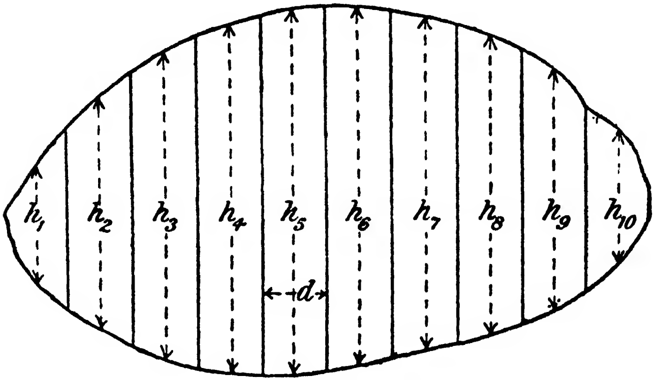


FIG. 2.—Area of an irregular figure.

Rules for the Measurement of Volumes. (See Fig. 3.)

Cube, edge s; volume $= s \times s \times s = s^3$.

Prism having its ends perpendicular to its axis; volume = area of one end multiplied by the length of the prism.

Cylinder.—Diameter, d; length, l; volume $= \frac{\pi d^2}{4} \times l$.

Pyramid. volume = area of base multiplied by one-third the perpendicular height (h).

Cone.—A cone is a special case of the pyramid in which the base is a circle; volume $= \frac{\pi d^2}{4} \times \frac{h}{3}$.

Sphere, radius r; volume $= \frac{4}{3}\pi r^3$, or, in terms of its diameter d, volume $= \frac{\pi d^3}{6}$.

NOTE.—If the linear dimensions are in inches, the volume is expressed in cubic inches.

Laboratory Exercises.

(1) Measure the volume of a casting by calculation from its

dimensions. Calculate its weight when given the weight of a cubic foot of the metal, and check the result by weighing the casting.

(2) **Density.**—Measure the volume of a sphere or cylinder. Weigh it, and then calculate the weight of unit volume of the material. *i.e.* its density.

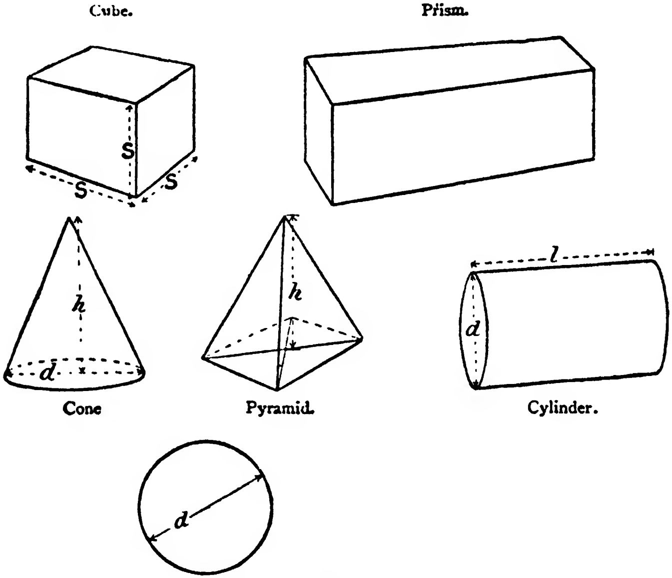


Fig. 3.

Measurement of Angles.—Angles may be measured either in *degrees* or in *radians*.

A right angle is divided into 90 degrees, written 90°, or a *degree* may be defined as the angle at the centre of a circle subtended by an arc of $\frac{1}{360}$th of the circumference (Fig. 4).

A radian is the angle at the centre of a circle subtended by an arc equal in length to the radius of the circle (Fig. 4); it is equal to 57·29 degrees, *i.e.* 2π radians = 360°, hence 1 radian $= \frac{360}{2\pi}$ $= 57{\cdot}29°$. Any angle $= \frac{\text{arc}}{\text{radius}} =$ radians.

Trigonometrical Ratios of Angles.

Let ABC (Fig. 5) be the angle, then—

$$\text{Sine } A\hat{B}C = \frac{AC}{AB} \text{ and is usually written } \sin A\hat{B}C$$

$$\text{Cosine } A\hat{B}C = \frac{BC}{AB} \text{ and is usually written } \cos A\hat{B}C$$

$$\text{Tangent } A\hat{B}C = \frac{AC}{BC} \text{ and is usually written } \tan A\hat{B}C$$

For values of these ratios, see table on p. 367.

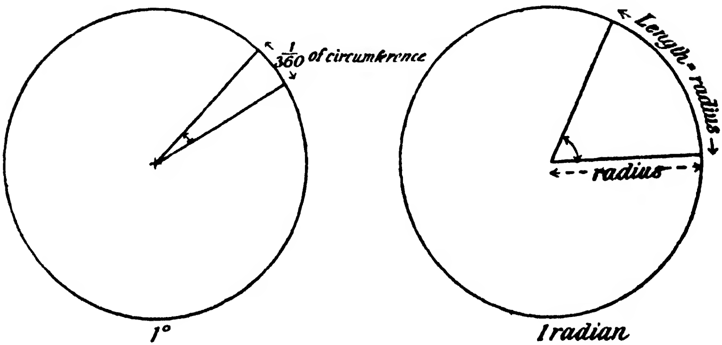


FIG. 4.

Relation between the Sides and Angles of a Triangle. —In an acute-angled triangle (Fig. 6) we have the following relations :—

$$a^2 = b^2 + c^2 - 2bc \cos B\hat{A}C$$
$$b^2 = a^2 + c^2 - 2ac \cos A\hat{B}C$$
$$c^2 = a^2 + b^2 - 2ab \cos A\hat{C}B$$

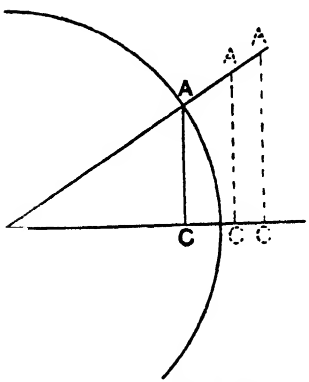


FIG. 5.—Sine, cosine, and tangent.

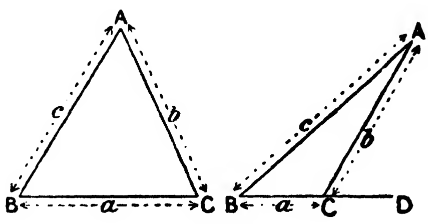


FIG. 6.

If one of the angles of the triangle is obtuse, the side opposite to the obtuse angle (Fig. 6) is given by—

$$c^2 = a^2 + b^2 + 2ab \cos \text{ACD}$$

If one of the angles of the triangle is 90°, the side opposite to the right angle is given by—

$$c^2 = a^2 + b^2$$

We also have in *any* triangle (Fig. 6)—

$$\frac{a}{\sin \text{BAC}} = \frac{b}{\sin \text{ABC}} = \frac{c}{\sin \text{ACB}}$$

Use of Squared Paper.—The use of squared paper in recording the results of experiments, etc., will be appreciated by the student on reading Chapters VII. and VIII. The method of plotting graphs will be understood from the following simple example.

It is found by experiment with a lifting tackle that an effort P lbs. is required to lift a weight of W lbs.; the values of P for different weights W being shown in the following table:—

P . .	1·5	3·0	4·2	5·0	5·9	7·0	7·6	9·6	10·5
W . .	0	10	15	20	25	30	35	45	50

Now, distances measured horizontally from the *origin* O are called *abscissae*, and distances measured vertically from the origin are called *ordinates*. Suppose we plot W horizontally (abscissae), and P vertically (ordinates). Choose any convenient scale, and mark on the horizontal and vertical axes the various values of W and P, as shown in Fig. 7. Then, to plot the graph connecting W and P, proceed as follows: From the table we see that when W = 0, P = 1·5; hence, above the point, where W = 0, *i.e.* at the origin (O) make a cross (+), corresponding to P = 1·5 on the scale of ordinates. Repeat this for each value of W given in the table, and obtain the series of points shown. Now draw an *average* curve through these points. In this case the curve is found to be a straight line. *It should be noticed that we do not join the points plotted by a series of straight lines*, but we draw an *average* line through the points, so that there are as many points on one side of the line as there are on the other side.

If now we wish to see what effort will be required to lift a weight of 40 lbs., we find the point on the graph corresponding to W = 40 and see what the corresponding value of P is. Following the dotted lines in the figure we see that this value is 8·6 lbs.

Workshop and Laboratory Methods of Measurement.

For turning and boring work in the workshop the standards of reference formerly consisted solely of the cylindrical, external, and

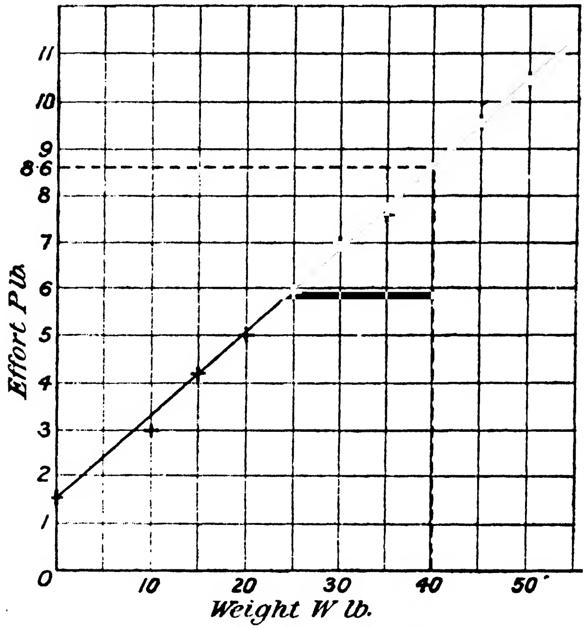


FIG. 7.—Use of squared paper.

internal gauges, one pair of which is shown in Fig. 8. These gauges are manufactured true to $\frac{1}{10000}$ inch. The workman sets his

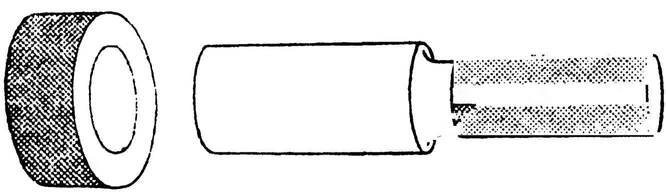


FIG. 8.—Internal and external gauges.

calipers (Fig. 9) to the standard gauge by his sense of touch, and

then transfers them to the work, the finished diameter of the work being estimated the same as the gauge by his sense of touch also.

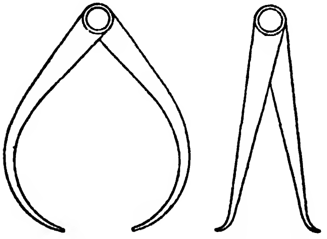


FIG. 9.

It will be easily seen that the accuracy of the work will depend upon the skill and experience of the workman, and that therefore the work cannot be guaranteed to any specified degree of accuracy.

Limit Gauges.— Modern practice demands that machine parts shall be interchangeable, and this can only be obtained when the parts are machined to a definite degree of accuracy. To obtain this degree of accuracy *limit* gauges are used. Fig. 10 shows an internal limit gauge, one

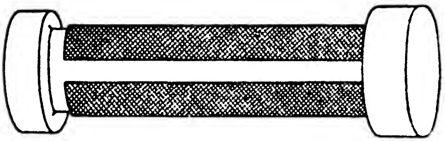


FIG. 10.—Internal limit gauge.

end of which is made slightly smaller in diameter than the other, the difference in the diameters being determined by the degree of accuracy to which it is desired to work. The smaller end must *go in* the hole, but the larger end must *not go in*. By the use of these gauges any number of parts can be made to size within the limit of accuracy desired, very frequently to $\frac{1}{1000}$ inch.

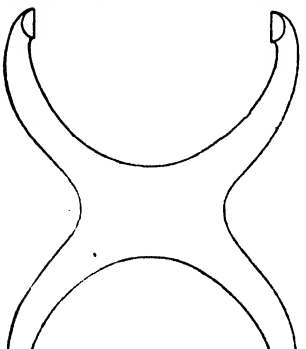


FIG. 11.—External limit gauge.

The external limit gauge (Fig. 11) is used in exactly the same way, being used for turning cylindrical pieces.

Micrometer Screw Gauge.— When great accuracy is required in measurement, a micrometer screw gauge is used (Fig. 12). In the gauge illustrated the pitch of the screw is $\frac{1}{40}$ inch, and the longitudinal scale A is graduated in tenths, each tenth being sub-

divided into 4 parts.' The screw is rotated by turning the sleeve B, the bevelled edge of which is divided into 25 equal parts. One revolution of the sleeve B therefore moves the screw forward $\frac{1}{40}$ inch, corresponding to one of the small divisions on the longitudinal scale, and $\frac{1}{25}$ of a revolution moves it forward $\frac{1}{25}$ of $\frac{1}{40}$, namely $\frac{1}{1000}$ inch. Hence, with this particular instrument it is possible to read definitely to $\frac{1}{1000}$ inch. The object to be measured is placed between the stop C and the end of the screw. The diameter of the piece shown in Fig. 12 will be seen to be—

$$0{\cdot}4'' + \tfrac{1}{40}'' + \tfrac{16}{25} \text{ of } \tfrac{1}{40}, \textit{ i.e. } 0{\cdot}4 + 0{\cdot}025 + 0{\cdot}016 = 0{\cdot}441 \text{ inch.}$$

The stop C is screwed into the frame of the instrument, and may be moved longitudinally to adjust the zero of the instrument,

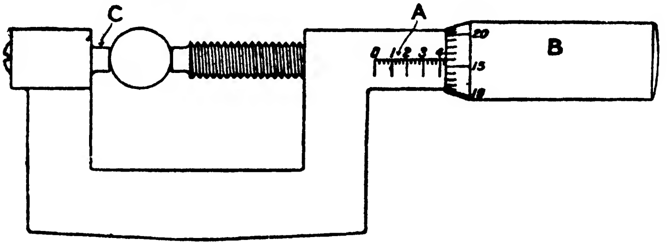


FIG. 12.—Micrometer screw gauge.

i.e. when the end of the screw is in contact with C the instrument should read zero; if it does not, then the zero error must be corrected or allowed for in all measurements made.

Verniers.—If a steel rule were graduated directly in, say, hundredths of an inch or hundredths of a centimetre, it would be practically impossible to read off a length with the naked eye to one of these small divisions. Indeed, an experienced man would get more accurate results by using a rule graduated in tenths only, and estimating the second place of decimals. The use of a vernier enables an unskilled man to read *definitely* to a greater degree of accuracy. A vernier which reads accurately to $\frac{1}{100}$ inch is shown in Fig. 13. The main scale is graduated in tenths of an inch; the vernier consists of a sliding piece, which may be moved along the main scale. This sliding piece is also graduated. In the particular vernier illustrated, ten divisions on the vernier scale are made equal to nine divisions on the main scale, hence the difference between the lengths of a main scale and a vernier division is $0{\cdot}1 - \frac{1}{10} \times 0{\cdot}9 = \frac{1}{100}$ inch. In using the instrument, the object to be measured

(shown shaded in Fig. 13) is placed as shown, and the sliding piece, or vernier, is brought into contact with it. The second decimal place of the length of the object is then read off by looking along the vernier scale to find where one division on it coincides with a division on the main scale, *i.e.* the fourth in Fig. 13.

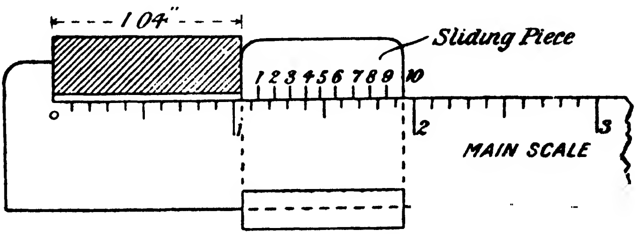


FIG. 13.—Simple vernier.

Hence, the length measured exceeds 1 inch by four times $\frac{1}{100}$ inch, and is 1·04 inches.

Laboratory Exercise.—Construct a scale about 6 inches long on a strip of drawing-paper, dividing it into tenths of an inch. Then make a vernier scale, as above, on another short strip of paper. Use the vernier so constructed to measure a length of, say,

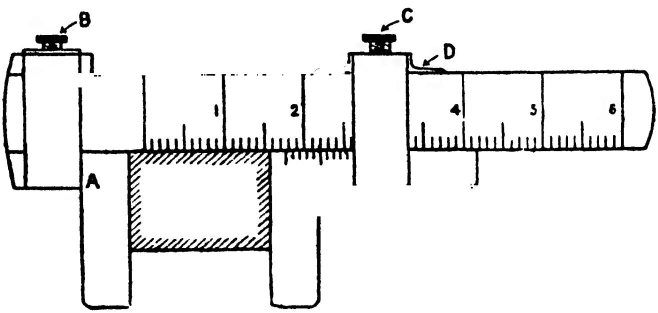


FIG. 14.—Vernier calipers.

10 cms., taken from a rule graduated in centimetres. From your result calculate the number of centimetres in a length of 1 inch.

Vernier Calipers.—Calipers for use in the workshop or laboratory are often fitted with verniers, such instruments being called vernier-calipers. The one shown in Fig. 14 is graduated on one side to read to $\frac{1}{100}$ inch, and on the other side to $\frac{1}{100}$ cm. In the

instrument shown it will be seen that the object being measured has a length of 1·75 inches. The stop A is clamped to the main scale by means of the screw B. The movable scale is brought into contact with the object as shown, and the length read off as already described. If a number of finished pieces of work of the some size are to be tested, the vernier scale is set to the desired position and clamped there by means of the screw C. To prevent the edge of the main scale from getting burred over by the continual setting in different positions, the end of the screw C bears on the flat spring D. This protects the edge of the scale and at the same time distributes the pressure over the length of the spring instead of concentrating it at the end of the screw C.

Laboratory Exercise.—To obtain practice in the use of the vernier calipers and the micrometer screw gauge, a series of balls (such as those used in ball bearings) of different diameters may be measured by both instruments and the results compared. Weigh each ball and calculate the weight of the material in pounds per cubic foot, using the diameters obtained from the more accurate instrument. Tabulate the results as follows :—

Diameter of ball as measured by		Volume from screw gauge measurement $\frac{1}{6}\pi d^3$.	Weight of ball in pounds.	Weight of one cubic foot of material.
Vernier calipers.	Screw gauge			

Spherometer.—The spherometer is an instrument used chiefly for the measurement of the radius of curvature of different surfaces. It consists of a horizontal circular plate H (Fig. 15) fitted with three vertical legs terminating in sharp points ABC. The points ABC form the corners of an equilateral triangle. The centre of the plate H is screwed and acts as a nut for a fine-threaded screw E which terminates in a sharp point D. The point D lies above the centre of the equilateral triangle ABC (Fig. 16). The other end of the screw E carries a circular bevelled disc F, the whole being rotated by turning the milled head K. To the plate H is attached a graduated scale G which just clears the bevelled edge of F. In the instrument illustrated the pitch of the screw E is $\frac{1}{32}$ inch, and each of the small divisions of G is $\frac{1}{32}$ inch. The bevelled disc F is divided into 100 equal parts, so that the instrument will read to

$\frac{1}{100}$ of $\frac{1}{32}$, *i.e.* $\frac{1}{3200}$ inch. A piece of plane plate glass is supplied with the instrument, forming a plane flat surface.

Method of Use.—Place the spherometer on the plane glass plate and turn the head K (Fig. 15) until all four points ABCD bear equally on the glass. This adjustment is decided by pushing one of the outer legs A, B, or C obliquely; if the instrument simply rotates, the centre point D is too prominent; if the instrument slides without rotation, the point D is too high. The adjustment being made, the readings of both scales G and on F are noted. Next, take

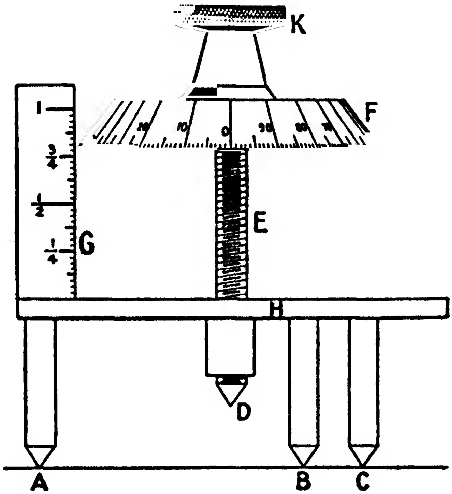


FIG. 15.—Spherometer.

a reading with the spherometer on the spherical surface whose radius of curvature is to be measured.

Let a = the difference of these two readings.

Now measure the distance between each pair of the three points A, B, and C. Call this distance l. Then if R denotes the radius of curvature of the surface

$$R = \frac{l^2}{6a} + \frac{a}{2}$$

$$= \frac{l^2}{6a} \text{ nearly when } a \text{ is very small.}$$

Proof.—Let EHGK (Fig. 17) be a great circle of the spherical

surface whose radius R is to be determined. Let the points of the three fixed legs of the spherometer touch the surface in the plane perpendicular to that of the paper shown at EFG.

Let HF $= a$, and FG in Fig. 17 or BD in Fig. 16 $= r$.

$$\text{Then HF} \times \text{FK} = \text{FG} \times \text{FG}$$

$$a(2\text{R} - a) = r^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let ABC (Fig. 16) be the equilateral triangle formed by the three fixed legs of the spherometer, the movable centre point being over the point D.

$$\text{Then BD} = \frac{\frac{\text{AB}}{2}}{\cos 30^\circ} = \frac{\frac{\text{AB}}{2}}{\frac{\sqrt{3}}{2}} = \frac{\text{AB}}{\sqrt{3}}$$

$$\therefore \; r = \frac{l}{\sqrt{3}}$$

Substituting this value of r in (1),

$$a(2\text{R} - a) = \frac{l^2}{3}, \text{ from which R} = \frac{l^2}{6a} + \frac{a}{2}$$

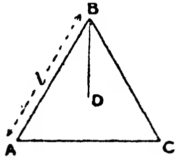


Fig. 16.

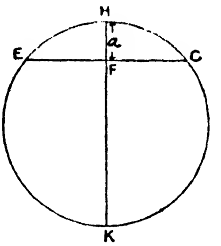


Fig. 17.

Surface Plate.—A surface plate consists of a rigid plate of cast iron, usually having three feet on the under side, so that its weight is always distributed on them in the same manner. By this means any tendency towards warping is minimized. The upper face is first planed, and then finished by scraping. If a standard surface plate is not available, three plates are made at the same time, and, after planing, are scraped, until on smearing a *little* "raddle," *i.e.* a mixture of oil and colouring matter, usually red lead, on any one plate, and then rubbing that plate on the other two, it is seen that the surfaces show contact over a large number of spots evenly distributed. All these spots will obviously be in the same

plane, and any one of the three plates may then be used as a standard plate for the reproduction of others.

Straight Edge.—A straight edge consists of a long strip of metal—usually mild steel—with one edge bevelled. This edge is made as straight and as true as possible, being planed and finished by scraping exactly in the same way as the surface plates above.

Useful Constants.

1 inch = 2·54 centimetres.
1 metre = 39·37 inches.
5280 feet = 1 mile.
6 feet = 1 fathom.
69 miles = 60 nautical miles.
1 knot = 1 nautical mile per hour = 6080 feet per hour.
1 chain = 66 feet.
80 chains = 1 mile.
1 square inch = 6·45 square centimetres.
1 cubic inch = 16·39 cubic centimetres.
1 square metre = 1550 square inches.
1 cubic metre = 61,025 cubic inches = 35·31 cubic feet = 1·308 cubic yards.
1 cubic foot of pure water at 62° F. weighs 62·3 lbs.
1 gallon of pure water at 62° F. weighs 10 lbs.
1 lb. avoirdupois = 7000 grains = 453·6 grams.
1 kilogram = 1000 grams = 2·204 lbs.

GREEK LETTERS USED IN THIS BOOK.

$\left.\begin{array}{l}\alpha \text{ Alpha} \\ \theta \text{ Theta} \\ \phi \text{ Phi}\end{array}\right\}$ for angles.
μ Mu . . . . for coefficient of friction.
ω Omega . . . for angular velocity.

CHAPTER I

FORCE

Force.—It is usual to define force as that which tends to produce or alter the motion of a body. To the beginner this probably conveys very little notion of the nature of force, but the idea of a push or pull is for the present a sufficient conception of force. Familiar examples of forces are to be found in the pull of a horse on a cart, of a locomotive on a train, of a weight on a rope which supports it, and the push or thrust of the muscles on a bicycle tyre pump.

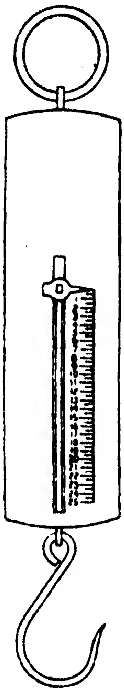


FIG. 18.—Spring balance.

Measurement of Force.—All bodies can exert vertically downwards the force of their own weight, and forces are usually measured in pounds. The weight of bodies results from the attraction of the earth upon them, and varies slightly in different parts of the world. We take the unit, a force of one pound, as equal to the weight of a standard pound mass in London; by standard pound mass we mean the quantity of matter in a certain piece of platinum carefully preserved by the Board of Trade.

We can often arrange to measure a force by balancing it against the weight of a body or by some effect which it produces.

Effects of a Force.—If a force acts on a body which is at rest it may set the body in motion; but if the force is balanced by another equal and opposite one, no motion results, but the body may be strained, that is, altered in shape or dimensions. Thus a pull may stretch a helical spring and the amount of the stretch may be used to measure the force. This is the principle of the spring balance (Fig. 18), which is sometimes used for measuring force. The spring balance may be graduated by hanging different known weights on it and registering the amount of stretch of the spring; its accuracy may also be tested by hanging standard weights on it.

Bodies such as strings, wires, chains, bars, etc., when transmitting a pulling force are said to be in tension, and the force is spoken of as a tension or a tensile force. Similarly, bars, pillars, columns, etc., which bear a thrust or push are said to be in compression, and the force is spoken of as a compressive force.

Forces represented by Lines: Vectors.—Before we can deal with the effect of a particular force on a body we must know—

(*a*) The magnitude of the force—usually stated in pounds.

(*b*) The point in the body at which the force is applied.

(*c*) The direction of the force.

These three things completely specify the force.

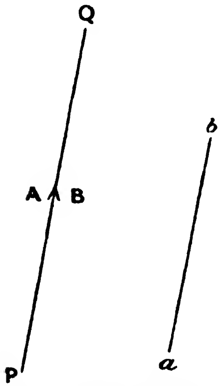


FIG. 19.—Force represented by a line.

Let P (Fig. 19) be the point of application of a force, and PQ through P the direction of the force as shown by the arrow. Put letters A and B on each side of the line PQ. Then we speak of the line PQ as the line of action of a force AB. We call all space to the left of PQ the space A, and all to the right of PQ the space B. Draw a line *ab* parallel to PQ, the line of action of the force AB and representing the magnitude of the force to some convenient scale. For example, if the force is 11 lbs. and we use a scale of 4 lbs. to the inch, the length of the line *ab* would be $\frac{11}{4} = 2\frac{3}{4}$ ins. The line *ab* is called a *vector*; it represents the force AB both in magnitude and direction.

Triangle of Forces.

Experiment.—Attach 3 strings to a small ring, and by a weight and 2 spring balances, or by weights and small freely moving guide pulleys as shown in Fig. 20, arrange that the strings are each pulled by a definite measured force and allow the little ring to come to rest. The 3 forces are then said to be balanced or in equilibrium. Three such forces acting on a body would not cause it to move; the body would remain at rest or *in equilibrium*. In the particular experiment shown in Fig. 20 the force AB was $2\frac{3}{4}$ lbs. hung over the pulley, BC was $1\frac{3}{4}$ lbs. hung over the pulley, and CA was 3 lbs. Mark the directions of the strings on a sheet of paper, which may be pinned to a vertical board supporting the pulleys, and note the pull in each string. Also put letters A, B, C in the spaces. Remove the paper, and from any point *a* draw a vector *ab* to represent the force AB, that is parallel to the line AB, and representing the pounds of force in AB to scale. From *b* draw the vector *bc* parallel to BC, and representing the

pounds of force in BC to the same scale as used before. If a vector from c be similarly drawn to represent in magnitude and direction the force CA it will be found that it ends at the starting-point a, that is, if ca be joined it will be found that the line ca is parallel to the line of the force CA, and its length represents the pounds of force in CA.

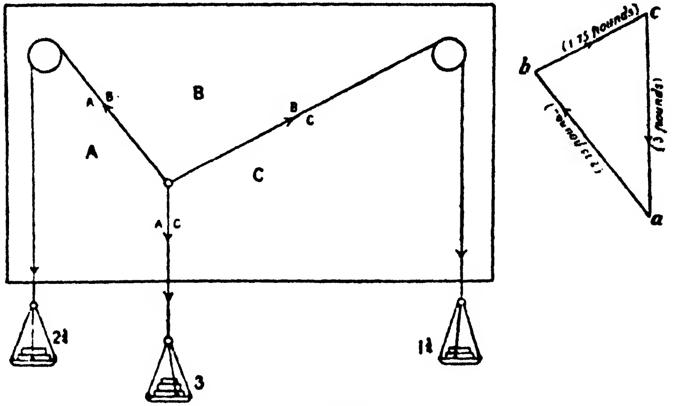


FIG. 20.—Triangle of forces.

This experiment, which may be repeated with any number of different weights, shows that if three forces acting at a point are balanced or in equilibrium, their three vectors may be put together to form a triangle with its sides parallel to the three forces and their lengths proportional to the forces.

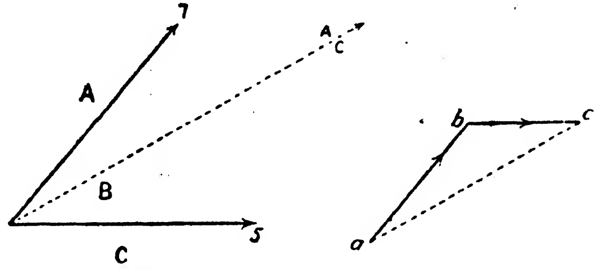
Addition of Forces by Vectors.—We may put the result of the foregoing experiment in another way. We may say the forces AB and BC together are balanced by the downward force CA of 3 lbs. Now evidently an upward force, or force in direction ac (not ca), would be balanced by the downward force ca, so that the joint effect of AB and BC is the same as a force represented completely by the vector ac (that is, ca reversed in direction). We call the vector ac the *resultant* or *vector sum* of ab and bc, and write—

$$ab + bc = ac$$

This is called geometric or vector addition, and is the rule for the addition of forces by drawing vectors. The vector ca (opposite to the resultant) is called the equilibrant of ab and bc, and represents the force CA which with AB and BC is necessary for equilibrium or no unbalanced force.

To find the *resultant* of two forces AB and BC of 7 lbs. and

5 lbs. respectively in the given directions (Fig. 21), we therefore choose a scale, say, $\frac{1}{4}$ inch to 1 lb.; then draw in vector ab $\frac{7}{4}$, that is $1\frac{3}{4}$ inches long, parallel to AB; and then from b draw a vector parallel to BC and $\frac{5}{4}$, that is, $1\frac{1}{4}$ inches long. Join ac, which is found to measure $2\frac{3}{4}$ inches, which represents $2\frac{3}{4} \times 4$ or 11 lbs. The



Space diagram. Force or vector diagram.

FIG. 21.—Resultant of two forces.

direction of AC may be shown dotted on the original figure by drawing a line parallel to ac. The diagram to the left of Fig. 21 shows the direction and positions of the forces, and is called the *space diagram*; the portion to the right is called the force or vector diagram, which shows the magnitudes and directions of the forces.

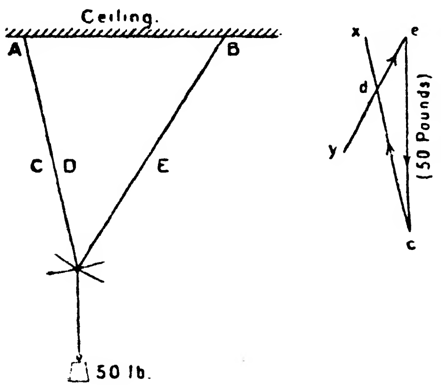


FIG. 22.—Tension in cords.

Example 1.—A weight of 50 lbs. is suspended from two points, A and B, on a horizontal ceiling, A being 10 feet from B. The suspension cord from A is 12 feet long, and that from B is 14 feet long. Find the pull or tension in the two cords.

We first draw to a convenient scale the space diagram. Choose a scale of, say, $\frac{1}{4}$ inch to 1 foot and make AB (Fig. 22) $\frac{10}{4}$, *i.e.* $2\frac{1}{2}$ inches long. With centre A and radius $\frac{12}{4}$ = 3 inches draw an arc, and with centre B and radius $\frac{14}{4}$ or $3\frac{1}{2}$ inches draw another arc. The intersection of these two arcs gives the point of application of the weight of 50 lbs. Placing the letters CDE in the spaces as shown, we can proceed to draw the force or vector diagram.

Consider the point of application of the weight of 50 lbs. At this point there are *three* forces acting, namely, the weight of 50 lbs. vertically downwards (EC), the force in the cord CD, and the force in the cord DE. To draw the force diagram we proceed as follows :—

Choose a convenient scale, say $\frac{1}{20}$ inch to 1 lb., and draw in the vector *ec* $\frac{50}{20} = 2\frac{1}{2}$ inches long, parallel to EC. Then from *c* draw vector *cx* parallel to cord CD, and from *e* draw vector *ey* parallel to cord ED, intersecting *cx* in point *d*. Then the vector *cd* represents the force in the cord CD, and the vector *de* represents the force in the cord DE. Measure the lengths of *cd* and *de*. It will be found that *cd* measures $1\frac{7}{8}$ inches, which represents $1\frac{7}{8} \times 20 = 37\frac{1}{2}$ lbs., and that *de* measures $\frac{3}{4}$ inch, which represents $\frac{3}{4} \times 20 = 15$ lbs.

The directions of the forces at the junction of the cord should be carefully noted. The force in CD given by *cd* acts from *c* to *d*, *i.e.* it is pulling at the junction ; the force in DE given by *de* acts from *d* to *e*, that is, it is also pulling at the junction, and both forces are tensile forces.

Example 2.—In a simple jib crane the length of the tie rod is 10 feet, the length of the jib 14 feet, and the crane post 5 feet. Find the forces in the jib and tie rod when a weight of 8 tons is suspended from the crane head.

Choose a convenient scale and set out the space diagram as in Fig. 23, lettering the spaces A, B and C. Now draw the vector diagram

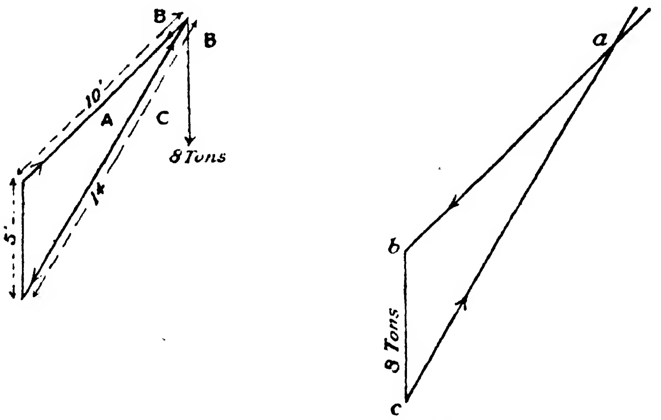


FIG. 23.—Forces at head of jib crane

of forces acting on the hinge at the crane head to a scale say of $\frac{1}{5}$ inch to 1 ton. First draw vector *bc* $\frac{8}{5}$, *i.e.* 1·6 inches long, parallel to BC, from *b* draw a vector parallel to BA, and from *c* draw another vector parallel to CA. Call the intersection of these two vectors the point *a*. Measure the length of *ab* ; it will be found to be 3·2 inches, which represents

$3 \cdot 2 \times 5 = 16$ tons. Measure the length of *ca*: it will be found t
4·48 inches, which represents $4 \cdot 48 \times 5 = 22 \cdot 4$ tons. Hence the forc
in the tie rod is 16 tons, and the force *ca* in the jib is 22·4 tons.

To find whether these Forces are Tensile or Con sive.—The directions of the forces at the crane head are foun the force diagram as in Example 1. Starting with the downwar *bc* and following the forces continuously round the triangle *b* *c* to *a*, and *a* to *b*, it will be seen that in the tie rod the force a from *a* to *b*, that is, it is *pulling* at the crane head, while in the the force acts from *c* to *a*, that is, it is thrusting or pushing at crane head. Hence the force in the tie rod is *tensile* and equal 16 tons, and the force in the jib is a thrust or is *compressive* ar equal to 22·4 tons.

Polygon of Forces.—The resultant, or the equilibrant several forces all acting at one point may be found by adding vec just as in the case of two forces.

To find the resultant of four forces, AB, BC, CD, and shown in position and magnitude in Fig. 24, first draw a vecto

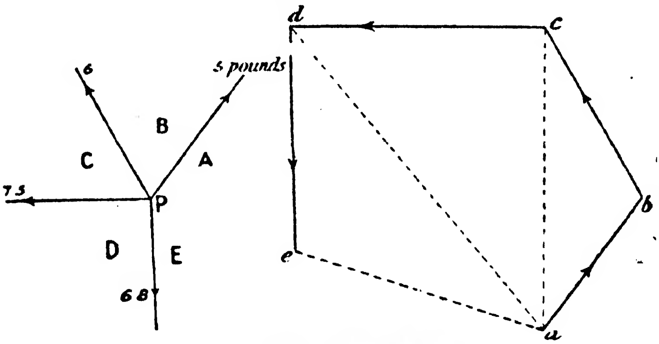


FIG. 24.—Resultant of several forces.

parallel to the line of action AB, choosing any convenient sc say $\frac{1}{2}$ inch to 1 lb., in which case *ab* would be $2\frac{1}{2}$ inches lo From *b* draw the vector *bc* 3 inches long (representing 6 lb parallel to the force BC; then, as before, the vector *ac* repres a force through P, which is the resultant of AB and CB. To resultant add the force CD by drawing a vector *cd* from *c* par to the line CD and 3·75 inches long (representing 7·5 lbs.). T *ad* represents the resultant of AB, CB, and CD, and is the ve sum of—

$$ab + bc + cd$$

Finally, to this resultant *ad* add the force DE by drawing a vector *de* from *d* parallel to the line DE and 3·4 inches long (representing 6·8 lbs.). Then *ae* represents the resultant of the four forces AB, CB, CD, and DE, the addition of vectors being—

$$ab + bc + cd + de = ae$$

It is not necessary to draw the dotted lines *ac* and *ad*, to draw the *vector polygon* of forces *abcde*. We simply draw in turn the vectors representing each force in magnitude and direction, beginning each vector at the end of the previous one, namely, draw *ab*, *bc*, *cd*, and *de*. This is shown in Fig. 25.

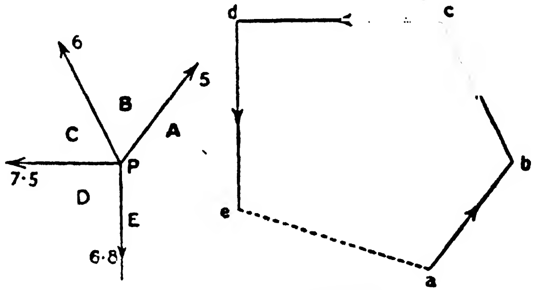


FIG. 25.—Polygon of forces.

The direction of the resultant is from *a* to *e*, and it acts through P. An equal and opposite force through P (in the direction *e* to *a*) would be the equilibrant. If we draw the polygon for the *five* forces AB, BC, CD, DE, and EA instead of the previous *four* forces only, the last side of the polygon (*ea*) ends at *a*. The resultant of these five forces is then nil, or, in other words, the five forces are in equilibrium. In this case the polygon of forces, drawn with each side representing a corresponding force, is a closed figure. Hence, when the vector polygon of forces for several forces acting at the same point forms a *closed* figure (starting and ending at the same point), the forces represented are in equilibrium, or balanced, and if they alone act on a body they keep it at rest.

Also, when any number of forces, acting through one point of a body, keep it at rest, the vector polygon drawn in this way for the various forces always forms a closed figure, ending at the starting-point.

Experiment.—The polygon of forces may be verified by direct experiment by the apparatus shown in Fig. 26. Arrange that the five

strings are each pulled by a definite measured force, and allow the little ring to come to rest. The five forces are then in equilibrium. In the particular experiment shown by Fig. 26 the force AB was $2\frac{1}{8}$ lbs., BC was $1\frac{7}{16}$ lbs., CD was $1\frac{9}{16}$ lbs., DE was $\frac{5}{8}$ lb., and EA $\frac{15}{16}$ lb.

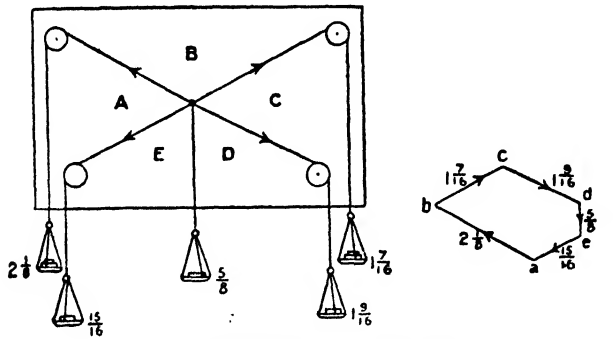


FIG. 26.—Experiment on polygon of forces.

Mark the directions of the five strings on a sheet of paper in the same way as in the experiment on the triangle of forces previously described. Now choose a suitable scale and draw the vector polygon as described above. Starting from point *a*, we draw *ab*, *bc*, *cd*, *de*, and finally *ea* which bring us back to the starting-point *a*.

If in any particular experiment it is found that the force or vector polygon is not quite closed, the error is due to the fact that the pulleys do not work freely enough.

This experiment therefore shows that if five forces acting at a point are in equilibrium, their five vectors may be put together to form a polygon, with its sides parallel to the forces, and their lengths proportional to the forces.

Example 1.—Five bars of a steel roof-frame, all in one plane, meet at a point; one is a horizontal tie-bar carrying a tension of 40 tons; the next is also a tie-bar inclined 60° to the horizontal tie-bar and sustaining a pull of 30 tons; the next (in continuous order) is vertical and runs upward from the joint and carries a thrust of 5 tons; and the remaining two in the same order radiate at angles of 135° and 210° to the first bar. Find the forces in the last two bars and state whether they are in tension or compression, that is, whether they pull or push at the common joint.

First draw the space diagram as shown in Fig. 27. Letter the spaces A, B, C, D and E. Then the first horizontal tie-bar is AB. AB and BC are in tension, and therefore are pulling at the common joint. CD is in compression, and therefore pushes at the common

joint. Place arrows on AB, BC, and CD, showing the directions of these forces at the common joint as shown. We are required to find the magnitude and direction of the forces in DE and EA.

Choose a convenient scale, say, $\frac{1}{10}$ inch to 1 ton, and draw the vector diagram as follows:—

First draw the vector *ab* parallel to AB, $\frac{40}{10}$ = 4 inches long. From *b* draw vector *bc* 3 inches long (representing 30 tons) parallel to BC. From *c* draw *cd* $\frac{1}{2}$ inch long (representing 5 tons) parallel to CD and in the same direction, viz. *downwards*. Through *d* draw a vector *df*

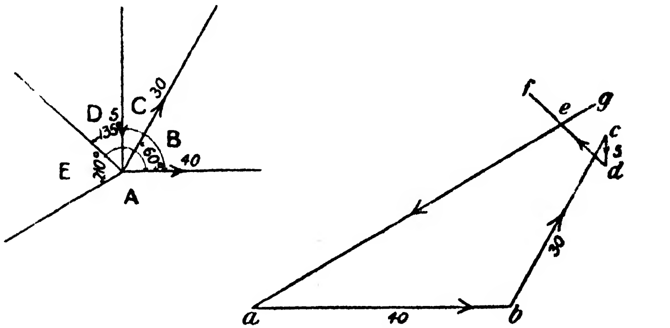


FIG. 27.

parallel to DE, and through the starting-point *a* draw a vector *ag* parallel to EA. Call the point of intersection of the two vectors *df* and *ag*, the point *e*. Now the five forces acting at the common joint in the space diagram are in equilibrium, therefore the force or vector polygon must close. Hence, the vector diagram is *abcde*, and *ea*, which measures 5·56 inches, which represents 5·56 × 10 = 55·6 tons, represents the force in the member EA; whilst *de*, which measures 0·96 inch, corresponding to 0·96 × 10 = 9·6 tons, represents the force in the member DE.

To find whether these are tensile or compressive forces, following the arrows round the force diagram in order, we see that the force *de* acts from *d* to *e*, and is therefore *pulling* at the common joint; the force *ea* acts from *e* to *a*, and is therefore *pulling* at the common joint also. Hence, the member DE is in *tension* with a force of 9·6 tons, and the member EA is in *tension* to the extent of 55·6 tons.

EXAMPLES I.

1. Two forces of 10 and 15 lbs. respectively act at a point. If the angle between the lines of action of the forces be 30°, find their resultant.

2. A push of 18 lbs. acts at a point, and inclined to it at an angle of 135° a pull of 35 lbs. acts on the same point. Find the resultant of the two forces.

3. If the forces in problem 2 are inclined at an angle of 135°, find the magnitude and direction of their resultant.

4. A weight of 25 lbs. is suspended from two points A and B on a horizontal ceiling, A being 8 feet from B. The suspension cord from A is 6 feet long, and that from B is 4 feet long. Find the pull or tension in the cords.

5. A machine 3 tons in weight is supported by two chains attached to the same point on the machine; one of these chains goes to an eye-bolt in a wall and is inclined 30° to the horizontal; the other goes to a hook in the ceiling and is inclined 45° to the horizontal. Find the tensions in the chains.

6. In a simple jib crane, the vertical crane post is 8 feet high, the jib is 13 feet long, and the tie is 7 feet long. Find the forces on the jib and tie-rod when a weight of $2\frac{1}{2}$ tons is supported at the crane head.

7. The following four forces act at a point:—a force of 16 lbs. in a direction due East, 20 lbs. due North, 30 lbs. in a direction North-West, and 12 lbs. in a direction 30° South of West. Find the magnitude and direction of their resultant.

8. Four forces acting at a point are in equilibrium, the magnitude and direction of three of them are: 8 lbs. acting due South, 15 lbs. acting in a direction North-East, and 18 lbs. acting in a direction 30° East of South. Find the magnitude and direction of the fourth force.

9. Five forces in equilibrium act at a point, the magnitude and direction of three of them being: One force of 90 lbs. in a direction due West, one force of 50 lbs. in a direction South-East, and one force of 35 lbs. in direction 30° West of South. The remaining two forces act in directions North-West, and 60° East of North respectively. What are their magnitudes?

10. At a certain joint in a roof truss five bars meet; one is vertical and carries a thrust of 15 tons; the next carries a thrust of 18 tons and runs upwards from the joint at an angle of 45° to the vertical; the next (in continuous order) carries a tension of 35 tons and runs downwards from the joint at an angle of 120° to the first bar; and the remaining two in the same order radiate at angles of 240° and 315° to the first bar. Find the forces in the last two bars, and state whether they are in tension or compression.

CHAPTER II

MOMENTS

Moment of a Force.—The effect of a force on a body does not only depend upon its magnitude and direction; the position or point of application of the force to the body is also important. For example, to move a hinged body such as a door, a much smaller force will suffice if it is applied at a considerable distance from the hinge than if applied close to the hinge. The effect of a push on a door will be greater the further it is from the hinge: the turning effect about the hinge is called the *moment* of the force about the hinge, and this moment depends entirely on the amount of the force and its perpendicular distance from the hinge.

Suppose AB (Fig. 28) is the line of action of a force of 8 lbs., what is its *moment* about P? Draw a line PN from P on to AB such that PN is at right angles to AB. Suppose PN measures 3 feet, then—

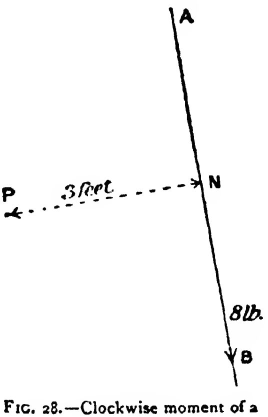


FIG. 28.—Clockwise moment of a force.

the moment of the force *about P*

$$= 3 \times 8 = 24 \text{ pound-feet}$$

To find the moment, in pound-feet, of a force about any point, multiply the amount of the force in pounds by its perpendicular distance from the point in feet. If we use inch units of length with pound units of force we obtain the moment in pound-inches; or, if F is the amount of a force in pounds and r feet its perpendicular distance from any point O, its moment about O is—

$$F \times r \text{ pound-feet}$$

Two Kinds of Moments.—The turning effect or moment of

the force of 8 lbs. about P in Fig. 28 is in the direction of the hand of a clock thus—

This is called a *clockwise moment* of 24 pound-feet.

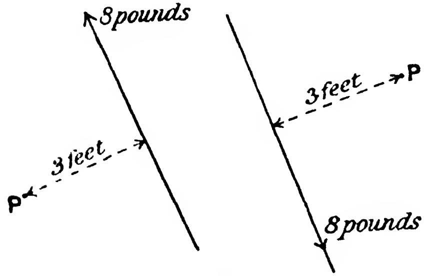


FIG. 29.—Contra-clockwise moments.

Fig. 29 shows t cases where a force 8 lbs. 3 feet from P h a turning effect in t contrary direction to that of the hands of a clock. thus—

This is therefore called a *contra-clockwise moment* of 24 pound feet.

Effects of Opposite Moments.—In Fig. 30 two forces ar shown having opposing moments about P. There is a—

Clockwise moment of 7 lbs. × 3 feet = 21 pound-feet
Contra-clockwise moment of 4 lbs. × 1·5 feet = 6 pound-feet

Of the 21 pound-feet clockwise moment 6 are neutralized or balanced by 6 pound-feet contra-clockwise moment, leaving 15 pound-feet effective clockwise moment about P as the net result of the two moments. We could arrive a the same result b another method.

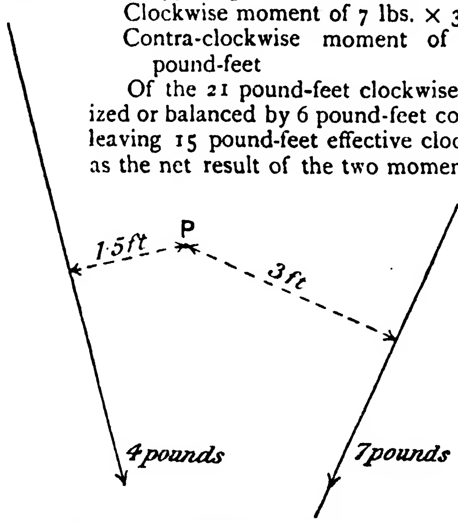


FIG. 30.—Resultant of two moments.

Find the resulta of the forces of 7 a 4 lbs. of Fig. 30 vectors as in Chapt I. That is, produ their lines of actio. to meet at *a*, Fig. 31; set off *ab* 7 units long in the direction of the 7 lbs. force, and *bc* parallel to the 4 lbs. force and 4 units long, then *ac* represents the resultant to scale; it measures 8·88 pounds. Produce

and measure its perpendicular distance PM from P; it scales off as 1·69 feet.

The resultant moment is 8·88 lbs. × 1·69 feet = 15 pound-feet

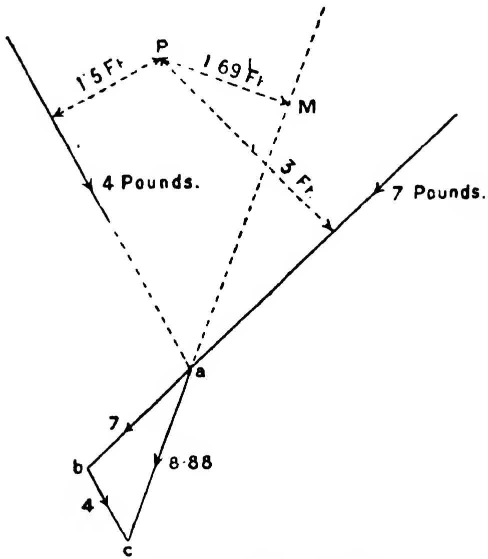


FIG. 31.—Moment of resultant of two forces.

...ckwise as before. Thus we find that the *moment of the resultant ... the two forces is equal to the resultant moment of the two forces.*

Resultant Moment on a Body at Rest.

Experiment.—Take a very thin piece of sheet metal and arrange so ... it hangs vertically under the action of a number of forces of known ...gnitude in its plane, as shown in Fig. 32. In the figure are shown ... forces applied by means of thin strings passing over freely moving ...eys, and a fifth force produced by hanging a weight by a string directly. Allow the sheet to come to rest under the action of these forces. In a particular experiment the forces were (Fig. 32) $6\frac{3}{4}$ pounds, ...pounds, $3\frac{3}{4}$ pounds, $2\frac{3}{4}$ pounds, and $8\frac{1}{2}$ pounds respectively. Draw on ...e sheet the lines of action of the forces in the same way as in the ...xperiment on the Polygon of Forces shown in Fig. 26. Now choose any point O and measure the perpendicular distances from that point on to the lines of action of the five forces; these distances are found to be 1·3 inches, 1·57 inches, 1·86 inches, 1·5 inches, and 3·26 inches respectively. Taking the moments of the forces about O, we find—

Clockwise moments are—

$(6·75 \times 1·3) + (5 \times 1·57) + (3·75 \times 1·86) + (2·75 \times 1·5)$
$= 8·77 + 7·85 + 6·97 + 4·13 = 27·7$ pound-inches

Contra-clockwise moments are—

$8·5 \times 3·26 = 27·7$ pound-inches

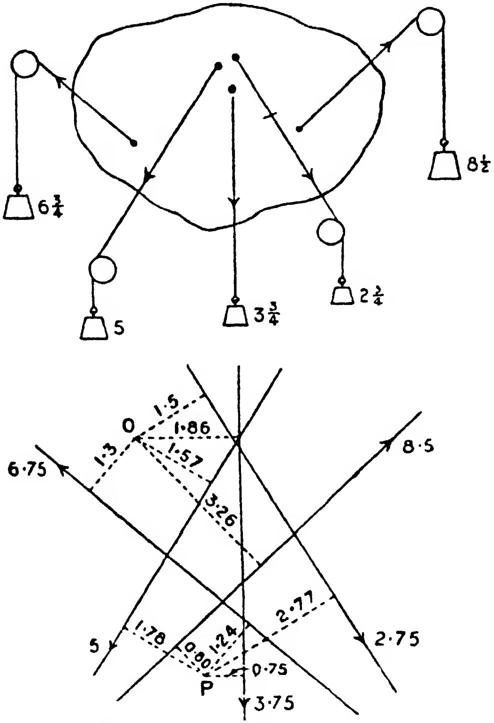


FIG. 32.—Experiment showing equality of opposite moments.

This experiment shows, therefore, that the total clockwise moment about the point O is equal to the total contra-clockwise moment about O.

If another point, P, be taken, and the perpendicular distanc measured in the same way, we find—

Clockwise moments are—

$(3·75 \times 0·75) + (2·75 \times 2·77) + (8·5 \times 0·80)$
$= 2·81 + 7·62 + 6·80 = 17·23$ pound-inch

Contra-clockwise moments are—

$(6{\cdot}75 \times 1{\cdot}24) + (5 \times 1{\cdot}78) = 8{\cdot}36 + 8{\cdot}90 = 17{\cdot}26$ pound-inches

Similarly, it will be found that about *any* point in the plane of the ·ces the total clockwise moment is equal to the total contra-clock-se moment; or, in other words, if we write clockwise moments ɔsitive and contra-clockwise moments negative, the sum of all the oments of the forces about any point in their plane is equal to ·ro, or Σ(moments) = o, the Greek letter Σ standing for the words "summation of."

Principle of Moments.—The above experiment illustrates the *principle of moments* for forces in one plane, namely: If a body is at rest under the action of several forces in the same plane, the total moment (say clockwise) of all the forces about every point in the plane is zero; that is, the contra-clockwise moments are equal to the clockwise moments.

This principle is of great use in finding the amount and direction of some unknown force on a body which is at rest.

Example.—A light horizontal rod AB, 10 feet long (of negligible weight), is pivoted at A (Fig. 33), and a weight of 7 lbs. is hung at

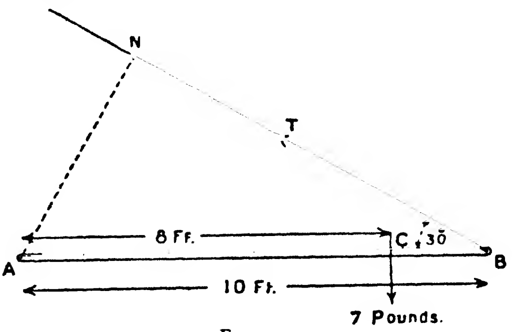


FIG. 33.

C 8 feet from A. The end B is supported by a cord inclined 30° to the horizontal and fastened to a point above A. Find the tension in the cord.

The only forces acting on the rod are (1) the weight at C; (2) the pull of the cord at B; (3) the force exerted by the hinge at A. We do not know this last force, but as its distance from the point A is zero its moment about A is zero. Then the clockwise moment of the weight at C about A must balance the contra-clockwise moment of the pull or tension T of the string at B about A.

$$T \times AN = 7 \text{ lbs.} \times 8 \text{ feet} = 56 \text{ pound-feet.}$$

The distance AN may be found by drawing the rod and cord scale; it is 5 feet, or it may be found thus—

$$\frac{AN}{AB} = \frac{AN}{10} = \sin 30^\circ \text{ (see p. 5)} = \tfrac{1}{2}$$
$$\therefore AN = 10 \times \tfrac{1}{2} = 5 \text{ feet}$$
$$\text{then } T \times 5 = 56 \text{ pound-feet}$$
$$T = \tfrac{56}{5} = 11{\cdot}2 \text{ pounds}$$

which is the required pull in the cord.

Parallel Forces.

Experiment.—We can verify the fact that the principle of mon holds good for parallel forces (as distinct from intersecting force: follows. The apparatus shown in Fig. 34 consists of a graduated

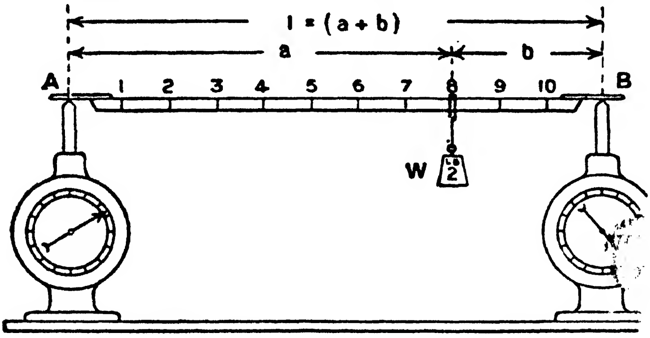


FIG. 34.—Moments of parallel forces.

supported at A and B on two spring balances. At any point of rod a known weight may be hung. The spring balances are adju to read zero with the weight of the rod only on them ; if not, ther initial readings of the balances must be subtracted from subse ones.

In a particular experiment a weight of 2 lbs. was hung at C 8 of length from A, and the readings of the spring balances were 0·54 at A and 1·455 lbs. at B. A and B are 11 units of length apart.

The moment of the upward supporting force at B about A is—

$$1{\cdot}455 \times 11 = 16{\cdot}0 \text{ contra-clockwise}$$

The moment of the weight 2 lbs. about A is—

$$2 \times 8 = 16{\cdot}0 \text{ clockwise.}$$

Hence we see that the total clockwise moment about A is eq the total contra-clockwise moment about A.

Similarly, if we take moments about B we find—

The moment of the upward supporting force at A about B is

$$0{\cdot}545 \times 11 = 6{\cdot}0 \text{ clockwise}$$

The moment of the weight 2 lbs. about B is

$$2 \times 3 = 6 \text{ contra-clockwise.}$$

Further trial with one or more weights hung at different points on e rod will show that the moment of the upward supporting force or *action* at B about A is equal to the moment of the weight or weights oout A, or, with the letters shown on Fig. 34—

$$l \times \text{supporting force at B} = W \times a$$

and similarly $l \times$ supporting force at A $= W \times b$

, if the *reaction* or supporting force at A is called R_A and that at B lled R_B—

$$l \times R_B = W \times a, \text{ or } R_B = W \times \frac{a}{l}$$

$$l \times R_A = W \times b, \text{ or } R_A = W \times \frac{b}{l}$$

The supporting forces are inversely proportional to their distance from the weight, and their sum is equal to the weight. The supporting forces act upwards, and, of course, the downward pressure on the supports at A and B is just equal in amount to the upward supporting force at the same point. Notice that the downward force W is the *equilibrant* of the upward forces R_A and R_B; the *resultant* R_A and R_B upward would be an upward force equal to W acting he same point C.

e may further verify the principle of moments for parallel , by the arrangement shown in Fig. 35. A rod AB supported

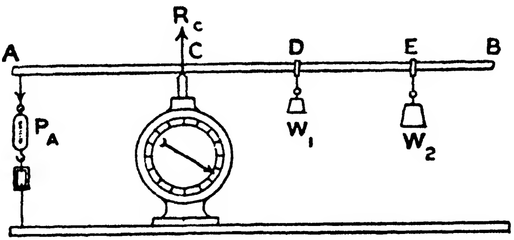


FIG. 35.

compression spring balance at C, loaded with weights to the nt of C, and the left end A held down by a spring balance so as keep AB horizontal.

For moments about A we should find—

$$R_C \times AC = W_1 \times AD + W_2 \times AE$$

for moments about C

$$P_A \times AC = W_1 \times CD + W_2 \times CE$$

ce, R_C may be called the *equilibrant* of P_A, W_1 and W_2.

Rules for Parallel Forces.—We can, from the above, state the rules for the resultant R (and equal and opposite equilibrant) of two parallel forces P and Q at any distance apart as follows. If P and Q are alike in direction (Fig. 35A),

$$R = P + Q$$

and if AB is any line perpendicular to the forces, R acts through C on AB so that

$$R \times AC = Q \times AB$$

or

$$AC = \frac{Q}{R} \times AB = \frac{Q}{P+Q} \times AB$$

and similarly—

$$BC = \frac{P}{R} \cdot AB = \frac{P}{P+Q} \times AB$$

and

$$\frac{AC}{BC} = \frac{Q}{P}$$

which is expressed by saying that the distances of C from the ends

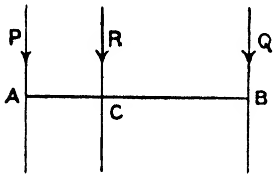


FIG. 35A.—Resultant of parallel forces.

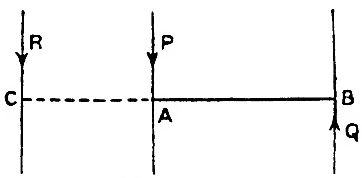


FIG. 35B.—Resultant of unlike parallel forces.

of AB are inversely proportional to the forces at the ends. The line of action of R is called the centre of the parallel forces P and Q.

If P and Q act in opposite directions to each other, and P is the greater force (Fig. 35B),

$$R = P - Q$$

and

$$R \times AC = Q \times AB$$

or

$$AC = \frac{Q}{R} \times AB = \frac{Q}{P-Q} \times AB$$

and similarly—

$$BC = \frac{P}{R} \times AB = \frac{P}{P-Q} \times AB$$

and

$$\frac{AC}{BC} = \frac{Q}{P}$$

That is, as before, the distances of C from the ends of AB are inversely proportional to the forces at the ends, but C is outside AB beyond the greater force. The line of action of R is again the centre of the parallel forces P and Q.

Couples.—When P and Q are equal and opposite $R = 0$, but the moment of the forces is equal to $P \times AB$ about *every* point in the plane of the two forces. Two such forces are said to form a *couple*, and AB is called the arm of the couple. A couple has then a definite moment, but no resultant force in any direction; it has no centre. If Q is nearly equal to P, AC (Fig. 35B) is very great, but if $Q = P$ we may say that AC is infinitely great.

Moment of the Weight of a Body.—In reckoning the moments of all the forces acting on a body we ought to include the moment of the weight of the body, unless the weight is very small in comparison with the forces, as was the case in the experiment described on p. 28. The resultant force of the weight acts through a point in the body which we call its *centre of gravity*; we defer the general consideration of the centre of gravity until Chap. XIV., but the reader will see that if the weight acts *through* the centre of gravity, the moment of the weight of a body about its centre of gravity will be zero. The centre of gravity of a straight uniform rod is at the centre of its length, and the weight of the rod may be taken as a vertical force acting there, which is sufficient for our present purpose.

Reactions or Supporting Forces of a Horizontal Beam.

Example.—A uniform horizontal beam 10 feet long (Fig. 36), rests on two supports A and B 7 feet apart, one, A, being at one end of the

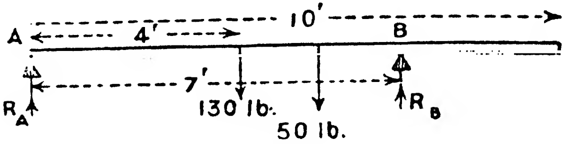


FIG. 36.

beam. The beam weighs 50 lbs., and a weight of 130 lbs. hangs from a point 4 feet from the supported end. Find the supporting forces at A and B.

Taking the weight as 50 lbs. weight at the middle of the beam, *i.e.* 5 feet from A, by moments about A we have—

$$R_B \times 7 \text{ (contra-clockwise)} = 130 \times 4 + 50 \times 5$$
$$= 770 \text{ pound-feet (clockwise)}$$
$$\therefore R_B = \tfrac{770}{7} = 110 \text{ lbs.}$$
$$\text{Total upward force } R_B + R_A = 130 + 50 = 180 \text{ lbs.}$$
$$\therefore R_A = 180 - 110 = 70 \text{ lbs.}$$

Checking this by taking moments about B—

$$R_A \times 7 \text{ (clockwise)} = 50 \times 2 + 130 \times 3$$
$$= 490 \text{ pound-feet (contra-clockwise)}$$
$$R_A = \tfrac{490}{7} = 70 \text{ lbs.}$$

Note on the application of the Principle of Moments.—In finding two or more unknown forces acting on a body at rest by the principle of moments, we may reckon the equal and opposite clockwise and contra-clockwise moments about *any* point in the plane of the forces, but we shall generally simplify a problem by taking the moments about a point in the line of action of one unknown force. The moment of this force about such a point is zero, and so we get a simpler problem.

Levers.—A lever is simply a rigid rod or bar capable of turning about a fixed point which is called the fulcrum; the lever may be straight or curved, and the forces exerted on or by the lever may be parallel or may be inclined to one another. The *principle of the lever* is practically the principle of moments.

Figure 37 shows three arrangements of a straight horizontal lever which can be turned by a force P (called the *effort*) about a

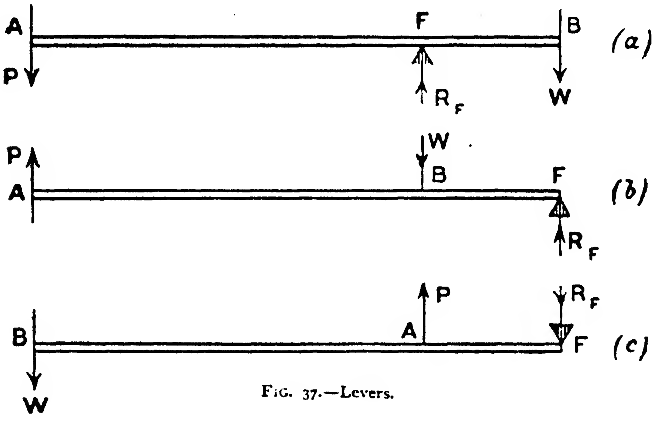


FIG. 37.—Levers.

fixed fulcrum F, against a resistance or weight W. In each case, equating clockwise and contra-clockwise moments about F,

$$P \times AF = W \times BF$$

$$\text{or } P = \frac{BF}{AF} \cdot W$$

Evidently, if F can be taken very near to W, the ratio $\frac{BF}{AF}$ can be made very small, that is, a small force, P, can thus be used to overcome a great resistance, W; (*a*) and (*b*) Fig. 37 show such arrangements, while (*c*) shows a large effort, P, overcoming

only a small resistance, W; this arrangement gives W a large movement for a small movement of the effort P. Note that the above forces are those exerted *on* the lever; the force exerted *by* the lever on the resistance is of magnitude W, and in the opposite direction to those shown in Fig. 37. The perpendicular distances (AF and BF) of the forces from the fulcrum F are called the ***arms*** of the lever, and sometimes the ratio of the arm of P (the effort) to the arm of W (the resistance) is called the *leverage*. The value of P, calculated above, is for equilibrium; but this value is sufficient to *balance* W, that is, to bring the lever just to the point of motion. The slightest excess over this value will be sufficient to cause motion.

Reaction at the Fulcrum.—At (*a*), Fig. 37, evidently the supporting force $R_F = P + W$, or, by taking moments about A,

$$W \times AB = R_F \times AF$$

$$\text{or } R_F = \frac{AB}{AF} \times W = \frac{AF + BF}{AF} \cdot W = \left(1 + \frac{BF}{AF}\right)W = W + P$$

Similarly at (*b*), Fig. 37—

$$W = P + R_F \text{ or } R_F = W - P$$

and at (*c*), Fig. 37—

$$R_F + W = P, \text{ or } R_F = P - W$$

The values of R_F shown are the pressures exerted *by* the fulcrum *on* the lever; the pressure on the fulcrum is equal and opposite.

Example 1.—A crowbar 8 feet long rests on a fixed fulcrum 3 inches from one end. What weight at the near end of the bar may be lifted by an effort of 100 lbs. at the other end?

The arm of the 100-lb. force is $(8 \times 12) - 3 = 93$ inches.

The arm of the weight W is 3 inches. Taking moments about the fulcrum—

$$W \times 3 = 100 \times 93 \text{ pound-inches}$$
$$W = \frac{9300}{3} = 3100 \text{ lbs.}$$

Example 2.—If the fulcrum is at the end of the bar, what weight 3 inches from that end would the 100 lbs. at the other end move?

Taking moments about the fulcrum—

$$W \times 3 = 100 \times 96 \text{ pound-inches}$$
$$W = \frac{9600}{3} = 3200 \text{ lbs.}$$

Example 3.—Forces of 10 lbs. and 35 lbs. are to be in balance at the end of a lever 10 feet long. How far must the fulcrum be placed from the end at which the 10 lbs. hang?

Let x inches be the distance required. Then the arm of the 35-lb. force (Fig. 38) is $120 - x$ inches.

Taking moments about the fulcrum—

$$10 \times x = 35(120 - x) = 4200 - 35x$$
$$45x = 4200$$
$$x = \frac{4200}{45} = 93{\cdot}33 \text{ inches}$$

and the distance from the other end $= 120 - 93{\cdot}33 = 26{\cdot}67$ inches.

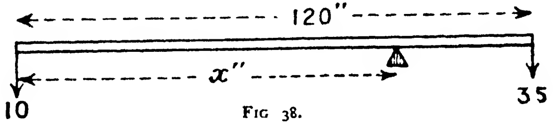


FIG 38.

Cranked Levers.—If a lever is of some such shape as shown in Fig. 39 (a bell-crank lever), the principle of moments is still applicable. To find the pull at B in the direction shown at Q, due

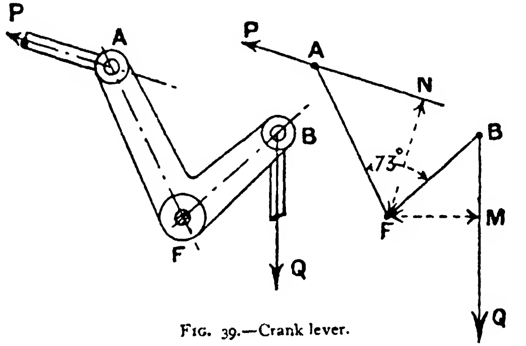


FIG. 39.—Crank lever.

to a force P at A, it is only necessary to find, by drawing a skeleton or centre-line diagram to scale, the perpendicular distances FN and FM of the force lines from the centre of the fulcrum, or pivot, at F; then, by equating moments about F—

$$Q \times FM = P \times FN$$

or

$$Q = P \times \frac{FN}{FM}$$

Example.—In the lever shown in Fig. 39, the length of AF is 2·6 inches, FB is 1·94 inches, and the angle AFB is 73°. The force P is 10 lbs. and its perpendicular distance FN is 1·92 inches. If the perpendicular distance of Q from F is 1·43 inches, find the pull Q acting at B.

$$Q = P \times \frac{FN}{FM} = 10 \times \frac{1{\cdot}92}{1{\cdot}43} = 13{\cdot}4 \text{ lbs.}$$

If a great leverage is required for any purpose, that is, if a large resistance is to be overcome by a small effort, it will be seen by reference to Fig. 37 (*a*) or (*b*) that the arm of the effort P must be very much greater than the arm of the resistance W. To obtain a great leverage without the use of excessively long pieces of material, compound levers are used. Fig. 40 shows a simple case of a com-

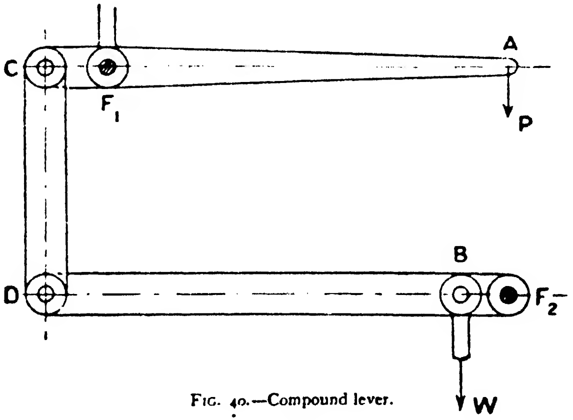


FIG. 40.—Compound lever.

pound lever made up of three straight pieces. The lever on which the effort P is applied is pivoted about the fulcrum F_1. One end of the second piece, or link, is attached by means of a pin-joint, C, to the first lever, as shown, the other end being attached to the third piece by a pin-joint at D. The third piece is a lever pivoted about the fulcrum F_2, and the resistance W is applied at B. The leverage in the case illustrated is—

$$\frac{AF_1}{CF_1} \times \frac{DF_2}{BF_2}$$

so that if CF_1 and BF_2 are made very small, the leverage is considerable, an effort P pounds being able to overcome a resistance W of—

$$P \times \frac{AF_1}{CF_1} \times \frac{DF_2}{BF_2} \text{ lbs.}$$

Instead of having a number of jointed levers, bell-cranked levers may be used. Fig. 41 shows a compound lever consisting of three pieces: a straight lever AC pivoted about the fixed fulcrum F_1 and jointed to the link CD. A cranked lever DF_2E is jointed to the link at D, and turns about the *fixed* fulcrum F_2; the resistance

to be overcome is applied at E. In the position illustrated the angles F_1CD, CDF_2, DF_2E, and F_2ES are all right angles, and the leverage is—

$$\frac{AF_1}{CF_1} \times \frac{DF_2}{EF_2}$$

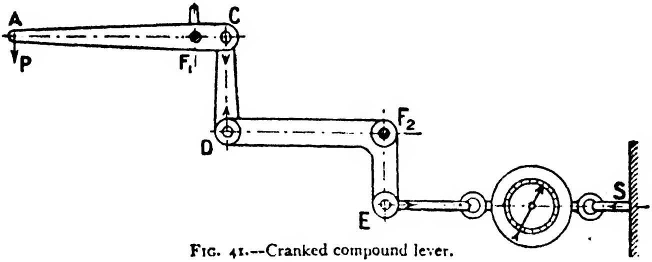
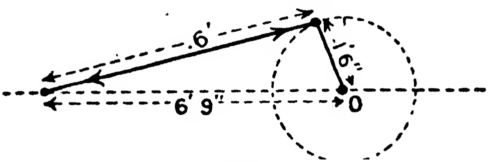


FIG. 41.—Cranked compound lever.

Note that in general the leverage of compound levers is leverage of first × leverage of second × leverage of third, and so on.

EXAMPLES II.

1. The connecting rod of a steam engine is 6 feet long, and the length of the crank 1 foot 6 inches. The thrust in the connecting rod is 2500 lbs.



Question 1.

If the rod and crank are in the positions shown, find the turning moment about the crank shaft O.

2. Two forces act as in the figure; P is of 14 lbs. in a direction 10° East of North, and Q is 18 lbs. in a direction North-West. Find the resulting moment about point A.

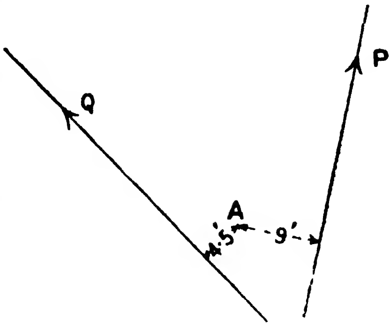
3. The three given forces keep a body at rest. A point A is on the line of action of force P, and its perpendicular distance from the 7-lb. force is 2 inches. What must be the moment of Q about A? Measure the perpendicular distance of the line of action of Q from A and hence calculate Q. The point B is on Q and 1½ inches from the line of action of the 7-lb. force. What must be the moment of P about B? Measure perpendicular distance of B from line of action of P and hence calculate P. Check your result by drawing a vector

diagram with sides parallel and proportional to 7 lbs. P and Q as calculated, and seeing if it forms a closed triangle.

4. A light bar 6 feet long is hinged at one end and held in horizontal position by a rope attached to the free end and making an angle of 45° with the rod. A weight of 1 cwt. is hung at a distance of 4 feet from the hinge. Find the tension of the string.

5. If the bar in the previous question is of uniform thickness and weighs 50 lbs., find the tension in the string.

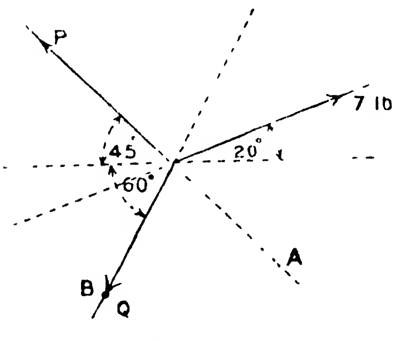
6. The figure represents a pump handle to which a force of 50 lbs. is applied in a line 3 feet from the pin or fulcrum. Find the vertical lifting force F exerted on the pump bucket by the handle.



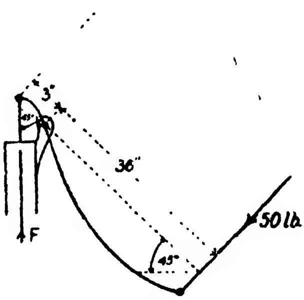
Question 2.

7. A horizontal beam 12 feet long is supported at the ends and carries two loads, one of $2\frac{1}{2}$ cwt. 3 feet from the left-hand end, and another of 6 cwt. $7\frac{1}{2}$ feet from the left-hand end. Neglecting the weight of the beam, calculate the reactions or upward forces of the supports.

8. A uniform horizontal beam 25 feet long is supported at the ends and carries the following loads: Two tons 4 feet from left-hand support, 8 tons



Question 3



Question 6

13 feet from the right-hand support, and $6\frac{1}{2}$ tons 22 feet from the left-hand support. If the beam weighs 10 cwt., calculate the reactions of the supports.

9. A uniform horizontal beam AC of weight 85 lbs. is 35 feet long, and rests on two supports A and B 27 feet apart, A being at one end of the beam. It carries the following loads: 5 cwt. 3 feet from A, 7 cwt. 20 feet from A, and $2\frac{1}{2}$ cwt. at C. Calculate the reactions of the supports.

10. A uniform horizontal beam AC 22 feet long is carried on two supports D and E, D being 4 feet from A and E 7 feet from C. If the beam weighs

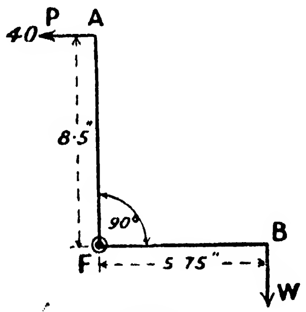
70 lbs. and carries the following loads: 75 lbs. at A, 120 lbs. between D and E and $1\frac{1}{2}$ feet from D, 186 lbs. between D and E and 4 feet from E, and 29 lbs. at C; find the reactions of the supports D and E.

11. A light horizontal lever AB of length 45 inches is pivoted about the fulcrum F, 5 inches from B. An effort of 25 lbs. is applied vertically at the end A: find the pressure at F and the resistance (W) overcome at B. If W and F change places, find the pressure at F and the resistance (W) overcome.

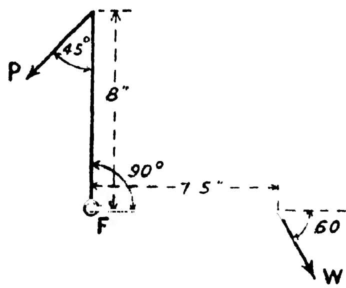
12. A lever is 36 inches long and is pivoted about a point $1\frac{1}{2}$ inches from one end. What weight must be hung at the end nearest to the fulcrum in order to keep the lever horizontal, if the weight of the lever is 15 lbs.?

13. A lever of weight 9 lbs. is 28 inches long. At one end is hung a weight of 6 lbs. and at the other end is hung a weight of 17 lbs. Find the position of the fulcrum so that the lever will rest in equilibrium in a horizontal position.

14. In the bell-crank lever shown P is equal to 40 lbs. at right angles to AF. If W is at right angles to BF, calculate its magnitude.



Question 14.



Question 15.

15. If P and W act on the bell-crank lever in the directions shown in the figure and P is 28 lbs., find W.

16. In the compound lever shown in Fig 40, AF is 11 inches, CF is 1 inch, CD $6\frac{1}{4}$ inches, DB $13\frac{1}{2}$ inches, and BF_2 $\frac{7}{8}$ inch. If P is equal to 56 lbs., what is the value of W?

17. In the compound lever shown in Fig. 41, $AF_1 = 18$ inches, $F_1C = 2\frac{1}{2}$ inches, link $CD = 6$ inches, $DF_2 = 9{\cdot}75$ inches, $F_2E = 3$ inches, angle $CDF_2 = 90°$. In the position shown AC is horizontal; CD and F_2E are vertical; DF_2 and ES are horizontal. What effort P vertically downwards will overcome a resistance W of 5 cwt.?

CHAPTER III

PRACTICAL APPLICATIONS

The Steelyard.—The common, or Roman, steelyard is a machine for weighing bodies, and consists of a lever AB (Fig. 42) which

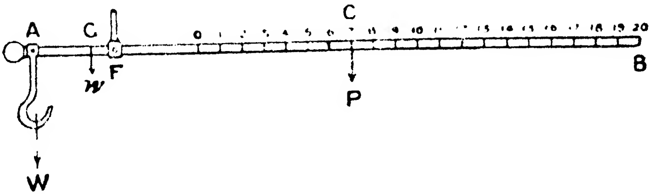


FIG. 42.—The steelyard.

turns about a fixed fulcrum F. The body to be weighed is hung from the end A nearest the fulcrum, and on the arm FB slides a moveable weight P. The point at which P must be placed, in order that the lever may rest freely in a horizontal position, determines the weight of the body. The arm FB has numbers engraved on it at different points of its length, so that the number at which the weight P rests gives the weight of the body.

To Graduate the Steelyard.—Let w be the weight of the steelyard, and let G be the point of the lever through which w acts. The lever is usually constructed so that G lies in the shorter arm AF. When there is no weight hung at A let o be the point at which P must be placed to balance w.

Taking moments about the fulcrum F, we have—

$$w \times GF = P \times oF \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$oF = \frac{w}{P} \times GF$$

This equation fixes the position of the point o; w, P and GF being known.

Now hang a known weight W at A, and let C be the point at

which P must be placed so that the lever rests in a horizontal position. Taking moments about F again, we have—

$$W \times AF + w \times GF = P \times FC \quad . \quad . \quad . \quad (2)$$

Subtracting equation (1) from equation (2), we have—

$$(W \times AF + w \times GF) - w \times GF = P \times FC - P \times oF$$

or

$$W \times AF = P(FC - oF)$$

$$W \times AF = P \times oC$$

$$\therefore oC = \frac{W}{P} \times AF$$

This equation fixes the point C; W, P and AF being known.

Suppose the movable weight P is 1 lb. and W is 1 lb., then o1 = AF, and the point C is marked with the number 1. If, now, a weight W of 2 lbs. be placed at A, then the distance of P from o will be $\frac{2}{1} \times AF$ or $2 \times AF$. Similarly, a weight of 3 lbs. at A requires P to be $3 \times AF$ from F, and its position is marked with the number 3, and so on. It will be seen, therefore, that if P is 1 lb. the distances between the successive graduations on the arm FB are all equal to AF. By a suitable choice of the distance AF and of the movable weight P, the steelyard may be graduated in whatever units are desired.

Example.—A common steelyard weighs 10 lbs.; the weight is suspended from a point 3 inches from the fulcrum, and the centre of gravity of the steelyard is $1\frac{1}{2}$ inches from the fulcrum, and on the same side of it. The movable weight is 10 lbs. Where should the graduation corresponding to 1 cwt. be situated?

Referring to Fig. 42, we have—

AF = 3 inches; GF = $1\frac{1}{2}$ inches; w = 10 lbs.; W = 112 lbs.; and P = 10 lbs. Taking moments about F gives—

$$112 \times 3 + 10 \times 1{\cdot}5 = 10 \times FC$$

$$336 + 15 = 10\,FC$$

$$\therefore FC = \tfrac{351}{10} = 35{\cdot}1 \text{ inches}$$

Hence the graduation corresponding to 1 cwt. should be 35·1 inches from the fulcrum.

Lever Safety Valve.—The safety valve consists of a lever FB (Fig. 43) pivoted at the end F. The valve is attached to the lever at some point V close to F. The centre of gravity of the lever is at point G, and the valve is held on its seat against the upward steam pressure by the weight W hung from some point A on the lever. The weight W and distance AF are so adjusted that when the steam pressure acting *upwards* on the valve reaches a certain value, it overcomes the *downward* force exerted on the valve by the weight W. The result is, the valve opens and steam

escapes until its pressure falls to the working value and the valve closes again.

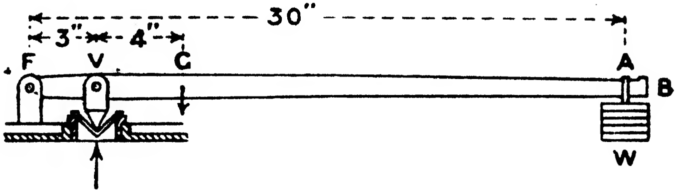


FIG. 43.—Lever safety valve.

Example 1.—The safety valve in Fig. 43 is 2 inches in diameter, and is just on the point of blowing off steam. The lever weighs 4 lbs., and its centre of gravity is at G. If the weight of the valve is $1\frac{1}{2}$ lbs., and the weight on the lever is 50 lbs., what is the pressure in the boiler?

Let p = steam pressure in pounds per square inch above atmospheric pressure. Then area of valve on which the steam pressure acts $= \frac{\pi}{4} \times 2^2 = \pi$ square inches. Then total upward pressure on valve $= \pi \times p$ pounds. The forces acting are, $\pi \times p$ pounds *upwards* at V, the weight of the valve $1\frac{1}{2}$ lbs. *downwards* at V, the weight of 50 lbs. downwards at A, and the weight of the lever 4 lbs. *downwards* at G.

Taking equal contra-clockwise and-clockwise moments about F—

$$\pi p \times FV = 1{\cdot}5 \times FV + W \times FA + 4 \times FG$$
$$\pi p \times 3 = 1{\cdot}5 \times 3 + 50 \times 30 + 4 \times 7$$
$$\tfrac{22}{7} \times 3p = 4{\cdot}5 + 1500 + 28 = 1532{\cdot}5$$
$$\therefore p = \frac{1532{\cdot}5 \times 7}{22 \times 3} = 162{\cdot}5 \text{ lbs. per square inch.}$$

Example 2.—A safety valve is just on the point of blowing off steam at a pressure of 180 lbs. per square inch above atmospheric. The valve is 3 inches diameter and weighs 2 lbs., being attached to the lever 4 inches from the fulcrum. The weight of the lever is 6 lbs., and its centre of gravity is 10 inches from the fulcrum. What weight must be hung on the lever 36 inches from the fulcrum?

Taking moments about the fulcrum—

$$\left(\frac{\pi}{4} \times 3^2 \times 180\right) \times 4 = 2 \times 4 + 6 \times 10 + W \times 36$$
$$\pi \times 9 \times 180 = 8 + 60 + 36W$$
$$5091 = 68 + 36W$$
$$\therefore 36W = 5091 - 68 = 5023$$
$$\therefore W = \tfrac{5023}{36} = 139 \text{ lbs.}$$

Body acted upon by Three Forces—Important.—If three forces alone keep a body at rest they must either (1) be all parallel

or (2) act along lines which all meet in one point. To realize this clearly the reader should sketch a number of cases of three straight lines showing the lines of action of the forces. If they are not all parallel (Fig. 44), suppose two, AB and BC, meet at P, then their resultant is a force passing through P and across the space B. Obviously, this resultant cannot balance the force CA unless the force CA is in the same straight line, in which case it passes through the point P, and all three forces pass through the point P. This will be found true for all possible cases except that of three parallel forces. We may also put the matter in this way. The forces AB and BC evidently have no moment about P, hence by the principle of moments the moment of CA about P must be zero, so that CA must also pass through P.

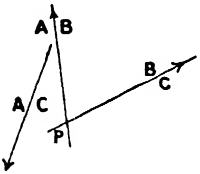


FIG. 44.

This important fact is very useful in showing the true direction of unknown forces, and when all the lines of action of the three forces are found we can use the triangle of forces. We shall illustrate the usefulness of this fact by some examples.

Fig. 45 shows the cranked lever problem already solved in

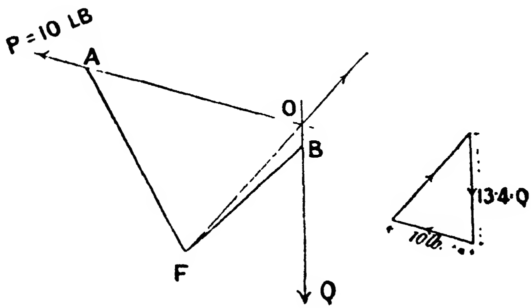


FIG. 45.

Chap. II., p. 36, and Fig. 39. The force P is given as 10 lbs., and it is required to find the pull Q at B in the direction shown.

Produce the lines of action of P and Q to meet in O. The third force on the lever is the force exerted *by* the fulcrum *on* the lever, and since P and Q pass through O, the third force must also pass through O, so that a line joining the centre of F to O gives the line of action of the third force. Drawing the triangle of forces

we find the magnitude of Q to be 13·4 lbs., thus confirming the previous result.

Hanging Chain.—A chain of total weight 120 lbs. hangs from two supports in such a manner that the inclination of its ends is 30° and 45° respectively. Find the tension of the chain at the ends.

Produce the lines of inclination of the ends to meet in some point P (Fig. 46). Then there are three forces acting at P, the

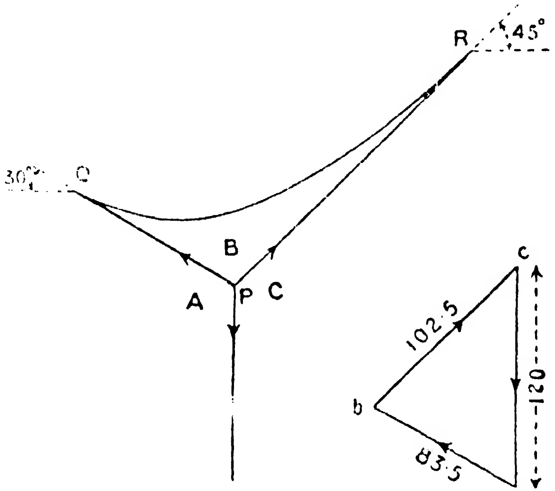


FIG. 46.

weight of the chain vertically downwards, and the tension at each end of the chain. Drawing the triangle of forces *abc*, we find the tension *bc* at the right-hand end is 102·5 lbs., and the tension *ab* at the left-hand end is 83·5 lbs. These results may be obtained by taking moments about R and Q or other points in the lines PR and PQ.

Foundry Crane.—In a foundry crane the load runs along a horizontal rail PQ (Fig. 47) of which the end P is hinged to a wall, and the outer end Q is supported by a tie-rod QR attached to the wall 5 feet above P. When a load of 1000 lbs. hangs from a point S on PQ 6 feet from P, find the pull or tension in the tie-rod, and the pressure on the hinge P, the length of PQ being 10 feet.

There are three forces acting on the rail PQ :—(1) the weight

of 1000 lbs. at S, (2) the pull of the tie-rod at Q, (3) the pressure of the hinge at P. The lines of action of the first two intersect at T, therefore the third force must pass through T, and its line of action must be PT.

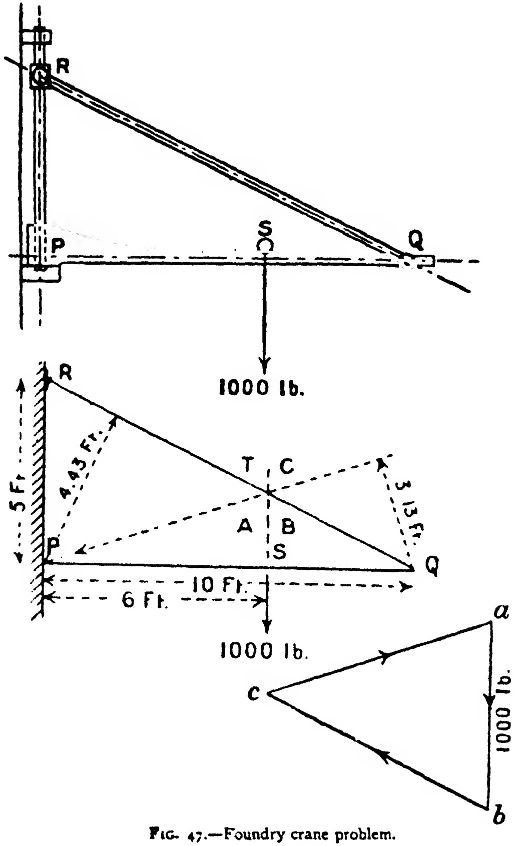


FIG. 47.—Foundry crane problem.

Place letters A, B and C in the spaces as shown on the centre line diagram in Fig. 47, and draw the triangle of forces for the point T. Choose any convenient scale and draw *ab* parallel to AB, the force of 1000 lbs.; from *b* draw *bc* parallel to BC, and from

a draw *ac* parallel to AC. The intersection of these two vectors gives the point *c*, and *bc*, which scales 1350 lbs., is the pull in the tie-rod RQ, whilst *ca*, which scales 1275 lbs., is the pressure exerted by the hinge P.

NOTE.—*ca* acts from *c* to *a*, and is the direction of the force exerted *by* the hinge to maintain equilibrium. The pressure *on* the hinge is 1275 lbs., but acts from *a* to *c*, that is, *towards* the hinge.

An alternative method of solving this problem is by the principle of moments. The perpendicular distance from P on to RQ is found by measurement to be 4·43 feet. Taking moments about P we have—

$$bc \times 4{\cdot}43 = 1000 \times 6$$
$$bc = \frac{6000}{4{\cdot}43} = 1353 \text{ lbs.}$$

The perpendicular distance from Q on to the line of action of PT is found by measurement to be 3·13 feet. Taking moments about Q we have—

$$ac \times 3{\cdot}13 = 1000 \times 4$$
$$ac = \frac{4000}{3{\cdot}13} = 1277 \text{ lbs.}$$

Another Case.—Suppose the same crane as in the previous example to carry a second load of 800 lbs. 3 feet from P (see Fig. 47A); find the pull in the tie-rod and the pressure on the hinge P.

In this case there is an additional force, making four forces altogether on PQ. We may, however, reduce these to three forces by finding the resultant of the two forces of 800 and 1000 lbs. in the way shown in Chap. II., and then consider the problem as if there were only three forces. Proceeding in this way, we find the resultant of the forces 800 and 1000 lbs. to be 1800 lbs. at a distance of $1\frac{2}{3}$ feet from the 800-lb. force. Drawing the triangle of forces in the same way as in the previous problem, we find *bc* the pull in the rod to be 1900 lbs., and *ac* the pressure *on* the hinge to be 1915 lbs.

This problem may also be solved by the principle of moments as follows: The perpendicular distance from P to RQ is 4·43 feet as before, whilst the distance of Q from PT is found by measurement to be 5 feet.

Taking moments about P we have—

$$bc \times 4{\cdot}43 = 800 \times 3 + 1000 \times 6$$
$$= 2400 + 6000 = 8400$$
$$bc = \frac{8400}{443} = 1896 \text{ lbs.}$$

Taking moments about Q we have—

$$ac \times 5 = 800 \times 7 + 1000 \times 4$$
$$= 5600 + 4000 = 9600$$
$$ac = \frac{9600}{5} = 1920 \text{ lbs.}$$

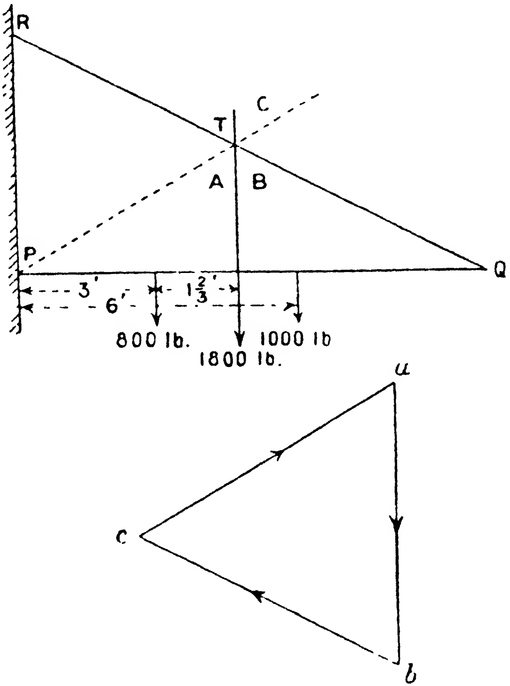


FIG. 47A.—Foundry crane problem

Toggle Joint.—This well-known arrangement is applied in many machines, and its principle is shown in Fig. 48. A moderate force P acting at the junction X of two rods or links causes a large thrust in the two links which is transmitted by them to their attachments at Z and Y. For simplicity let Z be a fixed pin and Y a pin in a block working between vertical smooth guides, P being horizontal and equal to 50 lbs. Suppose each link ZX and XY is 10 inches long and inclined, say, 10° to the vertical. On the pin X there are *three* forces acting: (1) the force P of 50 lbs.; (2) the

thrust of the link ZX; (3) the thrust of the link YX, and the triangle of forces may be drawn by setting off *ab* horizontally to represent 50 lbs. to scale, and drawing *bc* parallel to the force BC in XY, and *ac* parallel to the link ZX or the force AC to meet *bc* in *c*. Then *bc* represents the thrust of link XY on X, and *cb* represents the thrust of the link on Y. The pin Y is kept at rest by the thrust of the link XY, the vertical resistance W, and the horizontal

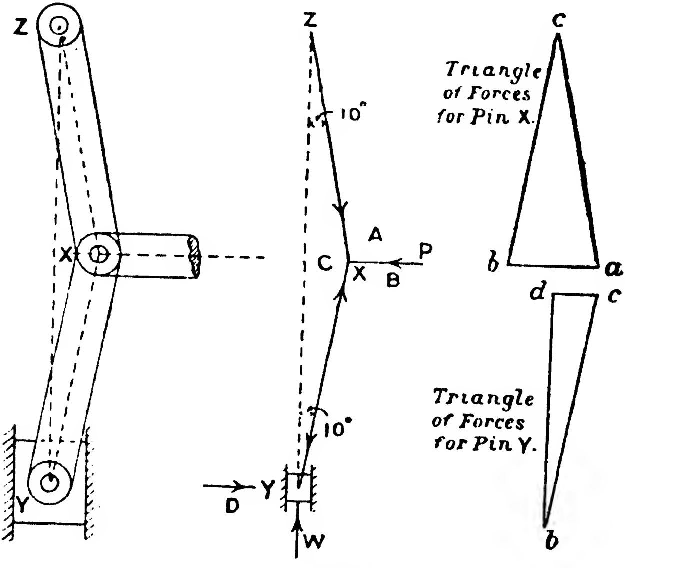


FIG. 48.—Toggle joint.

reaction DC of the vertical slide. Set off the vector *bc* again, and draw a vertical *db* and a horizontal vector *dc* intersecting at *d*. Then the vector *bd* gives the magnitude of the resistance W overcome by the force P of 50 lbs.; the resistance W is scaled off to be 141·7 lbs.

Now repeat the example with different inclinations of the links to the vertical, and we find the following results:—

Angle of links to the vertical.	Resistance W in pounds.
5°	286
10°	141·7
15°	93·3
20°	68·7
25°	53·6
30°	43·3

The results show how great W becomes as the links become more nearly in line with one another. The value of W is shown plotted for different angles of the links in Fig. 49.

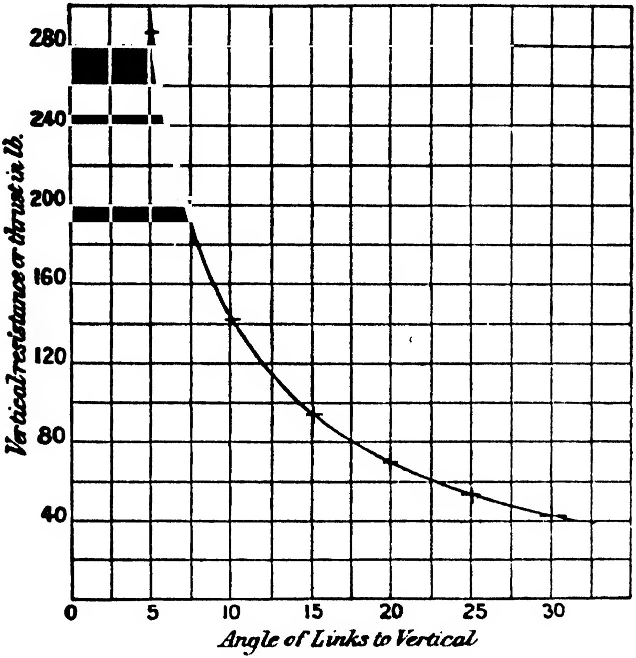


FIG. 49.—Force exerted in a toggle joint.

EXAMPLES III.

1. A common steelyard weighs 12 lbs., and its centre of gravity is 2 inches from the fulcrum and on the same side of the fulcrum as the weight. The movable weight or rider is 14 lbs., and the graduation corresponding to 1¼ cwts. is 3 feet 4 inches from the fulcrum. How far horizontally is the weight from the fulcrum?

2. The lever of a common steelyard weighs 8 lbs., and its weight acts 3 inches to the left of the fulcrum. The hook and links which carry the load weigh 3 lbs. and are attached 3 inches to the left of the fulcrum. If the graduation mark 3 feet from the fulcrum reads 112 lbs., find the weight of the rider. How far is the zero graduation from the fulcrum, and how much does the rider move per pound of graduation?

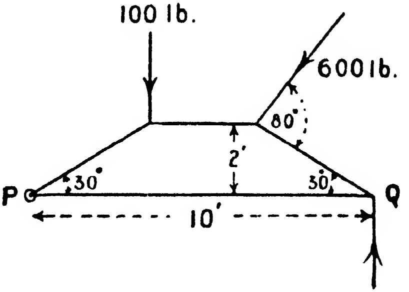
3. A safety valve is 2½ inches diameter and is just on the point of blowing

off steam. If the weight of the valve is 2 lbs. and the weight on the lever is 60 lbs. 28 inches from the fulcrum, what is the pressure of the steam in the boiler? The weight of the lever is 8 lbs. and acts 2 inches from the fulcrum towards the weight of 60 lbs., and the valve stem is pivoted 3½ inches from the fulcrum.

4. The lever of a safety valve weighs 8 lbs., and the distance between the fulcrum and the end of the valve spindle is 3 inches. If the length of the lever is 25 inches, and its centre of gravity 8 inches from the valve spindle, what weight must be put on the end of the lever so that steam will blow off at 150 lbs. per square inch? The weight of the valve and its spindle is 2½ lbs., and the diameter of the valve 3 inches.

5. A uniform balk of timber is 20 feet long and weighs 500 lbs. One end is raised by means of a rope inclined backwards over the balk, the other end of the balk remaining on the ground. When the inclination of the balk is 20° to the horizontal, and that of the rope 40° to the horizontal, find the tension in the rope and the pressure on the ground in magnitude and direction.

6. The frame carrying the given loads is supported at P and Q. The reaction at Q is vertical, the other reaction at P is not vertical but passes through a hinge at P. Find the reactions.



Question 5.

CHAPTER IV

SIMPLE FRAMES

Frames.—By *frames* or braced structures or trusses we understand structures consisting of rods, bars, pillars, chains, etc., hinged together to form a rigid whole. Frames generally consist of a number of braced triangles, and we shall apply the principles of Chap. I. and Chap. II. to find the stresses in the members of some simple examples. Braced structures are loaded at their joints only and the individual members are either in tension or compression. Even when the members are not actually hinged together we employ the same means to find the stresses in them approximately.

Jib Cranes.—We have already had a simple example of this in Chap. I., p. 19, and now consider some other arrangements of the rope or chain.

Example 1.—In the jib crane shown in Fig. 50 a weight of 4

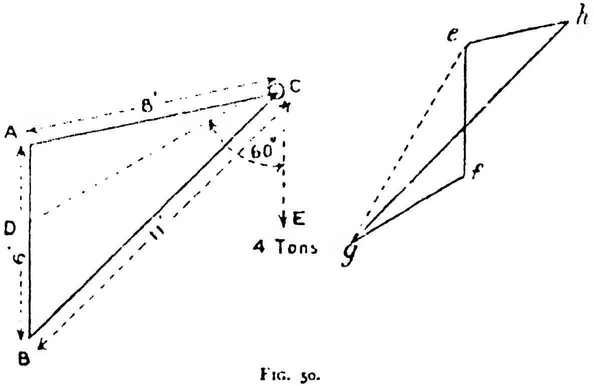


FIG. 50.

tons is supported by a rope passing over a pulley at the crane head C. The rope is then taken from the pulley to a drum or lifting winch, its direction being inclined 60° to the vertical, as shown. **Find the stresses in the tie-rod AC and in the jib BC.**

There are four forces acting at C, (1) the weight of 4 tons vertically downwards, (2) the pull of the rope equal to 4 tons in the direction from C to D, (3) the force or stress in the tie-rod, (4) the stress in the jib. The polygon of forces for the point C is drawn as follows. Choose any convenient scale, say, ½ inch to 1 ton, and draw the vertical vector *ef* 2 inches long to represent the weight of 4 tons; next draw *fg* 2 inches long parallel to the rope CD; from *g* draw the vector *gh* parallel to the jib BC, and from *e* draw *eh* parallel to the tie-rod AC. Then *gh* represents the stress in the jib, and *he* the stress in the tie-rod. Measuring these lengths, we find *gh* is 4·8 inches long representing 9·6 tons, and *he* is 1·72 inches long representing 3·44 tons.

It should be noted that if the tensions in the two parts of the rope CD and CE were replaced by their resultant *eg*, the problem would reduce to one on the triangle of forces, the vector diagram then consisting only of the triangle *egh*.

Example 2.—The crane of the previous example carries the

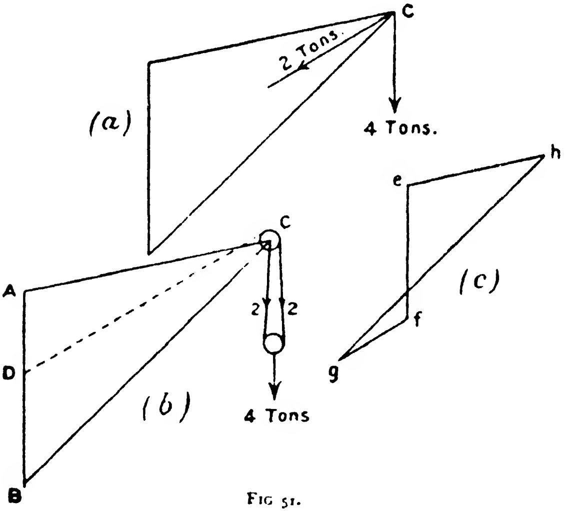


FIG 51.

weight of 4 tons by means of the snatch block shown at (*b*) in Fig. 51. Find the stress in the jib and tie-rod.

In this case the forces acting at C are shown at (*a*), Fig. 51, the tension in the rope being half the load, namely 2 tons. Draw a vertical vector *ef* to represent 4 tons as shown at (*c*), then *fg* parallel to CD to represent 2 tons, then *gh* parallel to the jib, and *eh* parallel to the tie-rod; *he* represents the stress in the tie-rod and scales 4·34 tons, whilst *gh* represents the stress in the jib and scales 8·5 tons.

Simple Roof Frame or Truss.

Example 3.—A simple triangular roof truss PQR (Fig. 52) has a span (PQ) of 20 feet, and a rise of 5 feet above the supports. It carries a load of 3000 lbs. at its apex. Find the stress in each member.

First find the reactions R_P and R_Q at P and Q; they will evidently be equal or each 1500 lbs., which we could find by taking moments about P and Q of the external forces 3000 lbs., R_P and R_Q, on the frame. Letter the space diagram ABCD as shown. Now draw the triangle of force for the point P, taking a scale of, say, 500 lbs.

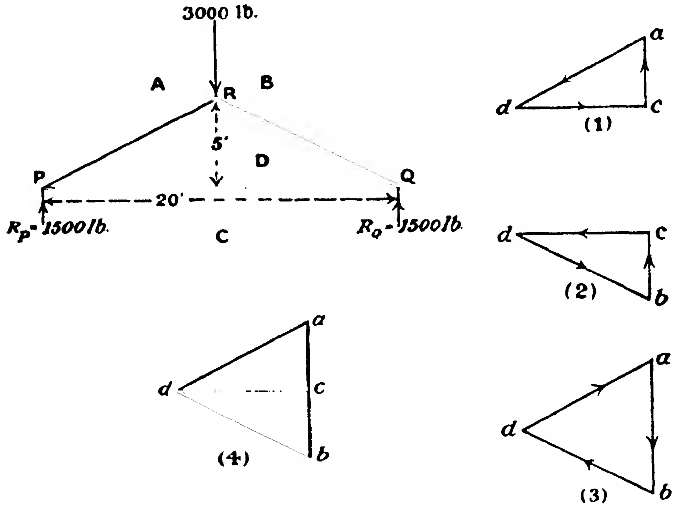


FIG. 52.—Force polygons and stress diagram for a simple roof.

to 1 inch as shown at (1). Set off *ca* upwards vertically to represent the force CA (or R_P) = 1500 lbs. to scale. Draw *ad* parallel to AD (or PR), and *dc* parallel to DC (or PQ) to meet it in *d*. Then *cad* is the triangle of forces required, and the direction of the vectors is *c* to *a*, *a* to *d*, and *d* to *c*. The force *ad* scales 3354 lbs. and its direction is from *a* to *d* (or R to P); the force *dc* scales 3000 lbs. and acts from *d* to *c* (or P to Q). Hence the force in AD or RP *thrusts* at P, that is, RP is in compression, and the force in DC or PQ pulls at P, that is, PQ is in tension.

Similarly, (2) Fig. 52 is the triangle of forces for the three forces at Q. It leads to the same conclusion with respect to PQ, namely, that it is in 3000 lbs. tension, and also shows RQ to be in 3354 lbs. compression.

We now draw the triangle of forces for the three forces meeting

at R as shown at (3), Fig. 52. This gives 3354 lbs. compression in PR and 3354 lbs. compression in RQ, thus checking the previous values found for those members and completing the solution of the problem.

Stress Diagrams.—If we put the diagrams (1) and (2) (Fig. 52) together, the side *dc* of diagram (1) falling on the side *cd* of (2), we get the diagram (4). This includes (3), and shows in a single diagram all the external forces AB, BC, and CA and all the stresses in the members BD, DC, and DA, and is called a *stress diagram.* When we want to find the stresses in all the members of a structure, instead of drawing the separate polygons for each joint we draw simply the stress diagram. For example, in the above case we first

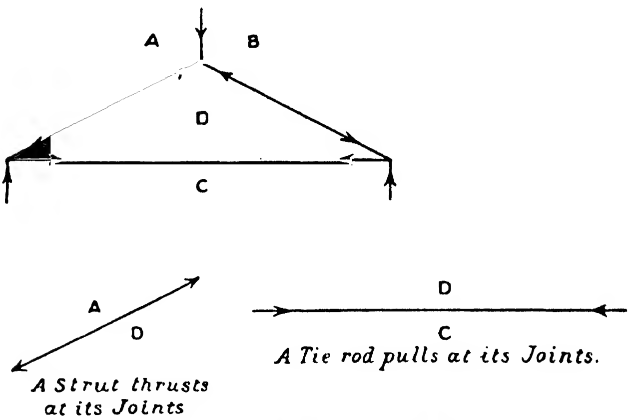


FIG. 53.—Forces in the members of a roof.

draw *ab* downwards to represent AB (3000 lbs. downwards) and then divide it equally at *c* so that *bc* and *ca* each represent 1500 lbs. upwards. Then on *ca* complete the triangle of forces *cad* for the point P, and then draw the triangle of forces for *either* point R or point Q, and we find that the vector diagram (4) contains the triangles for *both* the points R and Q.

When drawing a single closed polygon of forces we may show the direction of each vector by arrows placed on the sides of the polygon, as in (1) (2) and (3), Fig. 52. But in a stress diagram such as (4), Fig. 52, each line represents two opposite forces; for example, the line joining *a* to *d* represents the *thrust ad* on P, and also the thrust *da* on R. We therefore place arrows on the members of the frame on the space diagram as shown in Fig. 53.

Checking by Moments.—Since the three forces meeting at P, Fig. 52, are in equilibrium, we may apply the principle of moments thus: Taking moments about, say, R (in pounds-feet)—

$$1500 \times 10 \text{ (clockwise)} = \text{force in PQ} \times 5 \text{ (contra-clockwise)}$$

$$\text{force in PQ} = \frac{1500 \times 10}{5} = 3000 \text{ lbs.}$$

and to give a contra-clockwise moment about R it must *pull* at P, showing the stress in PQ is a tension.

Similarly, the force in PR may be found by moments about, say, Q, if the perpendicular distance of Q from PR be measured and the stress in RQ may be found by taking moments about P.

Experiment.—The calculated stresses in the above form of roof truss may be verified by experiment as follows:—An experimental form of the roof truss is shown in Fig. 54. Compression spring

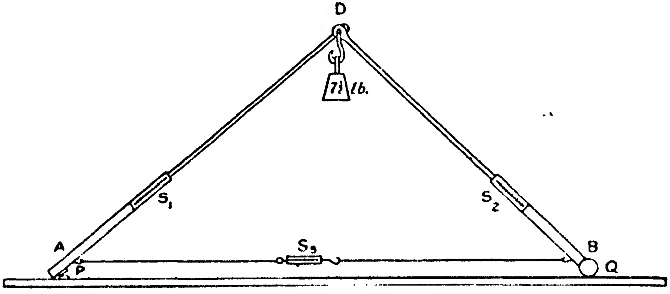


FIG. 54.—Experiment on forces in a roof truss.

balances S_1 and S_2 measure the thrust in AD and BD, whilst the tension in the tie AB is measured by means of the tension balance S_3. The end P is hinged to the wooden base PQ, and Q is free to move on the rollers shown. In a particular experiment a weight of $7\frac{1}{4}$ lbs. was hung from D, and the lengths of AB, AD and BD were found to be $38\frac{3}{8}$, $25\frac{1}{2}$ and $25\frac{1}{2}$ inches, respectively. The readings of the spring balances were; $S_1 = 5\frac{3}{4}$ lbs., $S_2 = 5\frac{3}{4}$ lbs., $S_3 = 4\frac{3}{8}$ lbs. By setting out the space diagram to scale and drawing the stress diagram from it as described above, the stresses in the members were found to be in AD and BD 5·8 lbs., and in DC 4·4 lbs., agreeing very closely with the observed readings of the spring balances.

Another Roof Truss.—Another form of roof truss is shown in Fig. 55, carrying 5000 lbs. at the apex. The stress diagram contains the following polygon of forces. For joint P, *cad*; for joint

Q. *bce*; for joint R *abcd*; and for joint S *cde*. The stresses in the different members are shown in the table below—

Member.	Stress (pounds).
PR or AD	7150 compressive
RQ or BE	7150 ,,
PS or DC	6250 tensile
SQ or EC	6250 ,,
RS or DE	2200 ,,

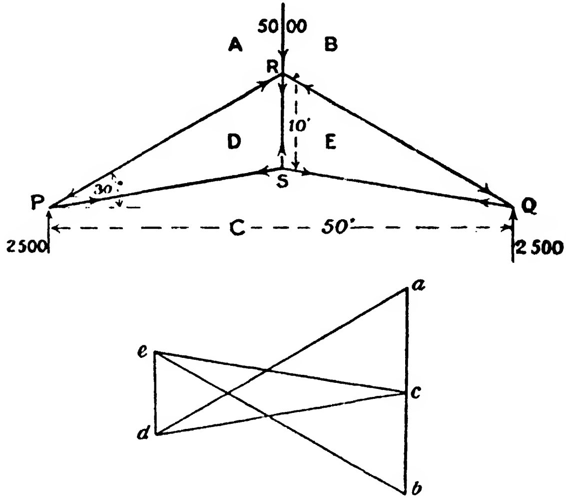


Fig. 55.

Other Frames.—Fig. 56 shows a braced support carrying a load of 800 lbs. at 10 feet from a wall. When the triangle *abc* for

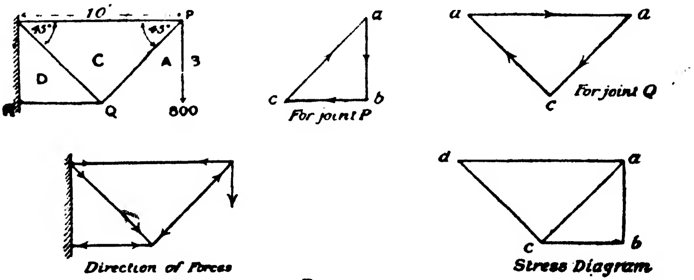


Fig. 56.

joint P has been drawn, by drawing *a* to *b*, *b* to *c*, and *a* to *c*, we know that the force exerted by PQ on P is *ca* = 1130 lbs., hence the force exerted by PQ on Q is the equal and opposite force *ac*, and this being known we can draw the triangle *acd* for joint Q. The stresses in the different members are—

Member.	Stress (pounds).
AC	1130 compressive
CD	1130 tensile
BC	800 ,,
DA	1600 compressive

Simple Braced Girders.—Fig. 57 shows a Warren girder consisting of three equilateral triangles. It is supported at the ends

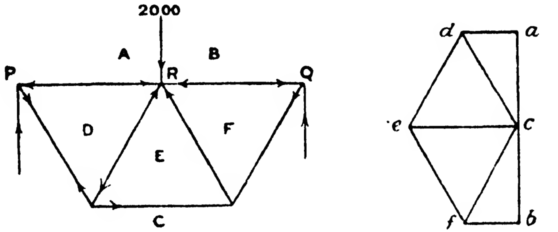


Fig. 57.

P and Q, and carries a load of 2000 lbs. at the joint R. Find the stresses in the different members.

The reactions at the supports are evidently *ca* at P and *bc* at Q, both being of magnitude 1000 lbs. Choosing any convenient scale, draw *ab* parallel to AB to represent 2000 lbs. and bisect it in *c*. Starting at the joint P or ADC, we draw *ad* parallel to AD, and from *c* draw *cd* parallel to CD; then from *d* draw *de* parallel to DE, and from *c* *ce* parallel to CE; from *e* draw *ef* parallel to EF, from *c*, *cf* parallel to CF, and from *b* draw *bf* parallel to BF. The last three vectors should all meet in the common point *f*. The stresses in the different members are—

Member.	Stress (pounds).
AD	578 compressive
BF	578 ,,
DE	1156 ,,
EF	1156 ,,
DC	1156 tensile
EC	1156 ,,
FC	1156 ,,

Fig. 58 shows the Warren girder in the above example inverted,

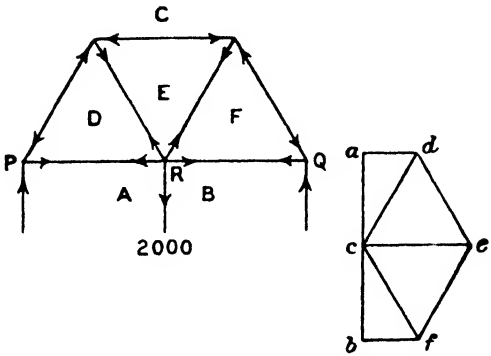


FIG. 58.

and carrying the load of 2000 lbs. at R as before. The stress diagram is drawn in the same way as before. It will be seen that although the stresses in the different members are of the same magnitude as before, those members which were in compression are now in tension and *vice versâ*, namely, the stresses in AD, BF, are now each 578 lbs. tensile, in DE and EF 1156 lbs. tensile, and the stresses in DC, EC and FC are now 1156 lbs. compressive.

Fig. 59 shows the Warren girder of Fig. 58 with two vertical members added; it is loaded as shown: find the stress in each member. The stress diagram may be drawn as follows:—First draw the load line *ad*, making *ab* to represent AB (500 lbs.), *bc* to represent BC (1000 lbs.), and *cd* to represent CD (500 lbs.). The reactions at the ends are evidently equal, hence bisecting *bc* or *ad* in *e* we have the reactions represented by *de* and *ea*. Now draw the triangle of forces *eaf* for the point EAF, so that *af* is parallel to AF and *ef* parallel to EF. From *f* draw *fg* parallel to FG and

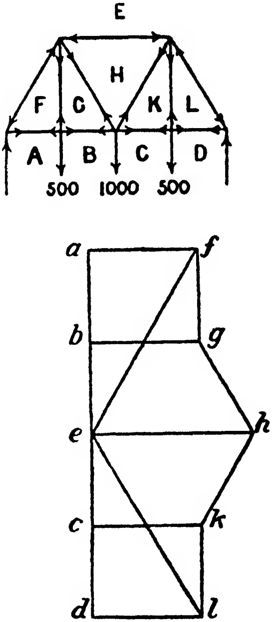


FIG. 59.

from b, bg parallel to BG. From g draw gh parallel to GH and from e, eh parallel to EH. This completes one half of the stress diagram; the other half may be drawn in a similar manner.

The stresses in the members are found to be—

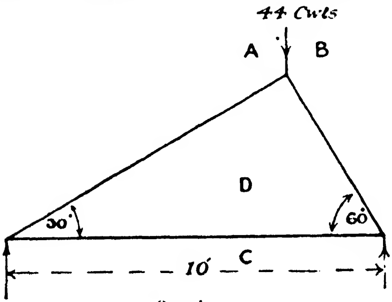
Member.	Stress (pounds).
AF, BG, CK, DL	578 tensile
FG and KL	500 ,,
GH and HK	578 ,,
EH	867 compressive
EF and EL	1156 ,,

EXAMPLES IV.

1. A crane jib measures 14 feet, the tie-rod 10 feet, and the vertical post 5 feet. A load of 3500 lbs. is attached to a chain which passes over a single pulley at the crane head and then parallel to the tie. Find the stresses in the tie-rod and jib.

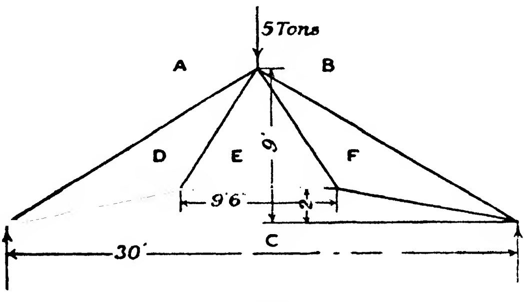
2. Solve Example 1, when the chain runs from the crane head in a direction parallel to the jib.

3. The crane of Example 1 carries the load of 3500 lbs. suspended from a snatch block as in Fig. 51. Find the stresses in the tie-rod and jib (1) when the chain runs parallel to the tie-rod, (2) parallel to the jib, (3) in a direction inclined downwards from the crane head at an angle of 37° to the vertical.



Question 4.

4. The simple roof truss shown carries a load of 44 cwt. at the apex. Find the stresses in the members.



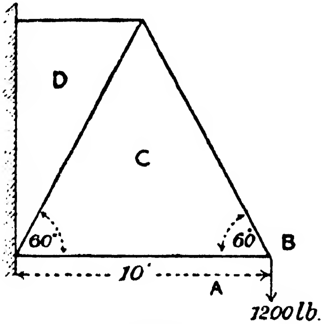
Question 6.

5. The roof truss shown in Fig. 55 carries a load of 3800 lbs. at the apex. Find the stresses in the members.

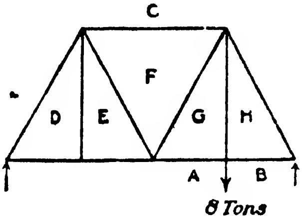
6. The given roof truss is loaded as shown. Estimate the stresses in the different members.

7. The braced support shown in the figure is loaded as shown. What are the stresses in the different members?

8. The Warren girder shown in Fig. 58 carries a load of $4\frac{1}{2}$ tons at the joint R. Find the stresses in the members.



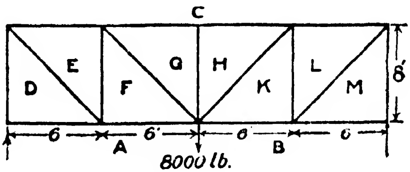
Question 7.



Question 9.

9. The Warren girder shown in the figure carries a load of 8 tons at the joint shown. Draw the stress diagram and determine the stresses in the members.

10. The truss shown in the figure has a span of 24 feet, divided into 4 equal panels each 8 feet high. The truss carries a load of 8000 lbs. at mid-span. Find the stresses in the different members.



Question 10.

CHAPTER V

WORK

WHEN a force acts on a body and causes it to move against a resistance the force is said to do work. If the force does not vary in magnitude and is in the direction of the motion, the amount of work done is equal to the product of the force and the distance moved, or—

$$\text{Work} = \text{force} \times \text{distance}.$$

Unit of Work.—If a force of 1 lb. acts through a distance of 1 foot, the amount of work done is—

$$1 \text{ foot} \times 1 \text{ lb.} = 1 \text{ foot-pound}$$

and one foot-pound is called the unit of work. If a weight of 1 lb. is lifted through a vertical distance of 1 foot, 1 foot-pound of work is done. If a weight of, say, 7 lbs. is lifted through a vertical height of 9 feet, the work spent in lifting is—

$$9 \times 7 = 63 \text{ foot-pounds}.$$

Other Units of Work.—Occasionally other units are used, for instance, the product of a force in tons and a distance in inches give inch-tons of work; similarly, tons and feet give foot-tons, pounds and inches give inch-pounds, and so on, but the foot-pound is the usual unit.

Example 1.—A horse pulling a cart exerts a steady horizontal pull of 80 lbs., and walks at the rate of 4 miles an hour. How much work does it do in 10 minutes?

Distance travelled in 60 minutes $= 4 \times 5280 = 21{,}120$ feet

" " 10 minutes $= \dfrac{21{,}120}{6} = 3520$ feet.

Work done $= 3520 \times 80 = 281{,}600$ foot-pounds.

Example 2.—How much work is done in raising 4 tons of coal from a mine 200 yards deep?

Height through which coal is raised is $200 \times 3 = 600$ feet.

Work done in *foot-tons* $= 600 \times 4 = 2400$ foot-tons,

or, lifting force in pounds $= 4 \times 2240 = 8960$ lbs.

Work done in foot-pounds $= 600 \times 8960 = 5{,}376{,}000$ foot-pounds.

Example 3.—If a horse is doing 33,000 foot-pounds of work in 1 minute when pulling a rope at a speed of 3 miles an hour, what pull must it exert?

$$\text{Distance moved in 1 hour} = 3 \times 5280 = 15{,}840 \text{ feet,}$$

$$\text{,, \quad ,, \quad 1 minute} = \frac{15{,}840}{60} = 264 \text{ feet.}$$

Work done in minute—

$$264 \times \text{pull} = 33{,}000 \text{ foot-pounds,}$$

$$\text{pull} = \frac{33{,}000}{264} = 125 \text{ lbs.}$$

Work done in Rotation.—Suppose a force of 7 lbs. (Fig. 60) is applied at right angles to a handle 2 feet long, or at the circumference of a pulley 2 feet radius, to turn a spindle at the other end

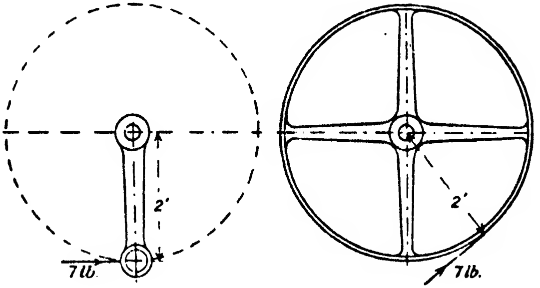


FIG. 60.

of the handle or at the centre of the pulley; in one revolution the work done would be—

$$\text{Force} \times \text{distance} = \text{force} \times \text{circumference} = 7 \times 2 \times 2\pi$$
$$= 28\pi = 88 \text{ foot-pounds.}$$

The moment of the force about the axis of rotation is—

$$7 \times 2 \text{ pound-feet} = 14 \text{ pound-feet,}$$

so that the work done may be written—

$$14 \times 2\pi = \text{moment} \times \text{angle in radians} = 88 \text{ foot-pounds,}$$

and for any amount of rotation either more or less than one revolution the same relation would be true, or—

work done = moment of force × angle turned through in radians.

This form is sometimes more convenient than the product of force and distance in considering motion of rotation. The moment of the turning-force is sometimes called the *torque;* so we may state—

$$\underset{\text{(foot-pounds)}}{\text{work done}} = \underset{\text{(pound-feet)}}{\text{torque}} \times \underset{\text{(radians)}}{\text{angle}}.$$

Example 1.—How much work is done in 300 revolutions of an electric motor if the torque on its shaft is 800 pound-feet?

$$\left.\begin{array}{l}\text{Total angle turned through in} \\ \quad 300 \text{ revolutions}\end{array}\right\} = 2\pi \times 300 = 1885 \text{ radians,}$$

$$\text{Total work done} = 800 \times 1885 = 1{,}508{,}000 \text{ foot-pounds.}$$

Example 2.—If a motor does 99,000 foot-pounds of work in one minute during which it makes 600 revolutions, what is the torque or turning moment on the motor shaft?

$$\text{Total angle in one minute} = 600 \times 2\pi = 1200 \times \pi \text{ radians,}$$

$$\text{Torque} \times 1200 \times \pi = 99{,}000 \text{ foot-pounds,}$$

$$\text{Torque} = \frac{99{,}000}{1200 \times \pi} = \frac{99{,}000}{1200 \times 3{\cdot}1416} = 26{\cdot}23 \text{ lb.-feet.}$$

Work represented by an Area.—The amount of work done by a uniform force acting through a distance in its own direction may be represented by a rectangular area or diagram of work, one side of which is proportional to the force and the other proportional to the displacement. For example (Fig. 61), if we represent a force of 50 lbs. by a length of say 2·5 inches, and a distance of 8 feet by 4 inches, the area 2·5 × 4 or 10 square inches represents the work done.

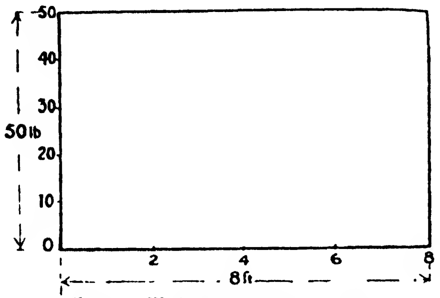


FIG. 61.—Work represented by an area.

$$\text{Work done} = 50 \times 8 = 400 \text{ foot-pounds}$$

$$\text{Area} = 4 \times 2{\cdot}5 = 10 \text{ square inches.}$$

So that the scale is $\frac{400}{10} = 40$ foot-pounds per square inch of area. The scale of work may best be found as follows: The force scale is 1 inch to $\frac{50}{2{\cdot}5} = 20$ lbs.; the distance scale is $\frac{8}{4} = 2$ feet to 1 inch, hence, a square of 1 inch side represents $2 \times 20 = 40$, or the work scale is 40 foot pounds to 1 square inch area.

If the force is not constant, but varies uniformly with the distance through which it acts, the diagram of work is not a rectangle, but a trapezoid, such as is shown in Fig. 62; the area in Fig. 62 represents the work done by a force, which increases uniformly from 20 lbs. to 45 lbs. during a displacement of 10 feet in its own direction; the scale is to be found as explained above.

The area is equal to the base multiplied by the mean or average height. The average height is equal to the height EF, midway between the extremes AB and CD. Note that—

$$\text{Length EF} = \frac{AB + CD}{2}$$

$$\text{Area} = AD \times EF.$$

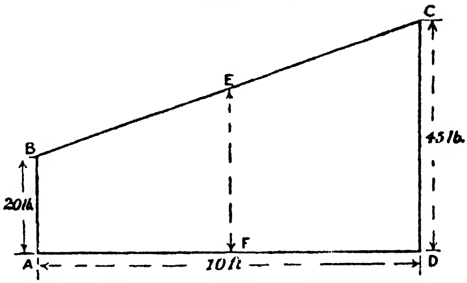


FIG. 62.—Work done by a force which varies uniformly.

Similarly, we might calculate the work done by multiplying the distance by the average force.

$$\text{Work done} = 10 \text{ feet} \times \frac{20 + 45}{2} = 10 \times 32{\cdot}5 = 325 \text{ foot-pounds.}$$

In cases where the force varies uniformly from zero to a maximum value, or falls uniformly from a maximum to zero, the

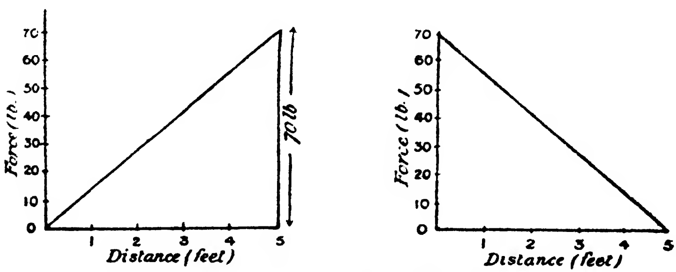


FIG. 62A.—Work done by a force which is proportional to distance.

same method may be adopted, the diagram of work being as shown in Fig. 62A, where the work is—

$$5 \times \frac{70 + 0}{2} = 5 \times 70 \times \tfrac{1}{2} = 175 \text{ foot-pounds.}$$

In all cases where the force varies uniformly to or from zero with distance the rule for finding the work done is—

Work = distance × mean force = distance × $\frac{1}{2}$ greatest force.

Example 1.—A spring originally not stretched is extended 4 inches. Find the work done in stretching the spring if the force required increases 5 lbs. per inch of stretch.

Maximum force required = 5 × 4 = 20 lbs.,
Average force = $\frac{20}{2}$ = 10 lbs.,
Work done = 4 (inches) × 10 (lbs.) = 40 inch-pounds.

Example 2.—How much work is required to stretch the above spring the last 1·5 inches?

Force required to stretch the last 1·5 inches after the spring is stretched 2·5 inches, is 2·5 × 5 = 12·5 lbs.

$$\text{Average force required in last 1·5 inches} = \frac{20 + 12·5}{2} = 16·25 \text{ lbs.}$$

Work done = 16·25 × 1·5 = 24·375 inch-lbs.

Check.—The work required for the first 2·5 inches = $\frac{12·5}{2}$ × 2·5 = 15·625 inch-pounds. And since 40 inch-pounds were required for 4 inches, the work in the last 1·5 inches must be—

40 − 15·625 = 24·375 inch-pounds as above.

Example 3.—A chain weighing 10 lbs. per foot, and 80 feet long, hangs vertically, and is then wound up on a drum at the top. Find the work done. How much work is done in winding up the first 20 feet?

Total weight of chain = 10 × 80 = 800 lbs.
Pull required at first = 800 lbs.
" " at end = 0

$$\text{Average pull} = \frac{800 + 0}{2} = 400 \text{ lbs.}$$

Work done = 400 × 80 = 32,000 foot-lbs.

Also after winding the first 20 feet—

Pull required = (80 − 20) × 10 = 600 lbs.

$$\text{Average pull in first 20 feet} = \frac{800 + 600}{2} = 700 \text{ lbs.}$$

Work done = 700 × 20 = 14,000 foot-lbs.

Check.—The work done in the last 60 feet = $\frac{600 + 0}{2}$ × 60 = 18,000 foot-pounds. Work in first 20 feet = 32,000 − 18,000 = 14,000 foot-pounds as above.

Work done in Machines used for Lifting.—There are many lifting machines, such as winches or crabs and pulley blocks of various sorts to be described and explained later, but in each a comparatively small force called the *effort* (P) is used to overcome a larger force (the weight lifted) called the load (W). Work is

done on the machine by the effort moving through a certain distance, and work is done by the machine in lifting the greater load, moving it through a shorter distance. In all actual machines some work is lost in resistances, called friction, in the mechanism, but in what immediately follows we are going to put this aside.

Principle of Work applied to a Machine.—In all machines the work done by the effort is equal to the useful work done on the load in the same time, plus the work done in overcoming the resistances in the machines. This is called the *principle of work.*

If we could make a machine to work so freely that its frictional resistance to motion were nothing, the work done by the effort would be just equal to the work done on the load. No machine is so perfect as this, but some simple machines such as a wheel and axle on ball bearings approach closely to it. When the resistances are so small as to be negligible and the motion is steady we may write for any given interval of time—

$$\text{Work done by effort} = \text{work done on load,}$$

or, in a given time—

$$\underset{\text{(in pounds)}}{\text{effort}} \times \underset{\text{(in feet)}}{\text{motion of effort}} = \underset{\text{(in pounds)}}{\text{load}} \times \underset{\text{(in feet)}}{\text{motion of load}}$$

Dividing by the effort and by the motion of the load, this may be put—

$$\frac{\text{motion of effort}}{\text{motion of load}} = \frac{\text{load (in pounds)}}{\text{effort (in pounds)}} = \frac{W}{P}$$

The ratio $\dfrac{\text{motion of effort}}{\text{motion of load}}$ denoted by V is called the *velocity ratio* of the machine, being the ratio of the effort's motion to the load's motion in the same time.

The ratio $\dfrac{\text{load}}{\text{effort}}$ may be called the *mechanical advantage* or force ratio of the machine, so that the above equation for an ideal perfect machine may be written—

$\dfrac{W}{P}$ (force ratio or mechanical advantage) = V (velocity ratio) or—

$$\frac{\text{force ratio}}{\text{velocity ratio}} = \frac{W}{PV} = 1.$$

To illustrate by numbers, if in order to lift the load 1 foot the effort has to move 15 feet, the velocity ratio or—

$$\frac{\text{effort's motion}}{\text{load's motion}} \text{ is } 15.$$

Then, if there were no frictional resistance, an effort of 1 lb.

would lift a load of 15 lbs., an effort of 2 lbs. would lift 30 lbs., an effort of 7 lbs. would lift 7 × 15 or 105 lbs., and so on. The force ratio or mechanical advantage would also be—

$$\frac{W}{P} = \frac{105}{7} = \frac{30}{2} = \frac{15}{1}.$$

Experimental Illustrations.—Fig. 63 shows a wheel and axle The wheel A and the drum B are keyed to the same spindle C; the spindle is mounted on ball bearings in order that the frictional resistance may be reduced to a minimum. The effort P is applied

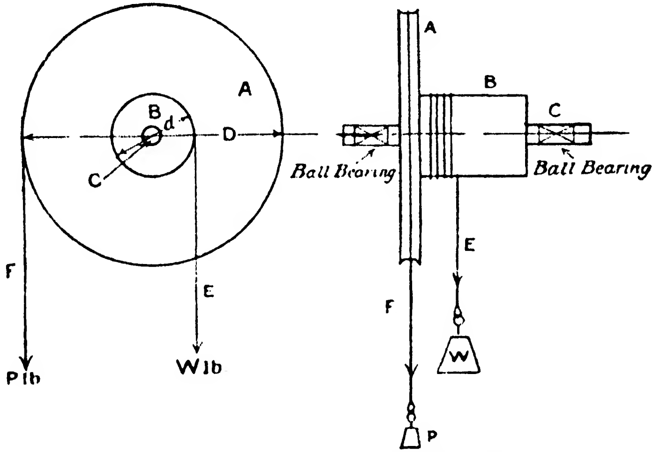


FIG. 63.—Principle of work for a frictionless machine.

by means of the cord F passing round the circumference of the wheel, and the load W is raised by another cord E coiled round the axle or drum, as shown.

First find the velocity ratio as follows:—Hang weights on the cords F and E so that they just balance, then move the effort weight P on cord F downwards, and measure the distance through which it must be moved in order to raise the load W on cord E through any convenient distance. In a particular experiment, in order to lift the load 1 foot, the effort had to be moved through 3 feet, hence the velocity ratio is 3. In another determination it was found that to lift the load 10 inches the effort had to move 30 inches, which gives a velocity ratio of $\frac{30}{10} = 3$ as before, the two determinations agreeing.

Different loads were then hung on the cord E, and the

corresponding effort applied by weights on the cord F, so that when just started downwards the resulting motion was steady. The results obtained are shown in the following table :—

Load W (lbs.).	Effort P (lbs.).	Ratio $\frac{W}{P}$.
5	1·68	$\frac{5}{1\cdot68} = 2\cdot98$
15	5·01	$\frac{15}{5\cdot01} = 2\cdot99$
25	8·34	$\frac{25}{8\cdot34} = 3\cdot0$

It will be seen that in each case the mechanical advantage or force ratio $\frac{W}{P}$ is practically equal to the velocity ratio ($V = 3$).

Fig. 64 shows a bicycle mechanism arranged to show that the

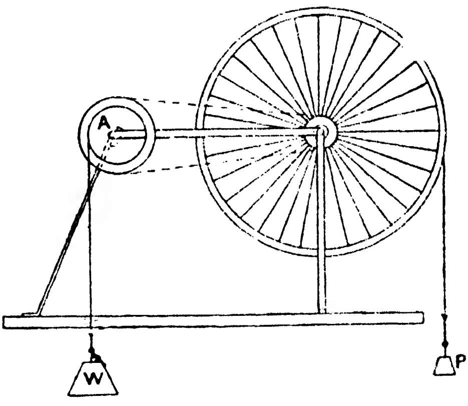


FIG. 64.—Bicycle mechanism.

mechanical advantage is nearly equal to the velocity ratio. The effort P is applied by hanging a weight on the string passing round the back wheel of the bicycle from which the tyre has been removed. The pedals are replaced by the pulley A, round the circumference of which is coiled another string on which the weight W is hung. In order to lift W 6 inches it was found that P had to be moved 51 inches, hence the velocity ratio is $\frac{51}{6} = 8\cdot5$

This value was checked by another determination which gave the same result, namely, to lift W 9 inches P had to move 76·5 inches, giving a velocity ratio of $\frac{76 \cdot 5}{9} = 8 \cdot 5$ as before.

The following results were obtained for the mechanical advantage at different loads; it should be noticed how closely they agree with the velocity ratio 8·5 :—

Load W (lbs.).	Effort P (lbs.).	Ratio $\frac{W}{P}$.
10	1·18	8·47
20	2·36	8·47
30	3·53	8·49
40	4·71	8·49
50	5·90	8·47

The above principle is also verified experimentally by means

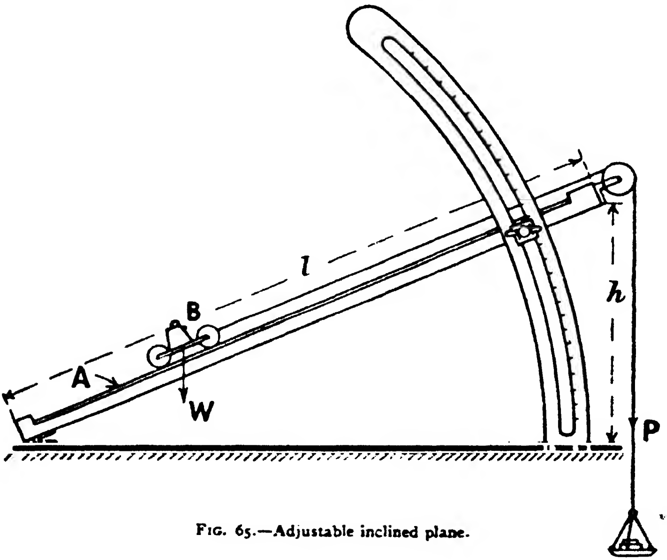


FIG. 65.—Adjustable inclined plane.

of the inclined plane shown in Fig. 65. The plane can be fixed at any angle to the horizontal, as shown; on the plane rests a sheet of

plate glass A. The carrier B is mounted on freely moving wheels, and is arranged to be pulled up the sheet of glass by means of the effort of a weight placed in the scale pan shown. In all the experiments made, the effort on the carrier was arranged to be parallel to the plane. To move the carrier (load W) from the bottom to the top of the plane through a *vertical* height h it is evident that the effort P must move through a distance equal to the length of the plane l. Hence the velocity ratio of vertical motion is equal to $\frac{l}{h}$.

The length l of the plane and its height h were measured for the corresponding values of W and P, the results obtained being tabulated below.

Load W (ozs.) (weight of carrier).	Effort P (ozs.) including scale pan.	Ratio $\frac{W}{P}$.	Height h (inches).	Length l (inches).	Velocity ratio V or $\frac{l}{h}$.
68·5	23·50	2·91	9·75	28·5	2·92
68·5	30·50	2·14	12·90	28·5	2·20
68·5	36·25	1·89	15·25	28·5	1·86
68·5	40·75	1·68	17·10	28·5	1·66
68·5	44·50	1·54	18·75	28·5	1·52
68·5	48·00	1·42	20·00	28·5	1·42

Again it is seen how closely the mechanical advantage $\frac{W}{P}$ agrees with the velocity ratio $\frac{l}{h}$ of vertical motion.

Work lost in Actual Machines.—In most cases a considerable part of the work put into a machine is lost in frictional resistances, so that the useful work got out of a machine, or done on the load, is not nearly equal to the work done by the effort. It is still however true that—

work done by effort = work done on load + work lost in friction

work lost in friction = work done by effort − useful work done on load.

Efficiency.—The fraction—

$$\frac{\text{Useful work (done on the load)}}{\text{Work expended (by the effort)}}$$

is called the *efficiency*. This proportion varies with different loads

on the machine, as we shall find later on. In a perfect frictionless machine the efficiency would be unity, that is—

useful work = work expended.

In actual machines—

$$\text{Efficiency} = \frac{\text{useful work}}{\text{work expended}} = \frac{\text{load's motion} \times \text{load}}{\text{effort's motion} \times \text{effort}}$$
$$= \frac{\text{mechanical advantage}}{\text{velocity ratio}}$$

Or again—

$$\text{Efficiency} = \frac{\text{load}}{\text{effort} \times \text{velocity ratio}} = \frac{W}{PV}$$

which is most simply understood as the useful work for 1 foot lift divided by work expended in the corresponding V-feet motion of the effort. The efficiency is always a proper fraction, and is usually multiplied by 100, and stated as so many *per cent.*

The velocity ratio V is fixed, but the mechanical advantage $\frac{W}{P}$ varies with the load.

Example 1.—If in a machine the effort moves 12·5 times as fast as the load, and a weight of 1 cwt. is lifted by an effort of 20 lbs., find the mechanical advantage and the efficiency of the machine at this load.

$$\text{Mechanical advantage} = \frac{\text{load}}{\text{effort}} = \frac{112 \text{ lbs.}}{20 \text{ lbs.}} = 5{\cdot}6$$

$$\text{Efficiency} = \frac{W}{P \times V} = \frac{112}{20 \times 12{\cdot}5} = 0{\cdot}448, \text{ or multiplying by } 100$$

$$\text{Efficiency} = 0{\cdot}448 \times 100 = 44{\cdot}8 \textit{ per cent.}$$

Example 2.—What load may be lifted by an effort of 25 lbs., if the velocity ratio is 18·5, and the efficiency is 55 per cent.?

For a frictionless machine the load would be $25 \times 18{\cdot}5 = 462{\cdot}5$ lbs., but actually it is $\frac{55}{100}$ of this, or—

$$0{\cdot}55 \times 462{\cdot}5 = 254{\cdot}4 \text{ lbs.}$$

Example 3.—If a lifting machine, having a velocity ratio of 23, lifts a load of 350 lbs. with an efficiency of 74 per cent., what effort would be required? What would be the mechanical advantage?

Useful work per foot of lift = 350 foot-pounds,

Work done by effort P per foot of lift = P × 23 foot-pounds.

$\frac{74}{100}$ or 0·74 of this being usefully applied,

$$\tfrac{74}{100} \times P \times 23 = 350$$
$$P = \frac{350}{23} \times \frac{100}{74} = 20{\cdot}6 \text{ lbs.}$$

$$\text{Mechanical advantage} = \frac{350}{20{\cdot}6} = 17{\cdot}0$$

$$\text{Or mechanical advantage} = \text{velocity ratio} \times \text{efficiency}$$
$$= \tfrac{74}{100} \times 23 = 17{\cdot}0.$$

EXAMPLES V.

1. (*a*) A force of 27 lbs. acts through a distance of 5 feet 3 inches. Find the work done in foot-pounds.

(*b*) A force of 800 lbs. does 1700 foot-pounds of work. Through what distance does it move?

(*c*) What force in pounds acting through a distance of 3 feet 9 inches will do 2 foot-tons of work?

(2) (*a*) Through what distance in feet must a force of 1800 lbs. move in order to perform 50 inch-tons of work?

(*b*) If 25,000 foot-pounds of work are done in lifting 2 tons of coal, through what height is it lifted in inches?

3. The average pull exerted by a locomotive on a train is 1·75 tons. How many foot-pounds of work are done per mile?

4. Find the work done in lifting 50,000 gallons of water from a lake to a tank which is 190 feet above the level of the water in the lake.

5. A steam engine cylinder is 15 inches diameter, and the mean pressure of the steam in it during a stroke is 40 lbs. per square inch. If the length of stroke is 18 inches, find the work done in one stroke.

6. A force pump has to deliver water at a uniform pressure of 750 lbs. per square inch. If the diameter of the pump cylinder is $4\frac{1}{2}$ inches, and the stroke 7 inches, how much work is done per stroke?

7. A man in turning a winch exerts a constant force of 50 lbs. at right angles to a handle 14 inches long. Find the work done in 25 revolutions of the handle.

8. A force of 85 lbs. is exerted at the circumference of a pulley 3 feet diameter which rotates at a uniform speed of 250 revolutions per minute. Find the work done per minute.

9. Find the force that must be applied at the circumference of a pulley 2 feet in diameter in order to do 220,000 foot-pounds of work in 70 revolutions.

10. If a motor does 156,000 foot-pounds of work in one minute when running at a speed of 860 revolutions per minute, what is the torque on the motor shaft?

11. A spiral spring when unstretched is 8 inches long. Its stiffness is such that the force required to stretch it is 10 lbs. per inch of stretch. Find the work done in stretching the spring to a length of 10·5 inches. How much work is done in stretching the spring the last $\frac{3}{4}$ inch?

12. A spiral spring 10 inches long is to be compressed to a length of 7 inches, a force of 15 lbs. being required per inch of compression. How much work must be done, and what work will be done in compressing the spring from a length of 9 inches to a length of 7·5 inches?

13. Calculate the work done in emptying a well 12 feet diameter and 25 feet deep, the weight of a cubic foot of water being 62·5 pounds.

14. A chain weighing 12 lbs. per foot of its length is 180 feet long, and hangs vertically: what work is done in winding up the chain on to a drum?

15. If the chain in Question 14 is used to lift a weight of $\frac{1}{2}$ a ton, what will be the total work done in raising this weight 170 feet by winding the chain on to a drum at the top?

16. A weight of 56 lbs. is to be lifted from the ground on to a table 3 feet above the ground by means of an elastic cord which stretches 1 foot for each 30 lbs. What will be the total amount of work done?

17. A lifting machine has a velocity ratio of 16, and a weight of 60 lbs. is lifted by an effort of 11·5 lbs. Find the mechanical advantage and efficiency of the machine at this load.

18. What load may be lifted by an effort of 20 lbs. if the velocity ratio is 115·5 and the efficiency is 34·6 per cent.?

19. If a lifting machine having a velocity ratio of 25 lifts a load of 40 lbs. with an efficiency of 53·8 per cent., what effort would be required and what be the mechanical advantage?

20. A man working at the rate of $\frac{1}{8}$ H.P. is raising a weight of $\frac{1}{2}$ a ton by means of a single rope system of pulley blocks. If the velocity ratio is 30 and the efficiency when lifting this load 25 per cent., what pull is the man exerting, and at what rate is he drawing the rope in?

CHAPTER VI

FRICTION AND LUBRICATION

Friction.—When a body moves over another body its motion is opposed by a resistance along the surface of contact of the two bodies. This resisting force is called friction. Its amount depends upon the materials which constitute the surfaces in contact, and upon whether they are rough or smooth.

When one body rests upon another one, and a force is applied to make one slide over the other, friction opposes and prevents the motion, provided the force applied is not large enough to overcome the friction. This friction is called *static friction* or the friction of rest, and its greatest amount when motion is about to begin is called the limit of static friction. The limiting value of static friction is generally greater than the sliding friction, or the friction of motion which opposes motion when sliding has begun.

Within certain limits the sliding friction between two surfaces in contact follows four simple rules or laws—

1. The magnitude of the friction is proportional to the total pressure between the two surfaces.
2. It depends upon the roughness of the surfaces and upon the material of which the surfaces are made.
3. It is independent of the areas of the surfaces
4. It is independent of the speed of sliding.

Although it is easy to prove that these laws are true within the pressures and speeds probable in simple laboratory apparatus, they are not actually true at very great speeds and pressures.

Experiment on the first Law of Friction.—The simple apparatus shown in Fig. 66 may be used. The board A is set in a horizontal position by means of a spirit level, and the slider B is pulled by a horizontal force applied by weights placed in the scale pan C, the force being transmitted from the scale pan by means of a strong thin string passing over a freely moving pulley D. Different weights may be placed on the slider B in order to vary the pressure between the surfaces of the slider and board.

Let W be the total load (including the weight of the slider) or vertical pressure between the surfaces, and P the horizontal pull exerted

on the slider, so that when motion is started by giving a downward jerk to the scale pan, the slider continues moving with *uniform* speed. It will be found that P varies proportionally to W, that is

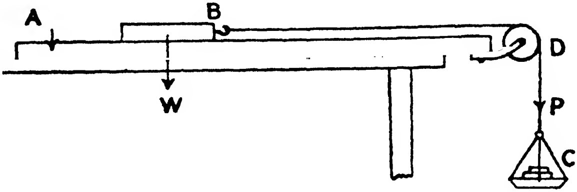


FIG. 66.—Experiment on friction.

to say, if W be doubled, P must be doubled, if W be increased three times, then P must be increased three times, and so on. In a particular experiment, using a brass slider and a truly planed oak board, both surfaces being *dry*, the following results were obtained—

Total load W (lbs.) (including weight of slider).	Horizontal effort P (lbs.) = friction.	Ratio $\frac{P}{W}$.
2·0	0·405	0·203
2·5	0·500	0·200
3·0	0·594	0·198
3·5	0·694	0·198
4·0	0·810	0·205
4·5	0·900	0·200
5·0	0·980	0·196

The above values of W and P should be plotted on squared paper as in Fig. 66A. Starting at the bottom left-hand corner, place the figure 0 and using any convenient scale write the figures 1, 2, 3, 4, 5, etc. on OX to represent different values of the load W. In a similar manner write the figures 0·1, 0·2, 0·3, 0·4, etc. on OY to represent the corresponding values of P. Now plot the values of W and P of the above table in the way described in the introduction (p. 6), and draw the best straight line which lies among the points. The fact that these points lie on a straight line which passes through O shows that the total friction P is proportional to W, or in other words, the friction P is equal to W multiplied by a constant. This constant is called the *coefficient of sliding friction*, and its value for the above pair of surfaces is found as follows:—

Take any two points A and B on the straight line some distance apart, then for the change in W represented by AC the corresponding change in P is represented by BC, and the coefficient of friction is $\frac{\text{force represented by BC}}{\text{force represented by AC}}$. For the two points taken in Fig. 66A

AC = 2·25 lbs., BC = 0·45 lbs., so that the coefficient of friction is $\frac{0{\cdot}45}{2{\cdot}25} = 0{\cdot}20$

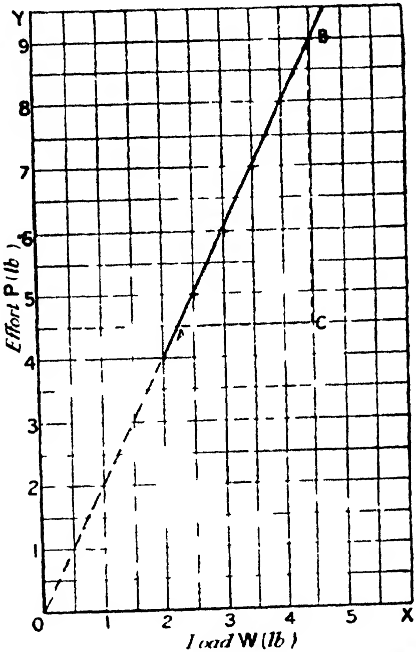


FIG 66A—Coefficient of friction from experiment with slider

In exactly the same way the coefficient (which is generally denoted by the Greek letter μ) for different pairs of surfaces may be found by using sliders and tracks of different materials

The following list gives average values of the coefficient of friction for various pairs of dry surfaces —

Surfaces	Coefficient of friction (μ)
Wood on wood	0 25–0 50
Metal on wood	0 20–0 60
Metal on metal	0 15–0 30
Leather on wood	0 25–0 50
Leather on metal	0 30–0 60

Resultant Force on a Sliding Body.—If, say, a block of wood is resting on a horizontal table (Fig. 67) the forces keeping it at rest are—

1. Its weight W acting vertically downwards.
2. The vertical upward pressure R of the table on the weight, which pressure must be just equal to W. R is distributed over the surface, but its resultant must be in the same line as the weight, that is, through the centre of gravity of the block.

If, now, a horizontal force P is applied in the direction shown, the block is kept at rest by the friction between the block and the table, providing of course that P is not large enough to cause motion. The force exerted on the block by the table is now the resultant of the vertical upward pressure R and the friction f, say,

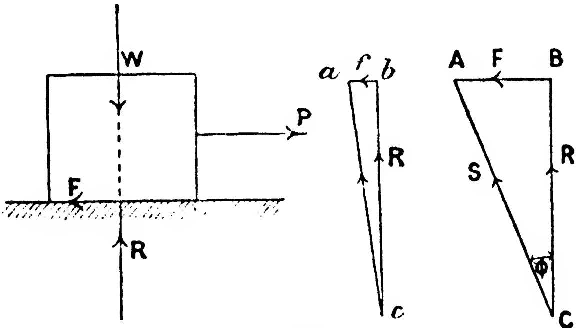


Fig. 67.—Sliding friction.

equal and opposite to P. This resultant is found by the usual method of adding forces as shown in the vector triangle abc, the resultant being parallel and proportional to ca. As P increases from zero the resultant upward pressure R will move to the right so that the contraclockwise moment of W and R balances the clockwise moment of P and F.

Angle of Friction.—If the horizontal pull P is increased till the limit of friction is reached, R remaining constant, and f increasing up to its greatest value, say, F, the resultant becomes more inclined to R. Its maximum inclination to the vertical is called the *angle of friction*. It is shown on the right-hand side of Fig. 67 as the angle $B\hat{C}A$, generally denoted by the Greek letter ϕ.

Now, tangent ϕ (see Introduction, p. 5) $= \frac{AB}{BC} = \frac{F}{R}$ or $\frac{F}{W} = \mu$,

so that we have the important relation—

tangent of the angle of friction = coefficient of friction.

The friction falls off when motion starts, so that just as we have two coefficients of friction we shall have two somewhat different angles of friction; one, the *maximum* angle which the resultant force between the two surfaces at rest makes with the perpendicular to the surface of contact, and the other the *constant* angle which the resultant makes with this perpendicular during sliding motion.

Body sliding down a Plane.

Small Slope.—When a body is at rest on a plane less steep than the angle of friction (see (*a*), Fig. 68) it is kept in equilibrium

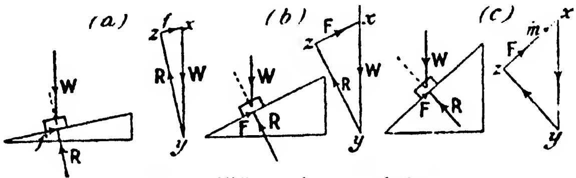


FIG. 68.—Sliding motion on rough slope.

by three forces, namely, its weight W vertically downwards, the perpendicular reaction or pressure of the plane R, and the friction f which is less than the limit of friction F. These forces are shown on the triangle of forces xyz.

Critical Slope.—If the inclination of the plane to the horizontal is increased up to the angle of friction (see (*b*), Fig. 68) the whole available friction F will be brought into play, and there will be no resultant force acting down the plane; the body can remain at rest on the plane, or, if started downwards it will not gain speed if the slope is equal to the angle of sliding friction. The forces are shown in the triangle of forces xyz (Fig. 68, (*b*)).

Steep Slope.—If the slope is steeper than the angle of friction the three forces W, R and F (Fig. 68, (*c*)) do not form a closed triangle but have a resultant, xm, down the plane, so that the body cannot remain at rest but slides down at an increasing speed.

Angle of Repose.—The angle of friction is also called the angle of repose, *i.e.* the steepest angle to the horizontal at which one of a pair of bodies can remain unsupported without sliding on the other.

Experiment.—To find the angle of friction and the coefficient of friction. Using the apparatus shown in Fig. 65, place a slider on the plane board or metal track, and then tilt the plane until its inclination to the horizontal is such that on just starting the slider it continues moving down the plane at uniform speed. Read off the angle of inclination ϕ of the plane to the horizontal, and from a

table of tangents look up the value of tan ϕ. Then tan ϕ is equal to μ, the coefficient of friction between the slider and the track.

In a particular experiment, using the same oak board and the same brass slider, the following result was obtained :—

ϕ.	$\tan \phi = \mu$.
$11{\cdot}25^\circ$	0·199
$11{\cdot}40^\circ$	0·202
$11{\cdot}25^\circ$	0·199
$11{\cdot}40^\circ$	0·202
Average . .	0·200

The average value of μ obtained by this method agrees with the value 0·200 found in the previous experiment described on p. 76.

Example.—A body weighing 112 lbs. is pulled along a horizontal surface at a steady speed by a horizontal force of 30 lbs. Find the work done in a distance of 10 feet. Also find the coefficient of friction between the body and the surface and the slope down which the body would just slide without assistance.

$$\text{Work done} = 30 \times 10 = 300 \text{ foot-pounds.}$$

$$\text{Coefficient of friction } \mu = \frac{P}{W} = \frac{30}{112} = 0{\cdot}268$$

Let ϕ be the angle of inclination of the plane to the horizontal, then $\tan \phi = 0{\cdot}268$. From the tables we see that $\phi = 15^\circ$.

Lubrication.—The sliding friction between two solid bodies may often be considerably reduced by arranging that their two surfaces shall not actually be in contact, but separated by a thin layer of a fluid or semi-fluid substance called a lubricant. Various kinds of oils are perhaps the best known lubricants, but grease of various degrees of stiffness is also used, the quality of the lubricant depending upon the pressure, speed and other circumstances. For heavy pressures thick grease is sometimes employed, but with high speeds oil is usually supplied to bearings under pressure from a pump to ensure a steady supply between the rubbing surfaces; such a system is called one of "forced lubrication."

In performing the above simple experiments on sliding friction, erratic results may be obtained due to the presence of a thin film of grease or moisture over part of one of the surfaces which are supposed to be dry. Even an imprisoned film of air may temporarily have a considerable effect. A repetition of the experiments with oiled surfaces will show lower coefficients of friction.

The resistances of liquids to motion are quite unlike those between solid surfaces, as they increase greatly with increase of speed, with increase of the area of surfaces in contact, and are independent of the pressure.

The total resistance to sliding between two solids separated

incompletely by a film of lubricant will not follow the simple laws stated for dry solid surfaces, for the resistance is partly that of a fluid and partly that of solids; its amount will depend upon the quantity and quality of the lubricant used and whether or not it gets squeezed out from between the surfaces. It may be well to point out that the frictional resistance of journal bearings between two curved (cylindrical) surfaces, such as a shaft in its bearing, does not materially differ from that between the flat slider and its track in the above explanations and experiments.

Heat generated at Bearings.—The work done against frictional resistances is converted into heat, and in bearings this heat represents a loss of work. In the imperfect conditions of lubrication present in the bearings of most machines for moderate speeds and pressures, the amount of friction follows fairly well the simple laws for sliding friction between dry solids, and if we know the suitable coefficient of friction we can calculate the work lost.

Example.—The pressure on a shaft or journal bearing 5 inches diameter is 2 tons, and the shaft makes 80 revolutions per minute. How much work is lost per minute in friction if the coefficient of friction is 0·03?

Total frictional force at the circumference of the shaft is—

$$2 \times 2240 \times 0{\cdot}03 = 134{\cdot}4 \text{ lbs.}$$

$$\text{The circumference of the shaft} = \frac{5}{12} \times \pi \text{ feet}$$

$$\text{Distance travelled per minute} = \frac{5}{12} \times \pi \times 80 = 104{\cdot}7 \text{ feet}$$

$$\text{Work lost per minute} = 104{\cdot}7 \times 134{\cdot}4$$
$$= 14{,}071 \text{ foot-pounds.}$$

Resistance to Rolling.—It is well known that the resistance offered to rolling a cylinder, a ball, or a wheel is often much less than the resistance to sliding them along a similar track. The resistance to rolling a very hard wheel or ball on a very hard track is very small, and on softer or more yielding tracks it is greater. The resistance to rolling between two bodies is not perfectly understood, but it results from the indenting of the bodies by one another; there is a certain amount of sliding motion when the two bodies are in contact. If both bodies are hard the area in contact will be very small, but if one or both yield considerably there will be a greater area of contact and more sliding motion and greater resistance to rolling.

Traction.—The resistances experienced when rolling trucks, carriages, etc., on tracks of various kinds are usually stated in pounds per ton. For instance, in the case of a train on the railroad,

an approximate value of the tractive resistance is 12 lbs. per ton, but the value varies greatly with the speed, for low speeds mechanical friction is the most important, while at high speeds the air friction is greater than the mechanical friction; for a cart on a good macadam road the tractive resistance is approximately 30 lbs. per ton.

Example.—Find the work done per minute against frictional resistances in hauling a train of weight 250 tons at a speed of 30 miles an hour, the frictional resistance being assumed constant and equal to 12 lbs. per ton.

$$\text{Total tractive resistance} = 250 \times 12 = 3000 \text{ lbs.}$$

$$\text{Distance moved in one minute} = \frac{30 \times 5280}{60} = 2640 \text{ feet;}$$

$$\text{Work done} = 2640 \times 3000 = 7{,}920{,}000 \text{ foot-pounds.}$$

Ball and Roller Bearings.—Advantage is taken of the small resistance to rolling in ball bearings, and also in roller bearings; a very common example of the use of ball bearings occurs in bicycles, the free running of which is well known. Fig. 69 shows a ball

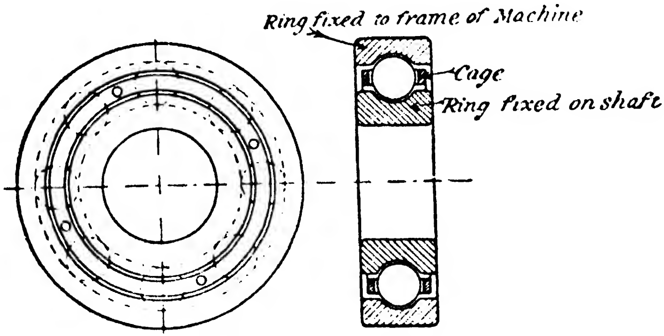


FIG. 69.—Ball journal bearing.

journal bearing for carrying a rotating shaft which has no end thrust, while Fig. 70 shows a single thrust ball bearing for carrying a shaft which has an end thrust always in the same (downward) direction. In both cases the hard steel balls are carried in a gun-metal cage in which they are loosely confined, kept at suitable distance apart and evenly distributed.

Fig. 71 shows a roller bearing used on the track wheels of many heavy waggons and agricultural machinery. The hard steel rollers are carried in gunmetal or malleable iron cages, and run

between a hard steel sleeve fitted on the shaft and a hard steel bush fitting inside the outer casing, as shown. In both ball and roller bearings oil is only required to prevent rusting.

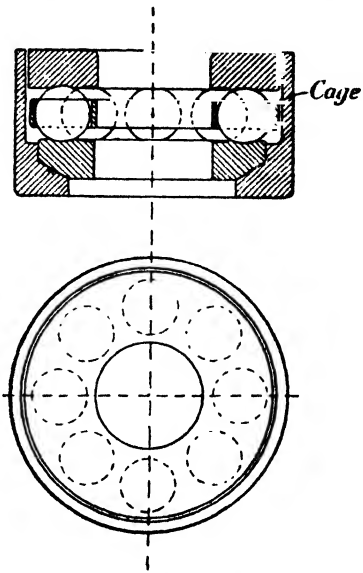


FIG. 70.—Single thrust ball bearing.

EXAMPLES VI.

1. A weight of 56 lbs. rests on a horizontal surface, and it is found that a horizontal force of 20 lbs. can keep the weight moving uniformly along the surface. Find the coefficient of friction.

2. A block of wood weighing 150 lbs. is to be pulled along a horizontal floor. If the coefficient of friction between the floor and the block is 0·45, what horizontal force will be required?

3. The sliding face of a steam engine slide valve measures 9 inches by 15½ inches, and the steam pressure on the back of the valve is 80 lbs. per square inch. If the coefficient of friction is 0·15, calculate the force required to move the valve.

4. A man weighs 150 lbs.: what is the greatest weight he can pull by a horizontal rope along a horizontal floor if the coefficient of friction between the weight and floor is 0·35, and between his boot soles and the floor 0·45?

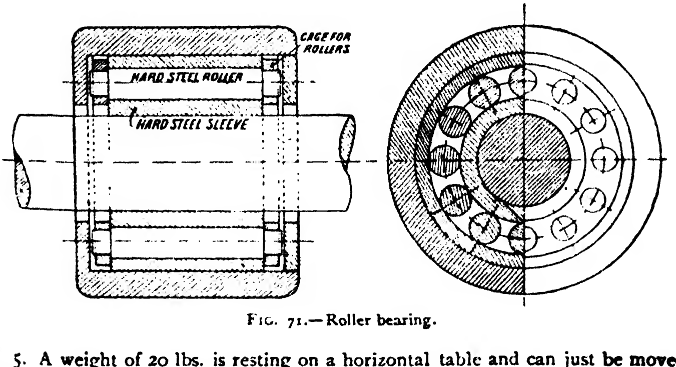


FIG. 71.—Roller bearing.

5. A weight of 20 lbs. is resting on a horizontal table and can just be moved by a horizontal force of 4·5 lbs; find the coefficient of friction and the direction and magnitude of the resultant reaction of the table.

6. A metal planing machine, the table of which weighs 2 cwts., makes 7 backward and 7 forward strokes in a minute. If the length of each stroke is 3 feet and the coefficient of friction between the sliding surfaces is 0·06, how many foot-pounds of work are done per minute on moving the table?

7. The crank of an engine is 9 inches long, and the connecting-rod 3 feet long. When the crank is at right angle to the line of stroke the total force on the piston is 8000 lbs. If the coefficient of friction between the crosshead stopper and guides is 0·05, find the frictional force opposing the sliding of the crosshead in this position.

8. A lathe spindle 3 inches diameter runs at 90 revolutions per minute. The load on the spindle is 5 cwts., and the coefficient of friction between the spindle and its bearings is 0·02. How many foot-pounds of work are wasted in friction per minute?

9. A body just slides down an inclined plane which is inclined to the horizontal at an angle of 15°. What is the coefficient of friction between the body and the plane?

10. If the body in Question 9 weighs 30 lbs. and the plane is tilted up until its slope is 40° to the horizontal, find the force acting down the plane on the body making it move.

11. A weight of 50 lbs. rests on an inclined plane whose slope is 20°. If the coefficient of friction between the weight and the plane is 0·45, find the force which, acting parallel to the plane, will just make the weight move up.

12. Find the force which, acting parallel to the plane in Question 11, will just make the weight of 50 lbs. move *down* the plane.

13. The pressure on a horizontal shaft or journal bearing 8 inches in diameter is 5 tons, and the shaft makes 100 revolutions per minute. How many British thermal units of heat are generated per minute by friction if the coefficient of friction is 0·02, and 778 foot-pounds are equivalent to one British thermal unit?

14. A horse pulls a cart which weighs 10 cwts. and is loaded with 1 ton at a uniform speed of 3 miles an hour. If the resistance is 40 lbs. per ton, what force does the horse exert on the level in hauling the cart, and how much work does he do in 5 minutes?

15. The weight on the driving wheels of a locomotive is 25 tons, and the coefficient of friction between the wheels and the rails is 0·09. Find the weight of the heaviest train it can draw on the level, and the work done per minute against frictional resistance when moving at a speed of 40 miles an hour if the tractive resistance is 12 lbs. per ton.

16. Find the work done in one minute against friction in pulling a train of 150 tons weight at a speed of 60 miles an hour if the tractive resistance is constant and equal to 12 lbs. per ton.

CHAPTER VII

SOME SIMPLE MACHINES

HITHERTO we have only considered ideal frictionless machines or machines in which the frictionless resistance was very small. In most actual practical machines used by engineers, the friction is a very considerable amount, and makes the amount of useful work done by the machine very much less than that spent by the effort in driving it. The friction occurs whenever one part of a machine moves over another, with which it is in contact; but at present we are going to consider its general effects on all classes of machines without inquiring as to exactly where it occurs.

On a particular lifting machine, the velocity ratio of which was 16 (that is, the effort moves through 16 feet in order to lift the load 1 foot) the efforts required to lift various loads were found to be as follows:—

Load W (lbs.).	Actual effort P (lbs.).
0	1·5
5	2·4
10	3·0
15	4·2
20	5·0
25	5·9
30	7·0
35	7·6
40	9·0

The effort plotted as ordinates on a base line of loads is shown by the line (*a*) Fig. 72. It should be noticed that the points thus showing the efforts determined by experiment lie fairly evenly on a straight line. This is found to be the case with all simple machines in good working order If there were no friction, the work done

by the effort would be equal to that spent in lifting the load, or for a lift of 1 foot—

$$\text{Effort} \times 16 = \text{load} \times 1$$
$$\text{or effort (without friction)} = \tfrac{1}{16} \times \text{load}$$

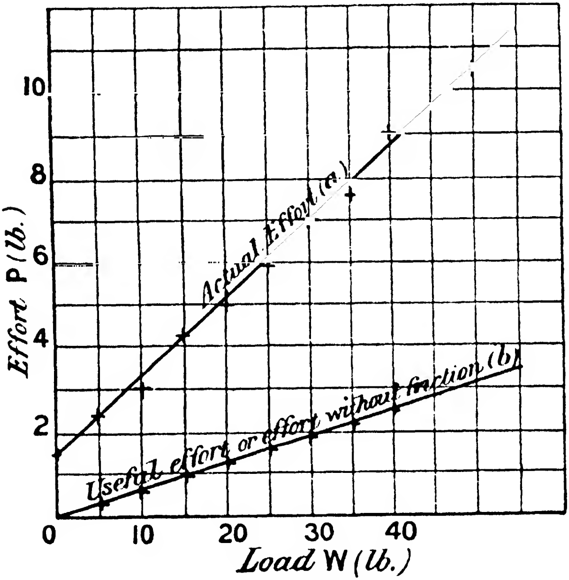


FIG. 72.—Relation of effort to load.

The smaller ideal effort found by dividing the load by the velocity ratio 16 for the same loads would be as follows :—

Load W (lbs.).	Ideal effort with no friction $= \frac{W}{16}$ lbs.
0	0
5	0·312
10	0·625
15	0·937
20	1·250
25	1·562
30	1·875
35	2·187
40	2·500

This ideal effort is shown plotted by the line (*b*) in Fig. 72. The distance between the actual effort and the ideal effort is shown by the vertical height of the line (*a*) above the line (*b*), and represents the amount of the actual effort which is lost in overcoming friction in the machine. A glance at Fig. 72 will show that for low loads the amount lost is nearly all the actual effort, while for higher loads the proportion of lost effort decreases. If P is the effort in pounds, W the load in pounds, and V the velocity ratio, the ideal effort for a frictionless machine being $\frac{W}{V}$, the effort wasted is—

$$P - \frac{W}{V}$$

We might also look upon the effect of friction as a decrease of load lifted with any given effort. In the table given above an actual effort of 5 lbs. lifts a load of 20 lbs., but in a perfectly frictionless machine it would lift $5 \times 16 = 80$ lbs., so that the effect of friction is to diminish the load lifted by—

$$80 - 20 = 60 \text{ lbs.}$$

Or, for any effort, the effect of friction in diminishing the load lifted is—

$$PV - W$$

For the above machine the effect of friction may be tabulated as follows:—

W (lbs.).	P (lbs.).	PV − W (lbs.).
0	1·5	24·0
5	2·4	33·4
10	3·0	38·0
15	4·2	52·0
20	5·0	60·0
25	5·9	69·4
30	7·0	82·0
35	7·6	87·0
40	9·0	104·0

This is shown plotted on a load base in Fig. 73. This curve drawn from the results of experiment will not generally show as good a straight line as the effort curve (*a*) in Fig. 72, because the discrepancies will be exaggerated after subtracting the actual load (W) from the ideal load (PV) of a frictionless machine.

Efficiency of Machines.—We have seen that some proportion of the work done on a machine by the effort is wasted in friction.

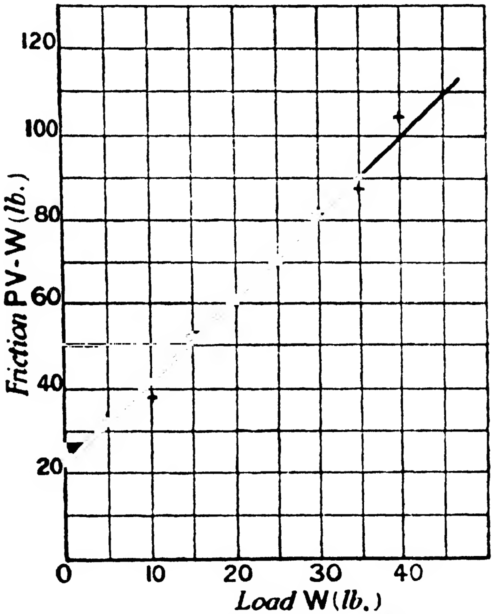


FIG. 73.—Relation of frictional loss to load.

The proportion usefully employed is called the mechanical efficiency of the machine, so that—

$$\text{Efficiency} = \frac{\text{useful work done}}{\text{work spent by effort in doing it}}$$

Taking 1 foot lift of the load W (lbs.) by an effort P (lbs.)—

$$\text{Efficiency} = \frac{W \times 1}{P \times \text{velocity ratio}} = \frac{W}{PV}$$

or—

$$\text{Efficiency} = \frac{\text{mechanical advantage } \frac{W}{P}}{\text{velocity ratio V}}$$

This may also be stated as the proportion which the actual load lifted (W) bears to the ideal load (PV), without friction, lifted by the effort P. Or, again, as the proportion which the ideal effort

$\frac{W}{V}$ (without friction) bears to the actual effort P required to lift a load W. The efficiency is *always* a proper fraction, and is often multiplied by 100 and stated as a percentage. The following are the efficiencies $\frac{W}{PV}$, for the loads and efforts given in the preceding tables :—

Load W.	Effort P.	Efficiency $= \frac{W}{PV} = \frac{W}{16P}$.
0	1·5	0 = 0 per cent.
5	2·4	0·130 = 13·0 ,,
10	3·0	0·208 = 20·8 ,,
15	4·2	0·224 = 22·4 ,,
20	5·0	0·250 = 25·0 ,,
25	5·9	0·265 = 26·5 ,,
30	7·0	0·268 = 26·8 ,,
35	7·6	0·287 = 28·7 ,,
40	9·0	0·278 = 27·8 ,,

This is shown plotted on a load base in Fig. 74. It should be

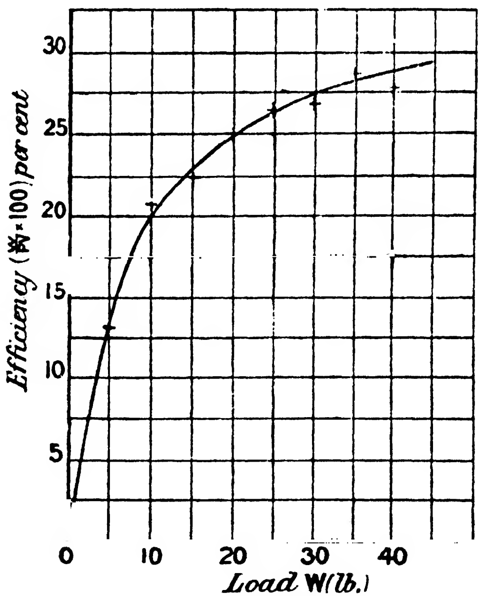


FIG. 74.—Relation of mechanical efficiency to load.

noted how low the efficiency is at small loads, and that as the load increases the efficiency increases also, but at a slower rate; in other words, the efficiency curve gets flatter or less steep as the load increases.

Experiments on Simple Machines.—To investigate by experiment the effect of friction on the efficiency of any lifting machine, the first thing to do is to find the velocity ratio; then, starting with the load (W) = o, find the effort (P) required, so that when once set in motion the load W is lifted at a uniform speed. Repeat the experiment with different loads, taking regular increments of, say, 5 or 10 lbs. up to the full load for which the machine is designed. Tabulate the results, working out the effect of friction (PV − W) and the efficiency $\left(\frac{W}{PV}\right)$ as explained above.

Simple Screw Jack.—Fig. 75 shows at (*a*) the screw jack as frequently used in practice for raising and holding up heavy pieces

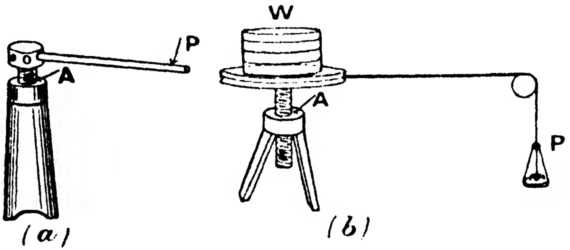


FIG. 75.

of machinery, etc. The effort is applied at the end of the lever, and the weight lifted directly by the head of the screw. For convenience in experimenting, the lever is replaced by a pulley, round which passes a string to which the effort is applied. The arrangement will readily be understood from Fig. 75 at (*b*). When experimenting with a particular machine it was found that the pitch of the screw (single thread) was $\frac{1}{3}$ inch, and the effective circumference of the pulley was 38·5 inches. Consider one revolution of the screw, and therefore of the pulley since the two rotate together. The effort will move a distance equal to the circumference of the pulley, whilst the load will be lifted a distance equal to the pitch of the screw: hence the velocity ratio will be—

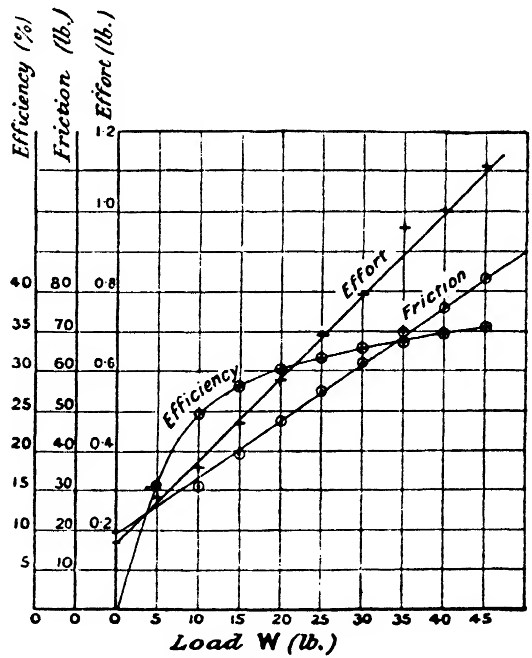
$$V = \frac{\text{circumference of effort circle}}{\text{pitch of screw}} = \frac{38 \cdot 5}{\frac{1}{3}} = 38 \cdot 5 \times 3 = 115 \cdot 5.$$

Starting with a load W = o, and increasing by regular increments

of 5 lbs., the following results were obtained, the effort P including the weight of the scale pan :—

Load W (lbs.).	Effort P (lbs.).	Friction (PV – W) lbs.	Efficiency per cent. $\frac{W}{PV} \times 100$.
0	0·172	19·86	0
5	0·282	27·48	15·4
10	0·359	—	—
15	0·469	—	—
20	0·578	46·77	29·9
25	0·688	—	—
30	0·797	62·04	32·6
35	0·960	—	—
40	1·000	75·50	34·6
45	1·110	83·13	35·1

In the above table several blanks are left in the third and fourth columns. The student is advised to fill them in himself for practice. The completed results are shown plotted on the same load base in Fig. 76.



Wheel and Axle.—Fig. 63 shows this machine, which has already been referred to in Chap. V. The following are the results obtained with a wheel and axle mounted on ordinary plain bearings. The effective circumference of the larger pulley or wheel was 42 inches, and that of the smaller drum or axle was 21 inches. Hence the velocity ratio $V = \frac{42}{21} = 2$.

The velocity ratio might also be found by the principle of the lever, for by taking moments about the axis of rotation, we have—

$$P \times \frac{D}{2} = W \times \frac{d}{2}$$

$$\therefore W = P \times \frac{D}{d} \quad \text{or} \quad \frac{W}{P} = \frac{D}{d}$$

Now, the mechanical advantage is equal to the velocity ratio for a frictionless machine, hence—

$$\text{vel. ratio } V = \frac{D}{d} = \frac{\frac{42}{\pi}}{\frac{21}{\pi}} = \frac{42}{21} = 2$$

Load W (lbs.).	Effort P (lbs.).	Friction PV − W (lbs.).	Efficiency per cent. $\frac{W}{PV} \times 100$.
0	0·8	1·60	0
5	4·30	3·60	58·2
10	7·14	4·28	70·1
15	9·91	4·82	75·7
20	12·81	5·62	78·0
25	15·63	6·26	80·0
30	18·50	7·00	81·2
35	21·50	8·00	81·4
40	24·45	8·90	81·8

Fig. 77 shows the above results plotted on a load base.

Rope Pulleys.—Fig. 78 shows at (*a*) (*b*) and (*c*) sets of rope pulleys the results of experiments on which are here given. In the set illustrated in Fig. 78 (*a*) there are two pulleys in the bottom block with four plies of rope running from it. Now, if the load be lifted one foot, each of the four lengths of rope shortens one foot, and therefore four feet of rope must be pulled off by the effort, since the same rope passes round all the pulleys in the top and bottom blocks. The velocity ratio of this set is therefore 4.

The results obtained are tabulated below and plotted on a load base in Fig. 79.

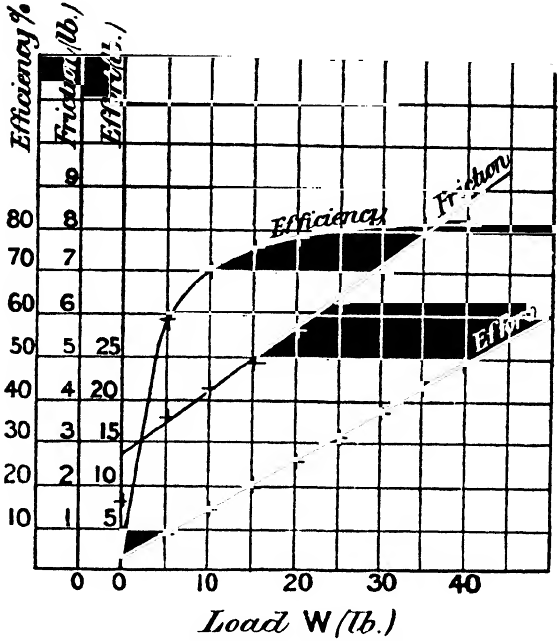


FIG. 77.—Results of test of wheel and axle

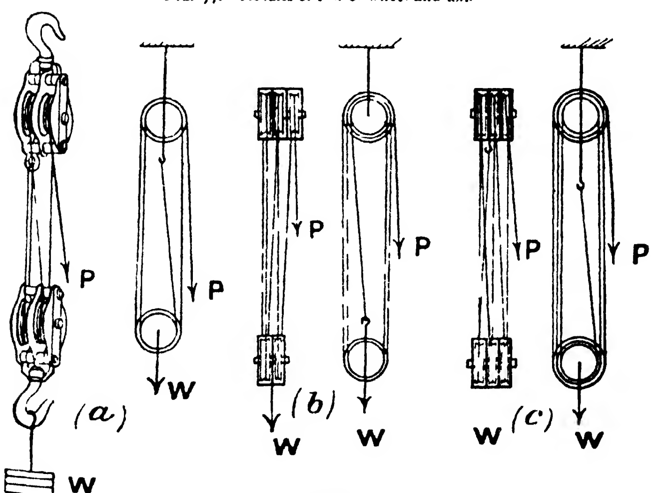


FIG. 78

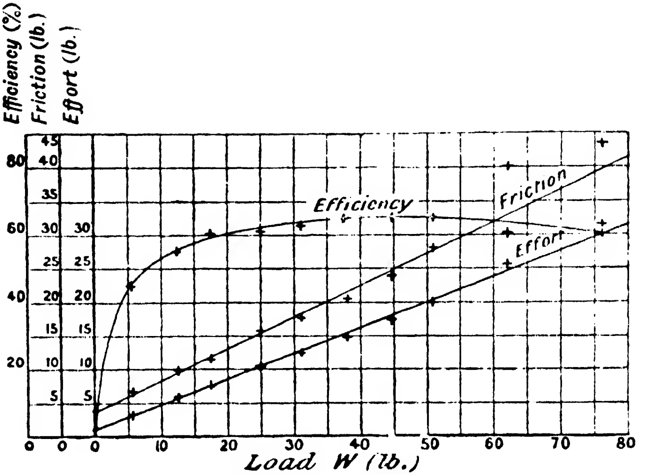


FIG. 79.—Results of test of rope pulley block.

VELOCITY RATIO = 4. FIG. 78 (a).

Load W (lbs.).	Effort P (lbs.).	Friction PV – W (lbs.)	Efficiency $\frac{W}{VP} \times 100$.
0	0·94	3·76	0
5·5	3·0	6·5	45·8
12·2	5·5	9·8	55·5
17·1	7·1	11·3	60·2
25·0	10·2	15·8	61·3
31·0	12·2	17·8	63·5
37·5	14·5	20·5	64·6
44·8	17·2	24·0	65·2
50·8	19·7	28·0	64·5
62·0	25·5	40·0	60·8
76·0	30·0	43·2	63·4

The set shown diagrammatically in Fig. 78 (*b*) has a velocity ratio of 5, and the following results were obtained :—

Load W (lbs.).	Effort P (lbs.).	Friction PV – W (lbs.).	Efficiency $\frac{W}{PV} \times 100$.
0	4·50	22·5	0
5	6·20	26·0	16·1
10	7·50	27·5	26·7
15	9·0	30·0	33·3
20	10·62	33·1	37·5
25	12·0	35·0	41·6
30	13·37	36·85	45·0
35	15·0	40·0	40·7
40	16·50	42·5	48·5
45	17·7	43·5	50·9
50	19·37	46·85	51·5
60	22·25	51·25	53·0
70	24·87	54·35	56·0

The above results are shown plotted on a load base in Fig. 80.

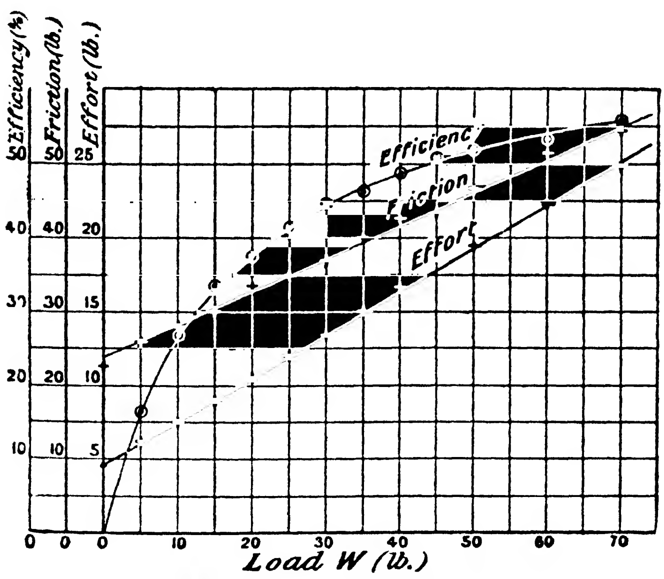


FIG 80.—Results of test of rope pulley blocks.

The set shown in Fig. 78 at (*c*) has a velocity ratio of 6, and the following results were obtained :—

Load W (lbs.)	Effort P (lbs.).	Friction PV − W (lbs).	Efficiency $\frac{W}{PV} \times 100$.
0	3·20	19·20	0
5	4·90	24·40	17·0
10	5·81	24·86	28·7
15	7·33	28·98	34·1
20	8·56	31·36	39·0
25	9·83	33·98	42·5
30	11·25	37·50	44·5
35	12·30	38·80	47·4
40	13·56	41·36	49·1
50	16·06	46·36	51·9
60	18·68	52·08	53·6
70	20·87	55·22	56·0

The above results are shown plotted on a load base in Fig. 81.

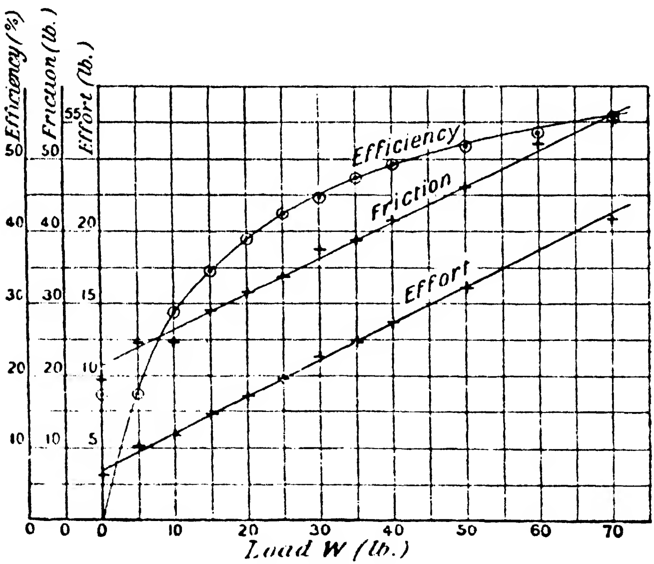


FIG. 81.—Results of test of rope pulley blocks.

When measuring the circumference of a pulley on a machine, as for example in the simple screw jack and the wheel and axle described above, the quickest and simplest way is to wrap the string or cord once round the pulley, unwind the string and measure its length. This will give the effective circumference of the pulley, *i.e.* the circumference of a circle the diameter of which is equal to the diameter of the pulley *plus* the diameter of the string. If the cord is of appreciable thickness its diameter (or circumference) must be measured as well as the diameter of the pulley. Let d = diameter of cord, D = diameter of pulley, then the effective diameter of the pulley will be $D + d$, and the effective circumference $\pi(D + d)$. The diameter of the cord was taken into consideration in obtaining the velocity ratios of the simple screw jack and wheel and axle described above.

EXAMPLES VII.

1. In a lifting machine the effort moves 120 feet for each foot the load is lifted. What effort will be required to lift a load of 1 ton if the efficiency at this load is 45 per cent.?

2. In a simple screw jack the pitch of the screw is ½ inch and the length of the lever at the end of which the effort is applied is 18 inches. What is the velocity ratio? If an effort of 9 lbs. applied at the end of the lever lifts a load of ½ ton, what is the efficiency?

3. In a wheel and axle the diameter of the wheel is 22½ inches and the diameter of the axle or drum is 7⅞ inches. The thickness of the cord on the wheel is ¼ inch, and that on the drum is ⅜ inch: find the velocity ratio of the machine. If the efficiency when lifting a load of 3 cwt. with a velocity of 1 foot per second is 78 per cent., how many foot-pounds of work must be supplied to the machine per minute?

4. The following results were obtained from an experiment with a certain lifting machine whose velocity ratio is 24·9:—

Load W lbs.	0	5	10	15	20	25	30	35	40
Effort P lbs.	0·094	0·45	0·81	1·17	1·53	1·88	2·25	2·61	2·97

Find the efficiency of the machine when lifting loads of (*a*) 8 lbs., (*b*) 18·5 lbs., (*c*) 26 lbs.

5. What will be the effect of friction in reducing the load lifted on the machine in Question 4 when lifting loads of (*a*) 12·5 lbs., (*b*) 38 lbs.

6. A crane similar to that shown in Fig. 50 is driven by an electric motor, and has to lift a load of 10 tons at a uniform speed of 2 feet per second. Assuming the efficiency of the motor to be 85 per cent. and the efficiency of the lifting mechanism 65 per cent., what horse-power must be supplied to the motor?

CHAPTER VIII

THE LINEAR LAW

IN the previous chapter we saw that when the curve of effort required to drive a simple machine is plotted on a base line showing the loads lifted, the points on the curve all lie in a straight line. The effort is then said to follow a *linear* law, and this linear or straight line relation between two quantities is so common that it is worth while considering it further.

At the beginning of Chap. VI. we saw that the effort P required to draw a weight W along a horizontal surface was proportional to the weight W, or, that the value of P was equal to the value of W multiplied by a constant. Referring to the table on p. 76 or to the results plotted in Fig. 66A, it was found in that case that for all values of the weight W—

$$P = 0{\cdot}20 \times W.$$

This is the relation between P and W for the straight line in Fig. 66A; evidently, when W = 0, P also is zero, so that the line passes through the origin O. The curve showing the relation between two quantities proportional to each other will always be a straight line passing through the origin or intersection of the axes on which the curve is plotted. Also the relation between two quantities which when plotted give a straight line passing through the origin is, that one is proportional to the other, that is, equal to the other multiplied by a constant. The constant 0·20 in the equation—

$$P = 0{\cdot}20W$$

expresses the number of pounds in P *per pound* of W. Similarly, if x is taken horizontally (instead of W) and y vertically, and—

$$y = mx$$

when m is a constant, the curve of y on a base x is a straight line through the origin O, and the constant m is simply the number of units of y *per unit* of x, or the value of y when $x = 1$.

For a given plotted straight line passing through the origin O the value of m in the equation such as $y = mx$ is found by dividing

any value of y on the line by the value of x for the same point, for—

$$m = \frac{y}{x} = \frac{\text{number represented by any vertical ordinate}}{\text{number represented by the corresponding horizontal distance}}$$

The quantity m is evidently proportional to the gradient or steepness of the straight line, or, in other words, to the tangent of the angle which it makes with the horizontal.

Example 1.—The following results were obtained by hanging a series of weights W on the free end of a spiral spring and thereby stretching it :—

Load W (lbs.).	Length of spring (inches).	Stretch of spring (l) (inches).
0	5·0	0
1	5·2	0·2
2	5·4	0·4
3	5·6	0·6
4	5·8	0·8
5	6·0	1·0
6	6·2	1·2
7	6·4	1·4
8	6·6	1·6
9	6·8	1·8
10	7·0	2·0

Find the law connecting the load W on the spring and the stretch l.

Plotting the load W vertically and the stretch l horizontally, we find the curve is a straight line passing through the origin o, as shown in Fig. 82. Hence W is proportional to l, or—

$$W = ml$$

where m is a constant.

Now, $m = \frac{W}{l}$, and taking the value of W = 5 lbs., for which $l = 1$ inch, we see that—

$$m = \frac{5}{1} = 5, \text{ and the law is}$$

$$W = 5l$$

or for every inch the spring is stretched the load required is 5 lbs., in other words, the *stiffness* of the spring is 5 lbs. pull per inch of stretch.

Example 2.—A tramcar is found to travel the distance y in x seconds, the distance moved in different times being measured and found to be—

Distance y (feet).	Time x (seconds).
0	0
7·5	1
13·0	2
20·0	3
27·0	4
34·0	5
42·0	6
49·5	7
57·5	8

Find the relation between y and x.

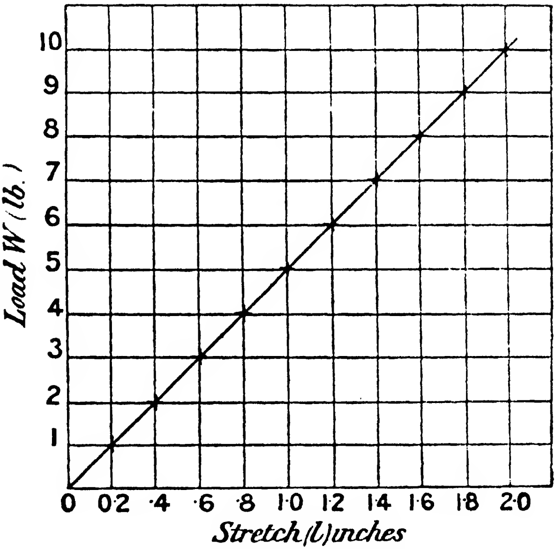


FIG. 82.—Simplest form of plotted straight line, vertical *proportional* to horizontal.

The above quantities are shown plotted in Fig. 83. Now take any two points on the straight line some distance apart, as at A and B. Draw the horizontal line AC and the vertical line BC. Then during the time represented by the length AC the tramcar has moved the distance represented by the length BC. The equation to the curve is—

$$y = mx$$

$$\text{and } m = \frac{y}{x} = \frac{BC}{AC} = \frac{30}{4 \cdot 15} = 7 \cdot 22$$

therefore the law is—

$$y = 7{\cdot}22x$$

and we see that when $x = 1$ second $y = 7{\cdot}22$ feet, or the car moves a distance of 7·22 feet per second.

We next consider straight lines which do not pass through the origin. For example, the effort-load curve plotted in Fig. 72 from the table at the beginning of Chap. VII. cuts the vertical axis at a point representing 1·5 lbs. effort, which is the effort required at no load. If we measure the efforts from this level, that is, in excess of 1·5 lbs., we get the following result which is shown plotted in Fig. 84 :—

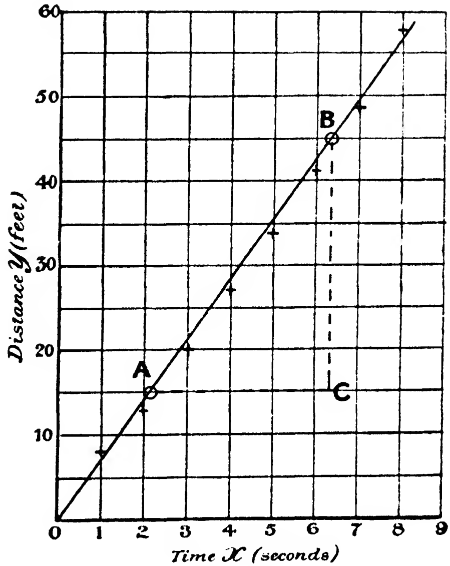
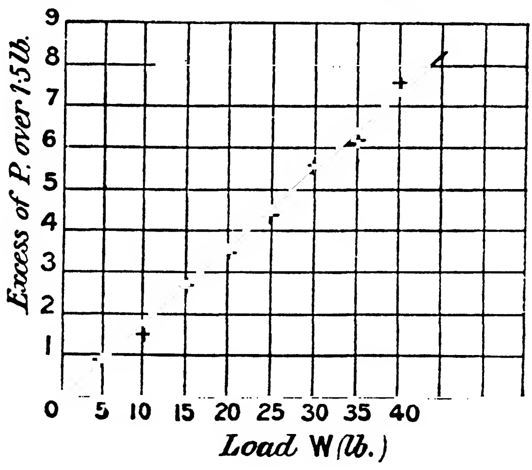


FIG. 83.



Load W (lbs.).	Actual effort P (lbs.).	Excess over 1·5 lbs.
0	1·5	0
5	2·4	0·9
10	3·0	1·5
15	4·2	2·7
20	5·0	3·5
25	5·9	4·4
30	7·0	5·5
35	7·6	6·1
40	9·0	7·5

It is simply the curve (*a*) Fig. 72 lowered by 1·5 lbs. Taking points on the straight line (not from the above table), we find the vertical ordinates are proportional to W, thus—

For $W = 15$ lbs. excess of P over $1·5 = 2·7$ lbs.

$$= \frac{2·7}{15} \text{ of } 15 = 0·18 \times 15 = 0·18W$$

For $W = 20$ lbs. excess of P over $1·5 = 3·6$ lbs.

$$= \frac{3·6}{20} \text{ of } 20 = 0·18 \times 20 = 0·18W$$

For $W = 38$ lbs. excess of P over $1·5 = 6·9$ lbs.

$$= \frac{6·9}{38} \text{ of } 38 = 0·18 \times 38 = 0·18W$$

and so on for every point exactly on the line in Fig. 84. The increase in the effort for every 1 lb. increase in W is 0·18 lb., and—

Excess of effort P over 1·5 lbs. = 0·18 × load W (pounds)

or $P - 1·5 = 0·18W$

and effort P (pounds) $= 1·5 + 0·18W$.

This is always the relation between the two quantities which when plotted give a straight line. It is—

effort = a constant + load × uniform increase per pound of load or $P = c + mW$.

The constant c represents the effort for no load, and the constant quantity m means the regular increase of P per unit increase of W. For any given plotted straight line we find c by reading off at the intersection of the straight line and the vertical axis (producing the line if necessary); we find m by taking any effort and subtracting c from it and then dividing the result by the corresponding value of W. We may put this—

$$m = \frac{P - c}{W}$$

The effort per unit of load $\left(\frac{P}{W}\right)$ is here not constant, but diminishes as W increases.

Thus for $W = 15$ lbs., $\frac{P}{W} = \frac{4\cdot2}{15} = 0\cdot28$

for $W = 38$ lbs., $\frac{P}{W} = \frac{8\cdot4}{38} = 0\cdot221$

Example 1.—What is the law for the effort in Fig. 76?

The straight line meets the vertical axis (where $W = 0$) at $P = 0\cdot172$ lb., hence $c = 0\cdot172$. Taking the point on the line where $P = 0\cdot9$ lb., we see that the corresponding value of W is 35·5 lbs., hence—

$$m = \frac{0\cdot9 - 0\cdot172}{35\cdot5} = \frac{0\cdot728}{35\cdot5} = 0\cdot0205$$

the increase in P per pound of W; therefore the law is P (pounds) = $0\cdot172 + 0\cdot0205W$ (pounds).

Example 2.—What is the law of the friction curve in Fig. 73?

The straight line drawn meets the vertical axis at $F = 22$ lbs. (note that we use 22 lbs. from the straight line, and not 24 lbs. as given in the table and shown by a cross near the line), hence $c = 22$. Referring again to the line in Fig. 73 (not to the table), at $W = 40$ lbs. $F = 100$ lbs.; hence the difference in level between $W = 0$ and $W = 40$ is $100 - 22$,

$$\text{and } m = \frac{100 - 22}{40} = \frac{78}{40} = 1\cdot95$$

and the law $F = c + mW$ becomes

$$F = 22 + 1\cdot95W$$

or Friction (lbs.) = $22 + 1\cdot95 \times$ (load in lbs.).

We might check the values of the constants by other points thus, reading from the plotted line—

$$\text{For } W = 30,\ F = 81$$
$$\text{and for } W = 10,\ F = 42$$

For the difference of $(30 - 10)$ or 20 lbs. in W the difference in F is $81 - 42 = 39$, hence—

$$m = \frac{\text{difference in F}}{\text{difference in W}} = \frac{39}{20} = 1\cdot95$$

Then substituting in the general relation—

$$F = c + mW$$
$$\text{for } W = \text{say, 30 lbs. and } F = 81$$
$$81 = c + 1\cdot95 \times 39 = c + 58\cdot5$$
$$c = 81 - 58\cdot5$$
$$= 22\cdot5$$

which agrees approximately with 22, the value previously found.

Similarly for any quantity y the values of which when plotted

vertically on a horizontal base scaling values of x give a straight line, the equation is—

$$y = c + mx$$

where c is the value of y at the intersection of the line with the vertical axis, and m is the increase of the vertical value y per unit increase of the horizontal value x. The reader should practise plotting such straight lines as are represented by $y = 5 + 3x$, $y = 1{\cdot}5 + 2{\cdot}3x$, and similar equations by calculating y for, say, any three values of x, and then testing the above rule for the constants. If the line cuts the vertical axis below the origin O, c is a negative quantity.

Further Examples of the Linear Law.—In a certain class of steam engine it is found that by conducting a series of trials at

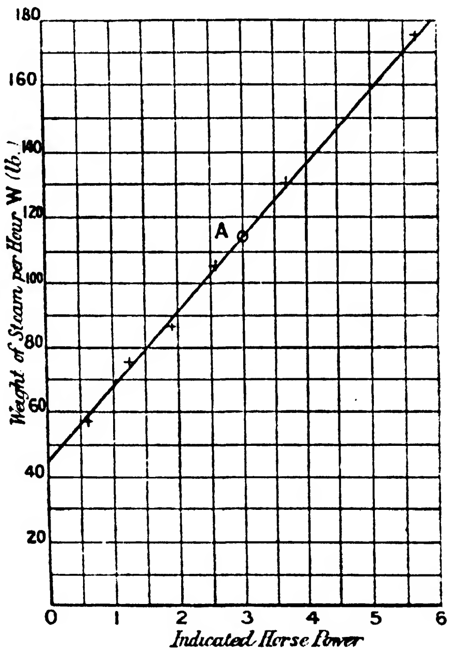


FIG. 85.—Typical example of straight line law.

different loads, and measuring the indicated horse-power (I.H.P.) and the weight of steam used by the engine per hour at each of the different loads, and then plotting these two quantities, the curve so

obtained is a straight line. For example, on such a series of tests with a small engine the following results were obtained :—

I.H.P.	0·56	1·25	1·90	2·57	3·68	5·70
Steam (lbs.) per hour W	75	75	86	105	130	175

Plotting these values the straight line shown in Fig. 85 is obtained.

The line crosses the vertical axis at the point where W = 45; considering any point such as A, we see that an increase in the I.H.P. of 3 corresponds to an increase of W equal to 114 − 45 = 69 lbs.

$$\text{The law is } W = c + m \times \text{I.H.P.}$$

$$\text{and hence } m = \tfrac{69}{3} = 23$$

and the law is—

$$W \text{ (lbs.)} = 45 + 23 \times \text{I.H.P.}$$

If a series of tests similar to the above be carried on on a gas engine, and a curve plotted connecting the "Brake horse-power," and the "number of cubic feet of gas" used per hour, that curve will in many instances be found to be a straight line.

Also the curve connecting the "Brake horse-power" and the "oil used per hour" for an oil engine will usually be a straight line, particulars from such a series of tests being given in Question 9 at the end of this chapter.

EXAMPLES VIII.

1. The following results were obtained from experiments on the coefficient of friction for various dry surfaces :—

W (lbs.).	F for steel on oak (lbs.).	F for brass on oak (lbs.).	F for oak on oak (lbs.).	F for brass on steel (lbs.).
2·5	0·45	0·47	1·20	0·40
5·0	0·80	0·90	2·50	0·75
7·5	1·25	—	3·65	1·10
10·0	1·70	1·80	5·0	1·50
12·5	2·05	2·20	6·30	1·70
15·0	2·40	2·70	7·74	2·25
17·5	2·85	3·10	8·77	2·10
20·0	3·30	3·60	10·05	3·05

Plot the four sets of values of F on the *same* load base W and state the equation

for each set of values of F. What is the value of the coefficient of friction for each of the four pairs of surfaces?

2. The following values of the length of a simple pendulum (l) and the time of one complete vibration (t) were obtained experimentally. Plot the curve connecting l and t^2, and find the constant c in the relation $t^2 = cl$.

l (inches) .	5	8	10	13	15	19	22	25	27
t (seconds)	0·715	0·904	1·011	1·150	1·238	1·398	1·50	1·598	1·661

3. In the following table p = pressure in pounds per square inch, w = weight in pounds per cubic foot of air at pressure p and temperature 32° F. Plot p and w and find the law connecting them ($w = p \times$ a constant).

p lbs. per sq. in.	14·70	30	50	70	90	110	130	150
w pounds	0·0807	0·1647	0·2745	0·3844	0·4941	0·6040	0·7137	0·8235

4. Plot the equations (a) $y = -1·5 + 2·8x$; (b) $P = 18 + 2·4W$.

5. Find the laws of the Effort and Friction curves shown in Fig. 77.

6. The results shown in the table were obtained experimentally from a lifting machine. Plot the two curves connecting P and W and F and W, and find their laws.

Load W (lbs.).	Effort P (lbs.).	Friction F = (PV − W) (lbs.).
0	0·094	2·34
5	0·45	6·32
10	0·81	10·31
15	1·17	14·29
20	1·53	18·28
25	1·88	22·26
30	2·25	26·25
35	2·61	30·24
40	2·97	34·21

7. The following figures show the total heat H (British thermal units) in one pound of steam at temperature t° F. Plot the curve and find the approximate law for H at any value of t.

H .	1146	1158·3	1167·6	1174·3	1179·6	1184·0	1188·7	1192·8	1195·8	1201
t° F	212	250·3	280·9	302·8	320·1	334·6	350·0	363·3	373·2	390·1

8. From a series of tests on a steam engine the following values of the weight

(W) of steam used per hour and the indicated horse-power (I.H.P.) were obtained :—

I.H.P. . .	210	290	334	405	530	600	650
W (lbs.). .	4020	5800	6650	8200	10,800	12,100	13,500

Plot the curve and find the law for W.

9. From a series of tests on an oil engine the following values of the weight of oil used per hour (W) and the brake horse-power (B.H.P.) were obtained :—

B.H.P. .	1·0	2·1	3·0	4·2	4·70	5·3
W (lbs.) .	1·07	2·16	2·85	3·91	4·40	4·90

Plot the curve and find the law for W.

CHAPTER IX

POWER

Power is the rate of doing work, or the work done per unit of time. A large amount of work can be done by a small engine or motor in a long time, but only a powerful engine or motor can do a large amount of work in a small time. Power is measured by the amount of work done per second or per minute, and the unit employed is 550 foot-pounds per second, or $550 \times 60 = 33{,}000$ foot-pounds per minute, which is called one horse-power. The work done in lifting, say, 330 lbs. 100 feet would be $330 \times 100 = 33{,}000$ foot-pounds, and this would be the same in whatever time the operation occupied —an amount of work has no reference to time. If the operation takes one minute, the power required would be one horse-power; if it takes only half a minute, the work per minute is $33{,}000 \div \frac{1}{2} = 66{,}000$ foot-pounds per minute, which is two horse-power. If the operation takes 5 minutes, the work per minute would be $\frac{1}{5} \times 33{,}000$, and the power would be $\frac{1}{5}$ of a horse-power.

To find the horse-power spent in any operation, we divide the work done in foot-pounds per minute by 33,000, or the work done in foot-pounds per second by 550.

Example 1.—What power would be spent in lifting 30 tons through vertical height of 40 feet in 15 minutes?

Total work done in 15 minutes $= 30 \times 2240 \times 40 = 2{,}688{,}000$ ft.-lbs.

Work done per minute $= \frac{2688000}{15} = 179{,}200$ ft.-lbs.

$\left.\begin{array}{l}\text{Horse-power (or number of times}\\ \text{33,000 ft.-lbs. per minute)}\end{array}\right\} = \frac{179200}{33000} = 5{\cdot}43$ horse-power, which is usually written 5·43 H.P.

Example 2.—How many foot-tons of work are done by an engine of 200 horse-power in 5 hours?

$$\text{Work done in 1 minute} = 200 \times 33{,}000 = 6{,}600{,}000 \text{ foot-pounds}$$
$$\text{Work done in 5 hours} = 6{,}600{,}000 \times 60 \times 5 \text{ foot-pounds}$$
$$= \frac{6{,}600{,}000 \times 60 \times 5}{2240} = 883{,}928 \text{ foot-tons.}$$

Example 3.—Find the horse-power required to drive a lift when raising a weight of 2 tons at a speed of 5 feet per second, if the efficiency of the machinery is 70 per cent.

Useful work per second = 2 × 2240 × 5 = 22,400 foot-pounds.

$$\text{Useful H.P.} = \frac{22400}{550}.$$

But this is only $\frac{70}{100}$ of the power required, the remainder being wasted, hence—

$$\frac{70}{100} \times \text{H.P. required} = \frac{22400}{550}$$
$$\therefore \text{horse-power} = \frac{100}{70} \times \frac{22400}{550} = 58{\cdot}2 \text{ H.P.}$$

Example 4.—A locomotive draws a train of total weight 250 tons along a level track at a speed of 40 miles an hour. If the tractive resistance is constant, and equal to 12 lbs. per ton, what horse-power must be developed in the engine cylinders if the efficiency of the machinery is 75 per cent.?

$$\text{40 miles an hour} = \frac{40 \times 5280}{60 \times 60} = \frac{176}{3} \text{ feet per second.}$$

Total resistance = 12 × 250 = 3000 pounds.

Useful work per second = $3000 \times \frac{176}{3}$ = 176,000 foot-pounds.

Useful horse-power = $\frac{176000}{550}$ = 320 H.P.

But this is only $\frac{75}{100}$ of the horse-power required;

$$\therefore \text{horse-power required} = 320 \times \frac{100}{75} = 426{\cdot}6 \text{ H.P.}$$

Power of Engines and Pumps.—The indicated horse-power of an engine is the actual power developed in the engine cylinder by the steam in the case of a steam engine, by the combustion of the gas in a gas engine, and by the combustion of the oil in an oil engine. In the case of a pump it is the actual power *expended* in the pump cylinder in pumping the water. The information required for the calculation of the indicated horse-power is: (1) The mean effective pressure on the engine piston during the stroke. (2) The area of the piston. (3) Length of stroke. (4) Number of working strokes per minute.

Mean or Average Effective Pressure.—This is found from the indicator diagram as follows:—

Let Fig. 86 be the indicator diagram, representing the pressure (in pounds per square inch) on one side of the piston during the stroke. Divide the diagram into, say, 10 strips of equal width, then draw vertical lines through the middle of each strip, as shown. Measure the length of each of these lines with a steel rule. Then the mean height of the diagram is the mean of these lengths. Adding up these lengths and dividing by the number of them give the mean height. If h is the mean height, then—

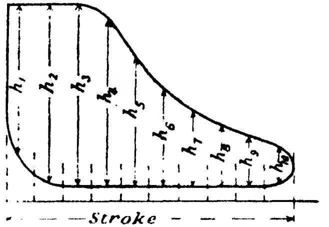


FIG. 86.—Mean effective steam-pressure.

$$h = \frac{1}{10}(h_1 + h_2 + h_3 + h_4 + h_5 + h_6 + h_7 + h_8 + h_9 + h_{10})$$

The mean effective pressure P is found by multiplying the mean height h by the scale of the diagram. If 1-inch vertical height on the diagram represents, say, 40 lbs. per square inch on the engine piston, and the mean height h works out to be, say, 0·90 inch, then mean effective pressure (P) $= 0{\cdot}90 \times 40 = 36$ lbs. per square inch.

The area of the piston in square inches $A = \frac{\pi d^2}{4}$, where d is the diameter in inches, and the length of stroke (L) in feet is measured.

The number of working strokes per minute (N) depends on the type of engine. If the engine is a single-acting steam engine, that is, the steam acts on one side of the piston only, N = the number of revolutions per minute of the engine crank shaft; if double-acting, N = twice the revolutions per minute, and the mean effective pressure is required on each side of the piston, there being two working strokes per revolution. For a gas or oil engine working on the common Otto cycle at full load, there is only *one* working stroke every *two* revolutions; hence, N = half the revolutions per minute.

Now, since P = mean effective pressure in pounds per square inch, and A = area in square inches, we have—

Mean force on the piston during the stroke $= P \times A$ pounds.

Work done per stroke $= P \times A \times L$ foot-pounds.

Work done per minute $= P \times A \times L \times N$ foot-pounds.

Indicated horse-power (I.H.P.) $= \frac{P \times A \times L \times N}{33000}$,

which is usually written in the easily remembered form—

$$\text{I.H.P.} = \frac{\text{PLAN}}{33000}$$

Example 1.—A single-cylinder single-acting steam engine running at 200 revolutions per minute has a cylinder 10 inches diameter and a stroke of 1 foot. If the mean effective pressure is 40 lbs. per square inch, what is the indicated horse-power?

$$\text{Area of piston } A = \frac{\pi d^2}{4} = \frac{3{\cdot}1416 \times 100}{4} = 78{\cdot}54 \text{ square inches.}$$

$$\text{I.H.P.} = \frac{40 \times 1 \times 78{\cdot}54 \times 200}{33000} = 19{\cdot}04 \text{ I.H.P.}$$

Example 2.—A single-cylinder double-acting steam engine running at 250 revolutions per minute has a cylinder 10 inches diameter, stroke 18 inches, and the diameter of the piston rod 2 inches. The mean effective pressure on the back end of the piston is 36 lbs. per square

inch, and on the crank end 40 lbs. per square inch. What is the indicated horse-power?

$$\text{Effective area of piston (back end)} = \frac{3{\cdot}1416}{4} \times 10^2 = 78{\cdot}54 \text{ square inches.}$$

$$\text{Stroke} = 18 \text{ inches} = \tfrac{18}{12} = 1{\cdot}5 \text{ feet.}$$

$$\text{I.H.P. from back end} = \frac{36 \times 1{\cdot}5 \times 78{\cdot}54 \times 250}{33000} = 32{\cdot}13 \text{ H.P.}$$

$$\begin{aligned}\text{Effective area of piston (crank end)} &= \text{area of cylinder} - \text{area of piston rod}\\ &= 78{\cdot}54 - \frac{3{\cdot}1416}{4} \times 2^2\\ &= 75{\cdot}40 \text{ square inches (practically).}\end{aligned}$$

$$\text{I.H.P. from crank end} = \frac{40 \times 1{\cdot}5 \times 75{\cdot}40 \times 250}{33000} = 34{\cdot}27 \text{ H.P.}$$

$$\begin{aligned}\text{Hence total indicated horse-power} &= 32{\cdot}13 + 34{\cdot}27\\ &= 66{\cdot}40 \text{ I.H.P.}\end{aligned}$$

The indicated horse-power could also be found as follows :—

$$\left.\begin{array}{l}\text{Average mean pressure on}\\ \text{both sides of piston}\end{array}\right\} = \frac{36 + 40}{2} \text{ lbs. per sq. inch}$$

$$= \tfrac{76}{2} = 38 \text{ lbs. per sq. inch.}$$

$$\left.\begin{array}{l}\text{Average effective area of}\\ \text{both sides of piston}\end{array}\right\} = \frac{78{\cdot}54 + 75{\cdot}40}{2}$$

$$= \frac{153{\cdot}94}{2} = 76{\cdot}97 \text{ square inches.}$$

$$\text{Working strokes per minute} = 250 \times 2 = 500$$

$$\text{I.H.P.} = \frac{38 \times 1{\cdot}5 \times 76{\cdot}97 \times 500}{33000} = 66{\cdot}4 \text{ H.P.}$$

Mechanical Efficiency.—All the I.H.P is not available for useful purposes, because some fraction of it is lost in overcoming the friction in the engine itself. The quantity

I.H.P − horse-power lost in engine friction

is called the Brake or Effective horse-power of the engine. The ratio $\dfrac{\text{Brake horse-power}}{\text{Indicated horse-power}}$ is called the Mechanical Efficiency of the engine and is always a proper fraction less than unity, being often expressed as a percentage thus—

$$\text{Mechanical Efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}} \times 100 \text{ per cent.}$$

In the case of a pump the efficiency is—

$$\frac{\text{Useful horse-power done in pumping water}}{\text{Actual horse-power expended in driving the pump}}$$

or—

$$\frac{\text{Useful work done by the pump on the water}}{\text{Actual work expended in driving the pump}}$$

Example 1.—A single-cylinder gas engine has a cylinder 9·5 inches diameter, stroke 19 inches, mean effective pressure 106 lbs. per square inch, number of explosions per minute 77. If the mechanical efficiency of the engine is 86·4 per cent., what is the brake horse-power?

$$\text{Area of piston} = \frac{\pi d^2}{4} = \frac{3{\cdot}1416 \times 9{\cdot}5 \times 9{\cdot}5}{4} = 71 \text{ square inches,}$$

$$\text{Stroke} = \tfrac{19}{12} \text{ feet,}$$

$$\therefore \text{ I.H.P.} = \frac{106 \times \frac{19}{12} \times 71 \times 77}{33000} = 28{\cdot}03 \text{ H.P.}$$

$$\text{Brake horse-power} = 28{\cdot}03 \times \frac{86{\cdot}4}{100} = 24{\cdot}22 \text{ B.H.P.}$$

Example 2.—A pump delivers 1000 gallons of water to a height of 100 feet every minute. If its efficieney is 70 per cent., what horse-power will be required to drive it? (A gallon of water weighs 10 lbs.)

$$\left.\begin{array}{c}\text{Work done by the pump on the}\\ \text{water per minute}\end{array}\right\} = 1000 \times 10 \times 100 \text{ foot-pounds,}$$

$$\text{Useful horse-power done in pumping} = \frac{1000 \times 10 \times 100}{33000} = 30{\cdot}3$$

But this is only $\frac{70}{100}$ of that required to drive the pump;

$$\therefore \text{ horse-power required to drive pump} = 30{\cdot}3 \times \tfrac{100}{70}$$
$$= 43{\cdot}23 \text{ H.P.}$$

Example 3.—An engine working at 5 brake horse-power drives a pump which delivers 30,000 gallons of water per hour to a height of $24\frac{1}{2}$ feet. What is the efficiency of the pump?

$$\text{Water delivered per minute} = \tfrac{30000}{60} = 500 \text{ gallons}$$
$$= 500 \times 10 \text{ or } 5000 \text{ lbs.}$$

$$\left.\begin{array}{c}\text{Useful work done by pump}\\ \text{per minute}\end{array}\right\} = 5000 \times 24{\cdot}5 = 122{,}500 \text{ foot-pounds,}$$

$$\text{Useful horse-power of pump} = \tfrac{122500}{33000} = 3{\cdot}71 \text{ H.P.}$$

$$\text{Hence, efficiency of pump} = \frac{\text{Useful horse-power in pumping}}{\text{Actual horse-power expended}}$$

$$= \frac{3{\cdot}71}{5} = 0{\cdot}742 \text{ or } 74{\cdot}2 \text{ per cent.}$$

Cylinder Volumes and Power.

We have seen (page 110) that the work done per stroke is—

$$P \times A \times L \text{ foot-pounds}$$

Suppose P = mean pressure in pounds per *square foot*,
A = area of piston in *square feet*,
L = length of stroke in feet as before.

Now, the product $A \times L$ is equal to the volume swept out by the piston per stroke; let this be equal to V, then we see that work done per stroke = $P \times V$ foot-pounds = PV foot-pounds, where P is the mean pressure in pounds per square foot, and V the volume swept out by the piston in cubic feet.

Consider, now, an engine supplied with water under the constant pressure P pounds per square foot, and suppose the volume (V cubic feet) swept out by the piston is equal to 1 cubic foot, we have the result that the work done by 1 cubic foot of water is equal to P foot-pounds, and generally, the work done by V cubic feet of water is equal to—

$$PV \text{ foot-pounds}$$

or, in other words, the energy stored up in V cubic feet at a pressure of P pounds per square foot is equal to—

$$PV \text{ foot-pounds.}$$

Example 1.—What will be the brake horse-power of a hydraulic engine which uses 2000 gallons of water per hour at a pressure of 700 lbs. per square inch, the mechanical efficiency being 72 per cent.?

Since 1 gallon of water weighs 10 lbs., and 1 cubic foot of water weighs 62·5 lbs., we have—

$$\text{Number of gallons in 1 cubic foot} = \frac{62{\cdot}5}{10} = 6{\cdot}25, \text{ and}$$

$$\left.\begin{array}{r}\text{Number of cubic feet of water}\\ \text{used per hour}\end{array}\right\} = \frac{2000}{6{\cdot}25}$$

$$\left.\begin{array}{r}\text{Number of cubic feet of water}\\ \text{used per minute}\end{array}\right\} = \frac{2000}{6{\cdot}25 \times 60}\text{ cubic feet.}$$

$$\text{Pressure per square foot} = 700 \times 144 \text{ lbs.}$$

$$\begin{aligned}\text{Work supplied per minute to the engine} &= PV \text{ foot-pounds}\\ &= 700 \times 144 \times \frac{2000}{6{\cdot}25 \times 60}\text{ ft.-lbs.}\\ &= 537{,}600 \text{ foot-lbs.}\end{aligned}$$

$$\text{Hence, horse-power supplied} = \tfrac{537600}{33000} = 16{\cdot}29 \text{ H.P.}$$

But of this only 75 per cent. or $\frac{75}{100}$ is available, the remainder being wasted.

$$\text{Hence, brake horse-power} = 16{\cdot}29 \times \tfrac{75}{100} = 12{\cdot}2 \text{ B.H.P.}$$

Example 2.—A hydraulic company supply water at a pressure of 700 lbs. per square inch. If they charge 1*s.* 6*d.* per 1000 gallons, what will be the cost of one horse-power for one hour?

$$\text{Number of cubic feet in 1000 gallons} = \frac{1000}{6{\cdot}25} = 160 \text{ cubic feet}$$

$$\left.\begin{array}{r}\text{Work stored in 1000 gallons or}\\ \text{160 cubic feet}\end{array}\right\} = 700 \times 144 \times 160 \text{ foot-pounds}$$

$$\text{Now, one horse-power for one hour} = 33{,}000 \times 60 \text{ foot-pounds}$$

$$\left.\begin{array}{l}\text{Hence, the number of thousands}\\ \quad\text{of gallons required per hour}\\ \quad\text{per horse-power}\end{array}\right\} = \frac{33000 \times 60}{700 \times 144 \times 160} = 0{\cdot}122, \text{ and}$$

$$\begin{aligned}\text{Cost of one horse-power per hour} &= 0{\cdot}122 \times 18\\ &= 2{\cdot}196 \text{ pence}\end{aligned}$$

Brake Horse-Power and its Measurement.—We have already seen that the B.H.P. = I.H.P. minus horse-power lost in friction, and is the useful power obtained from the engine, or the actual power which can be taken from the crank shaft.

In order to measure the brake horse-power an apparatus called a *Dynamometer* is used. There are two forms of dynamometers in common use, namely, *Absorption Dynamometers* and *Transmission Dynamometers*.

Absorption Dynamometers.—In this type all the brake horse-power is absorbed by the dynamometer or brake, the work

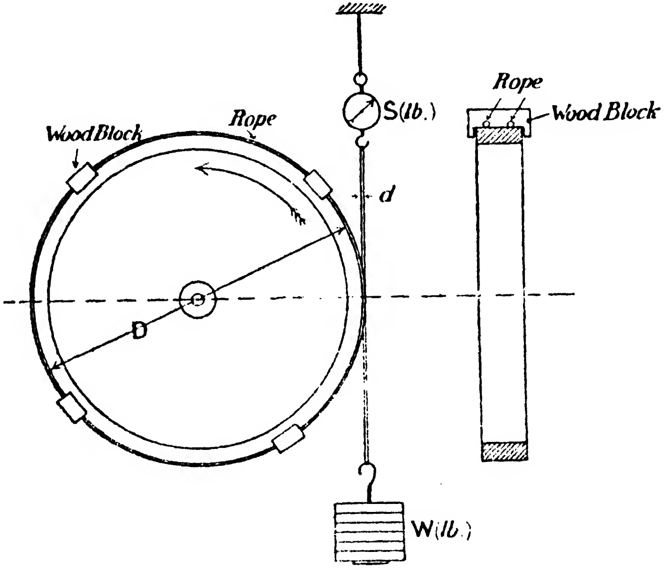


FIG. 87.—Rope brake, an absorption dynamometer.

being wasted in friction. If the engine is of moderate size a very common and effective arrangement is to put a rope brake on the fly-wheel of the engine, as shown in Fig. 87. The brake consists of two or more ropes (depending on the power to be absorbed) of equal length which pass once round the fly-wheel, one end being attached to a spring balance, and on the other end is hung a weight W. The ropes are prevented from slipping off the wheel by means of the wood blocks shown. The directing of rotation being as shown by the arrow, it will be seen that W *opposes* rotation, but the pull of the

spring balance S is in the direction of rotation. Hence, the effective frictional load on the rim of the wheel opposing rotation is W − S pounds.

Let D = diameter of wheel in feet
d = diameter of rope in feet
W = dead load in pounds
S = reading of spring balance in pounds
N = revolutions per minute made by the wheel.

In one revolution the work absorbed in friction between the ropes and the rim of the wheel in foot-pounds is—

$$\text{Nett load (pounds)} \times \text{Effective circumference of brake (feet)}$$
$$= (W - S) \times \pi(D + d) \text{ foot-pounds.}$$

$$\left.\begin{array}{r}\text{Work absorbed per}\\ \text{minute}\end{array}\right\} = (W - S) \times \pi(D + d) \times N \text{ foot-pounds}$$

$$\text{Brake horse-power} = \frac{(W - S) \times \pi(D + d) \times N}{33000} \text{ B.H.P.}$$

or stated in words—

$$\text{B.H.P.} = \frac{\text{Nett load} \times \text{speed in feet per minute}}{33000}$$

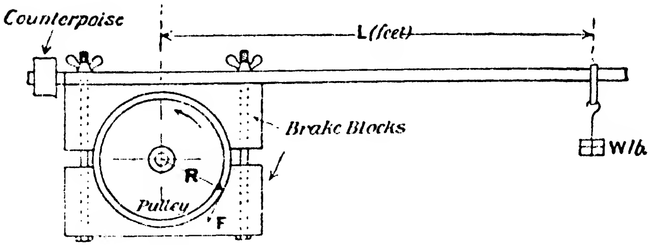


FIG. 88.—Prony brake, an absorption dynamometer.

In carrying out a test there is no need to measure the diameters of the wheel and rope. The effective circumference $\pi(D + d)$ is best found by passing a steel tape round the wheel and measuring the circumference in feet, then measure the circumference of the rope in the same way and add the two together; this will give πD(circumference of wheel) + πd(circumference of rope) or $\pi(D + d)$ feet.

Prony Brake.—Another form of the absorption dynamometer is the Prony brake, the principle of which will be understood from Fig. 88. A pulley on the shaft of the engine or motor is fitted with two brake blocks which can be pressed on to the pulley as

hard as may be required by the thumb screws shown. A weight W can be placed at a convenient point on the lever. When making a test, the pressure of the blocks on the pulley is adjusted by the thumb screws so that the lever carrying the weight W rests horizontally. To balance the weight of the lever itself a counterpoise is sometimes fitted; if the counterpoise is not fitted then an allowance must be made for the weight of the lever when calculating the brake horse-power.

Since the lever rests in equilibrium in a horizontal position the turning moment on the shaft in a contra-clockwise direction in Fig. 88 must be balanced by the moment of the frictional resistance of the blocks on the pulley in a clockwise direction. The moment of the friction resistance is—

$$W \times L \text{ pound-feet}$$

or—

$$F \times R$$

where F = frictional resistance in pounds, and R = the radius of the pulley in feet.

Hence the torque (p. 63) or turning moment on the shaft is—

$$W \times L \text{ pound-feet.}$$

$$\text{Work done in one revolution} = \text{torque} \times \text{angle in radians (p. 63)}$$
$$= WL \times 2\pi \text{ foot-pounds.}$$

$$\text{Work done per minute} = WL \times 2\pi N \text{ foot-pounds}$$

where N = revolutions per minute;

$$\text{Brake horse-power} = \frac{WL \times 2\pi N}{33{,}000}$$

or stating it in words—

$$\text{B. H. P.} = \frac{\text{torque (in pound-feet)} \times \text{radians per minute}}{33{,}000}$$

Heating of the Brake.—The energy absorbed by the brake due to friction is transformed into heat, resulting therefore in a rise in temperature. For engines that have to run on the brake for a considerable length of time a special brake pulley is usually fitted. This pulley has a hollow rim which is kept full of water, and prevents the temperature rising to a dangerous value.

Transmission Dynamometer.—In this type of dynamometer the energy is not wasted in friction, but is used for doing work in the usual way, the apparatus forming a connection between the engine, motor, or driving shaft and the machines it may be driving. The energy is transmitted through the dynamometer from the source of power to the machines, and the brake horse-power can be measured whilst being utilized for useful purposes.

Fig. 89 shows the Froude or Thorneycroft dynamometer. The pulley A on the engine shaft drives the pulley D on the machine by a belt which passes also round the two pulleys B and C as shown. The pulleys B and C are on the frame BCF which is pivoted at E. The tension or pull T_1 in the tight side of the belt is greater than the tension T_2 on the slack side; hence the total force $2T_1$ acting at B is greater than $2T_2$ acting at C. This then would cause movement of the frame BCF about E in a contra-clockwise direction, any rotation is, however, prevented by a weight W hung at a point on the lever EF at a distance L feet from E. Hence, we see that when the frame BCF is in equilibrium, neglecting friction the turning moment on it in a contra-clockwise direction due to the unequal

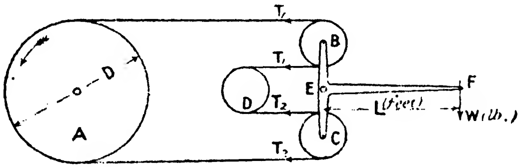


FIG. 89.—Transmission dynamometer.

forces $2T_1$ and $2T_2$, is balanced by the moment in a clockwise direction due to the weight W.

The contra-clockwise moment $= 2T_1 \times BE - 2T_2 \times EC$, and if BE is the same length as EC, T_1 and T_2 being measured in pounds and BE in feet, we have—

$$\text{Contra-clockwise moment} = 2T_1 \times BE - 2T_2 \times BE \text{ pound-feet}$$
$$= 2BE(T_1 - T_2) \text{ pound-feet.}$$

Clockwise moment $= W \times L$ pound-feet, and these are equal—

hence $$2BE(T_1 - T_2) = W \times L$$

and $$T_1 - T_2 = \frac{W \times L}{2 \times BE} \text{ lbs.}$$

Let D = diameter of pulley A in feet, and N = revolutions of A per minute.

$$\text{Then work done in one revolution} = (T_1 - T_2) \times \pi D \text{ foot-pounds}$$
$$\text{Work done in one minute} = (T_1 - T_2)\ \pi DN \text{ foot-pounds}$$
$$\text{Brake horse-power} = \frac{(T_1 - T_2)\pi DN}{33{,}000} \text{ or } \frac{W.L.\pi.D.N}{BE \times 66{,}000}$$

Actually the brake horse-power transmitted to the machine would differ from this amount because of the friction in the joints. The true values of $T_1 - T_2$ for various weights W can be found by

applying an absorption dynamometer to pulley D. If a few values of $T_1 - T_2$ are found, and a curve plotted on a base of loads W, other values of $T_1 - T_2$ for intermediate values of W can be found from the curve.

Example 1.—The following data was obtained in a test on an engine with the rope brake shown in Fig. 87. Circumference of brake wheel 117·875 inches, of rope 2·375 inches, dead load W = 520 lbs.; reading of spring balance S = 234 lbs., average speed 252 revolutions per minute: find the brake horse-power.

$$\begin{aligned}\text{Effective circumference of brake} &= 117{\cdot}875 + 2{\cdot}375 \\ &= 120{\cdot}25 \text{ inches} \\ &= \frac{120{\cdot}25}{12} = 10{\cdot}02 \text{ feet.}\end{aligned}$$

$$\text{Net load on brake } (W - S) = 520 - 234 = 286 \text{ lbs.}$$

$$\text{Brake horse-power} = \frac{286 \times 10{\cdot}02 \times 252}{33{,}000} = 21{\cdot}88 \text{ B.H.P.}$$

Example 2.—In a test with a Prony brake similar to Fig. 88, but without a counterpoise, the following particulars were obtained: The weight of the brake is equivalent to 7 lbs. on the lever at a distance of 4·35 feet from the engine shaft. The load W was 20 lbs., distance L was 4·35 feet, average speed of engine 170 revolutions per minute. Find the brake horse-power.

$$\begin{aligned}\text{Total torque} &= 20 \times 4{\cdot}35 + 7 \times 4{\cdot}35 \\ &= 4{\cdot}35 \times 27 = 117{\cdot}46 \text{ pound-feet.}\end{aligned}$$

$$\begin{aligned}\text{Brake horse-power} &= \frac{\text{torque} \times 2\pi N}{33{,}000} \\ &= \frac{117{\cdot}45 \times 2 \times 3{\cdot}1416 \times 170}{33{,}000} = 3{\cdot}80 \text{ B.H.P.}\end{aligned}$$

Example 3.—The Froude transmission dynamometer shown in Fig. 89 has BE = EC = 1 foot, EF = 6 feet, diameter of pulley A = 3 feet, speed of A = 250 revolutions per minute; W = 50 lbs. What is the horse-power transmitted?

Using the equation deduced on page 117,

$$T_1 - T_2 = \frac{W \times L}{2 \times BE} = \frac{50 \times 6}{2 \times 1} = 150 \text{ lbs.}$$

$$\text{Work done in 1 revolution} = 150 \times \pi \times 3 = 450 \times 3{\cdot}1416 \text{ foot-pounds}$$

$$\therefore \text{ Horse-power} = \frac{450 \times 3{\cdot}1416 \times 250}{33{,}000} = 10{\cdot}71 \text{ H. P.}$$

Electrical Power.—Many modern machine tools are driven directly by electric motors, and in order to estimate the power supplied to the motor (which corresponds to the indicated horse-power of an engine) we must have a voltmeter which registers the pressure of supply in "*volts*," and an ammeter which registers the current in "*ampères*." The electrical engineer's unit of work is the "*joule*," which is the work done when one *ampère* flows for one

second under a pressure of one *volt*; it is numerically equal to 0·7373 foot-pound. The electrical unit of power is the "*watt*," which is the rate of working when a steady current of one *ampère* flows under a pressure of one *volt*, and is equal to *one joule per second*. For example, if a motor takes a current of 100 ampères at a pressure of 200 volts, the watts supplied = 100 × 200 = 20,000 watts, or—

$$\text{Watts} = \text{ampères} \times \text{volts}.$$

It should be carefully remembered that the product of *ampères* and *volts* gives *power*, that is, the rate of doing work in joules per second, whereas the product of pounds and feet in mechanical units gives foot-pounds—being independent of time as already explained (p. 108).

Electrical Horse-power.—There are 746 watts in one horse-power, so that in order to convert watts into horse-power, we divide the number of watts by 746 thus—

$$\text{Electrical horse-power (usually written E.H.P)} = \frac{\text{number of watts}}{746}$$

or—

$$\text{E.H.P.} = \frac{\text{ampères} \times \text{volts}}{746}$$

The Kilowatt.—In estimating a large power the watt is an inconveniently small unit, and the practical unit for such purposes is 1000 watts, being called *one kilowatt* (K.W.), so that—

$$\text{Kilowatts} = \frac{\text{watts}}{1000} = \frac{\text{ampères} \times \text{volts}}{1000}$$

The relation between the kilowatt and the horse-power is rather important, and is—

$$\text{One kilowatt} = \tfrac{1000}{746} = 1·34 \text{ H.P.}$$

Board of Trade Unit.—The Board of Trade unit of electric supply is one kilowatt for one hour or one kilowatt hour, which is a quantity of work.

Example 1.—What is the equivalent in watts of the power spent in lifting 30 tons through a vertical height of 40 feet in 15 minutes?

We have seen in Example 1, p. 108, that the rate of working is 5·43 H.P.

Hence the equivalent in watts = 5·43 × 746 = 4050 watts.

Example 2.—How many joules are done by a motor of 200 H.P. in 5 hours?

Turning to Example 2, p. 108, we see that the work done in 5 hours is 6,600,000 × 60 × 5 foot-pounds.

$$\begin{aligned}\text{Hence the equivalent in joules} &= 6{,}600{,}000 \times 60 \times 5 \div 0·7373 \\ &= 2{,}686{,}000{,}000 \text{ joules.}\end{aligned}$$

Example 3.—A train of total weight 250 tons is drawn by an electric locomotive along a level track at a speed of 40 miles an hour. If the tractive resistance is constant and equal to 12 lbs. per ton, what will be the cost of running the train for one hour if the efficiency of the motors is 75 per cent., and the price of 1 Board of Trade unit is $1\frac{1}{2}d.$?

Turning to Example 4, p. 109, we see that the horse-power supplied to the motors (corresponding to the indicated horse-power in that example) must be 426·6 E.H.P., and this is equal to—

$$426·6 \times 746 \text{ watts}$$

or

$$\frac{426·6 \times 746}{1000} \text{ kilowatts.}$$

Hence, in running for one hour, the number of kilowatt-hours or Board of Trade units will be—

$$\frac{426·6 \times 746}{1000} = 318·24$$

and the cost at $1\frac{1}{2}d.$ per unit will be—

$$318·24 \times 1·5 \text{ pence} = £1\ 19s.\ 9d.$$

Example 4.—In Example 3, what will be the current taken by the motors if the pressure of supply is 500 volts?

$$\text{the watts supplied} = 426·6 \times 746$$
$$\text{and watts} = \text{volts} \times \text{ampères}$$
$$\text{hence ampères} = \frac{\text{watts}}{\text{volts}} = \frac{426·6 \times 746}{500} = 636·4 \text{ ampères.}$$

Example 5.—An engine delivering 250 B.H.P. drives a dynamo which generates 890 ampères at a pressure of 200 volts; what is the efficiency of the dynamo?

$$\text{Watts given out by the dynamo} = 890 \times 200 \text{ watts}$$
$$\text{Watts supplied to drive the dynamo} = 250 \times 746 \text{ watts}$$
$$\text{efficiency of dynamo} = \frac{\text{watts given out}}{\text{watts supplied}} = \frac{890 \times 200}{250 \times 746}$$
$$= 0·95 \text{ or } 95 \text{ per cent.}$$

The problem might also be solved as follows:—

$$\text{E.H.P. of dynamo} = \frac{890 \times 200}{746}$$
$$\text{H.P. supplied to drive it} = 250$$
$$\text{efficiency} = \frac{890 \times 200}{746 \times 250} = 0·95 \text{ as before.}$$

Example 6.—A pump driven by an electric motor delivers 100 gallons of water to a height of 100 feet every minute. If the efficiency of the pump is 70 per cent., and the motor takes 19 ampères, the pressure of supply being 200 volts, what is the efficiency of the motor?

Work done by the pump on the water per minute = $100 \times 10 \times 100$ foot-pounds.

$$\text{Useful horse-power done in pumping} = \frac{100 \times 10 \times 100}{33000} = 3·03 \text{ H.P.}$$

But this is only $\frac{70}{100}$ of that required to drive the pump;

$\therefore$ horse-power required to drive the pump,

$$\left.\begin{array}{l}\text{that is, the B.H.P.}\\ \text{of the motor}\end{array}\right\} = 3{\cdot}03 \times \frac{100}{70} = 4{\cdot}32 \text{ B.H.P.}$$

$$\text{Power supplied to the motor} = 19 \times 200 \text{ watts}$$

$$= \frac{19 \times 200}{746} = 5{\cdot}09 \text{ E.H.P.}$$

$$\text{Efficiency of motor} = \frac{\text{output}}{\text{input}} = \frac{\text{B.H.P.}}{\text{E.H.P.}} = \frac{4{\cdot}32}{5{\cdot}09}$$

$$= 0{\cdot}848 \text{ or } 84{\cdot}8 \text{ per cent.}$$

EXAMPLES IX.

1. What power would be spent in lifting 20 tons through a vertical height of 50 feet in 5 minutes?

2. A man weighs 168 lbs., and carries a cycle of weight 28 lbs. up a flight of steps 25 feet high in $\frac{1}{2}$ minute; at what horse-power does he work?

3. How many foot-tons of work are done by an engine of 115 horse-power in $2\frac{1}{2}$ hours?

4. A motor car weighs 2 tons. What must be the indicated horse-power of the engine when moving on the level at 30 miles an hour, the tractive resistance being 30 lbs. per ton and the mechanical efficiency of the engine 78 per cent.?

5. A single-cylinder gas engine has a cylinder 7 inches diameter, stroke 12 inches. The number of explosions per minute is 80, and the mean effective pressure is 65 lbs. per square inch. Find the indicated horse-power.

6. The diameter of a steam engine cylinder is 40 inches, and of the piston-rod 5 inches, and the stroke is 5 feet. The mean effective pressure on the back end of the piston is 40 lbs. per square inch, and on the crank end 42 lbs. per square inch. If the speed of the engine is 120 revolutions per minute, what is the indicated horse-power?

7. A single-cylinder oil engine has a cylinder 7 inches in diameter, stroke 13 inches, mean effective pressure 41 lbs. per square inch, number of explosions per minute 90, calculate the brake horse-power if the mechanical efficiency of the engine is 85·4 per cent.

8. A pump empties a shaft 10 feet in diameter and 250 feet deep which is full of water in 60 minutes. If the efficiency of the pump is 70 per cent., what horse power will be required to drive it?

9. A pump delivers 5000 gallons of water to a height of 150 feet every 5 minutes. If its efficiency is 75 per cent., what horse-power will be required to drive it?

10. An engine of 120 I.H.P. and efficiency 85 per cent. drives a pump which delivers 2000 gallons of water per minute to a height of 120 feet; what is the efficiency of the pump?

11. A hydraulic motor develops 25 B. H. P. with a mechanical efficiency of 75 per cent. It is supplied with water under a pressure of 700 lbs. per square inch. How many gallons will it require per minute?

12. What will be the cost of running the motor in Question 11 for 1 hour if the price of 1000 gallons is 6*d.*?

13. The following data was obtained in a test on an engine with the rope brake shown in Fig. 87: Circumference of brake wheel 14 feet $1\frac{1}{4}$ inches, of rope 2 inches; dead load W = 70 lbs.; reading of spring balance 10 lbs.; speed 168 revolutions per minute. What is the brake horse-power?

14. In a B.H.P. test the circumference of the pulley is 20 feet, and of the rope 2·4 inches. The effective load on the brake (W – S) was 795·4 lbs.; average speed 209·5 revolutions per minute. Find the B.H.P.

15. In a test with a Prony brake, the weight of the brake is equivalent to 7 lbs. 52·2 inches from the engine shaft. The load W on the lever was 55 lbs. 4·35 feet from the engine shaft. Find the brake horse-power if the speed is 200 revolutions per minute.

16. What is the equivalent in watts of the power spent lifting 10 tons through a vertical height of 15 feet in 2 minutes?

17. Express in joules the work done by a motor of 5 H.P. in one hour.

18. An electric tram car weighs 5 tons, the tractive resistance being 12 lbs. per ton on the level. If the efficiency of the motors and gearing is 65 per cent., what current will be taken at a speed of 15 miles an hour if the pressure of supply is 500 volts? What will be the cost of running the car a distance of one mile if the price is 1*d*. per Board of Trade unit?

19. The output of a dynamo is 250 ampères at 210 volts. The dynamo is driven directly by means of a steam engine whose efficiency is 85 per cent. If the efficiency of the dynamo is 93 per cent., what must be the indicated horse-power of the engine?

20. A pump is driven by an electric motor taking 25 ampères at 210 volts, and whose efficiency is 85 per cent. If the efficiency of the pump is 70 per cent., how many gallons of water will it deliver per minute to a height of 50 feet?

CHAPTER X

TRANSMISSION OF MOTION AND POWER

Belt-driving.—One of the commonest methods of driving a shaft or machine from a parallel shaft at a moderate distance away is by means of a flat belt, or strap, running over pulleys. Belts are made of leather and various special compositions.

Fig. 90 illustrates an open-belt drive, in which a pulley A, called the *driver*, drives another pulley B, called the *driven* pulley, or follower, in the same direction. In the crossed-belt drive, shown in

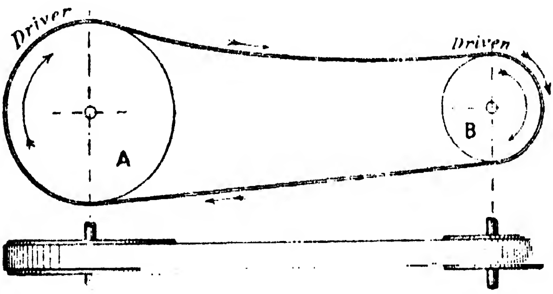


FIG. 90.—Open-belt drive.

Fig. 91, the *driver* A drives the *driven* pulley B in the opposite direction to its own motion. The grip between the pulleys and the belt is obtained by the friction which arises from the pressure between the belt and the pulleys. The pressure is increased by tightening the belt, and this increases the frictional grip.

Velocity Ratio.—If the driver A has a diameter of, say, 15 inches, and the driven pulley B has a diameter of 9 inches, and if there is no slipping, during one revolution of the driver its circumference and the belt move through 15π inches. If there is no slipping at the driven end carrying the follower B, this must also move

15π inches. But 9π inches of B is one revolution, so that the revolutions made by B while A makes one revolution are—

$$\frac{15\pi}{9\pi} = \frac{15}{9} = \frac{5}{3}, \text{ or } 1 \cdot 66.$$

That is—

$$\frac{\text{speed of pulley B}}{\text{speed of pulley A}} = 1 \cdot 6 = \text{velocity ratio of B to A}.$$

If the diameter of A is called D, and that of B is called d, and the revolutions of A per minute are called N, the revolutions of B being called n per minute—

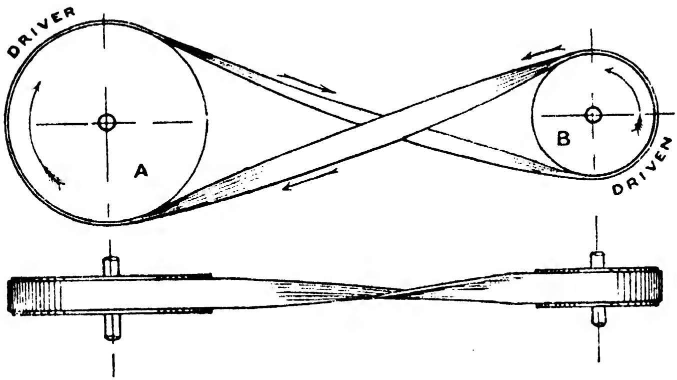


FIG. 91.—Crossed-belt drive.

Then for one revolution of A the belt moves $\pi \times$ D,
for each revolution of B the belt moves $\pi \times d$,

and for each revolution of A the revolutions of d are $\dfrac{\pi D}{\pi d} = \dfrac{D}{d}$

While A makes N revolutions (in 1 minute), B makes $\dfrac{D}{d}$ times as many. That is—

$$n = N \times \frac{D}{d}$$

or

$$\frac{N}{n} = \frac{d}{D}$$

or, in words, the velocity ratio—

$$\frac{\text{revolutions per minute of follower}}{\text{revolutions per minute of driver}} = \frac{\text{diameter of driver}}{\text{diameter of follower}}$$

Unless the belt is very thin compared with the diameters of the pulleys, the effective diameters of the pulleys should be reckoned to the centre of the thickness of the belt.

Slipping of Belts.—When a belt is transmitting power the frictional grip may prove insufficient, and some slipping *forward* of the driver may take place without carrying the belt with it; and some slipping *forward* of the belt over the follower may also take place. Apart from this slipping, there is a small, but continuous "creeping" of the belt on the pulleys, due to unequal stretching. A belt and pulley, then, do not give what is called a "positive" drive, such as is obtained from chains, which cannot slip; and belts should never be employed when an exact velocity ratio is of importance.

Example 1.—An engine drives a line of shafting by means of a belt. The diameter of the pulley on the engine shaft is 54 inches, and that on the shafting is 33 inches. If the speed of the engine is 140 revolutions per minute, what will be the speed of the shafting?

In this example the driving pulley is 54 inches diameter, and the follower is 33 inches diameter. Hence, for each revolution of the driver the follower makes $\frac{54}{33}$ revolutions, and the speed of the follower is—

$$140 \times \frac{54}{33} = 229 \text{ revolutions per minute.}$$

Example 2.—It is required to drive a shaft A at 620 revolutions per minute by means of a belt from a parallel shaft having a pulley B 30 inches diameter on it, and running at 240 revolutions per minute. What sized pulley will be required on the shaft A?

Let d = diameter of pulley required. Then for one revolution of B, A will make $\frac{30}{d}$ revolutions, and the speed of A will be $\frac{30}{d} \times 240$; and this is equal to 620 revolutions per minute.

Hence $$\frac{30 \times 240}{d} = 620$$

$$d = \frac{30 \times 240}{620} = 11 \cdot 6 \text{ inches.}$$

Power transmitted by Belts.—The follower B, Fig. 92, is pulled round by the frictional grip of the belt, because the pull of the belt on one side is greater than that on the other side. The turning moment exerted on the follower by the belt in one direction is greater than that in the other, hence work is done.

Let T_1 = tension in the tight side of the belt in pounds,

T_2 = tension in the slack side of the belt in pounds.

The effective turning force at the circumference of the follower is the difference of the forward and backward tensions, or the difference of the tensions in the tight and slack sides of the belt. The effective turning moment is the difference of the tensions multiplied by the radius of the follower.

In 1 foot motion of the belt the work done at the circumference of the following or driven pulley is—

difference of tensions in pounds × 1 foot-pound.

And if the belt moves V feet in 1 minute, the work done in one minute is—

difference of tensions × V foot-pounds.

Now T_1 is the greater tension on the "tight" side, and T_2 the

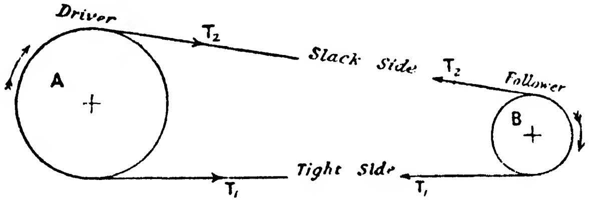


FIG. 92.

smaller tension on the "slack" side, the difference of tensions is $(T_1 - T_2)$ pounds, and the work transmitted per minute is—

$$(T_1 - T_2)V \text{ foot-pounds.}$$

Similarly, the driver A does this amount of work per minute on the belt.

The horse-power V transmitted, then, is—

$$\text{H P.} = \frac{(T_1 - T_2)V}{33000}$$

Example 1.—The tensions in the two sides of a belt are 120 lbs. and 50 lbs. respectively. If the speed of the driver is 240 revolutions per minute, and its diameter is 4 feet, what horse-power is transmitted by the belt?

Circumference of driver $= 4 \times \pi = 4\pi$ feet.

$$\text{Speed of belt} = \begin{cases}\text{circumference of driver} \times \text{revolutions of} \\ \quad \text{driver per minute}\end{cases}$$

$$= 4\pi \times 240 \text{ feet per minute.}$$

$$\text{H.P.} = \frac{(120-50)4\pi \times 240}{33000} = \frac{70 \times 4\pi \times 4}{550} = 6{\cdot}4 \text{ H.P.}$$

Example 2.—The speed of a belt is 2500 feet per minute, and it transmits 85 horse-power. Find the difference of the tensions in the two sides of the belt. Also, if the tension in the tight side is $2\frac{1}{2}$ times the tension in the slack side, find the two tensions and the width of belt required if the maximum tension is not to exceed 80 lbs. per inch width of belt.

$$\text{H.P.} = \frac{(T_1 - T_2)V}{33000}$$

Substituting for the horse-power and V, we have—

$$85 = \frac{(T_1 - T_2)2500}{33000}$$

$$T_1 - T_2 = \frac{85 \times 33000}{2500} = \frac{85 \times 66}{5} = 1122 \text{ lbs.}$$

Now, the tension in the tight side T_1 is to be $2\frac{1}{2}$ times the tension T_2 in the slack side, that is $T_1 = 2\cdot5T_2$.

Hence $$2\cdot5T_2 - T_2 = 1122$$
$$1\cdot5T_2 = 1122$$
$$T_2 = \frac{1122}{1\cdot5} = 748 \text{ lbs. in the slack side}$$
$$\text{and } T_1 = 748 \times 2\cdot5 = 1870 \text{ lbs. in the tight side.}$$

Now, the maximum tension allowed is 80 lbs. per inch width of belt, so that width of belt $= \frac{1870}{80} = 23\cdot4$ inches.

Example 3.—The width of a belt is 6 inches, and the maximum tension per inch width is not to exceed 80 lbs. The ratio of tensions on the two sides is $2\frac{1}{4}$, the diameter of the driver 3 feet 6 inches, and it makes 220 revolutions per minute, find the horse-power that can be transmitted.

The maximum tension in the belt $(T_1) = 80 \times 6 = 480$ lbs., and since $\frac{T_1}{T_2} = 2\cdot25$, $T_2 = \frac{480}{2\cdot25} = 213$ lbs.

Difference of tensions $(T_1 - T_2) = 480 - 213 = 267$ lbs.

$$\text{Speed of belt } V = \left\{ \begin{array}{l} \text{circumference} \times \text{revolutions per} \\ \quad \text{minute} \end{array} \right.$$

$$V = 3\cdot5 \times \pi \times 220 \text{ feet per minute}$$

$$\text{H.P.} = \frac{(T_1 - T_2)V}{33000}$$

$$= \frac{267 \times 3\cdot5\pi \times 220}{33000} = 19\cdot6 \text{ H.P.}$$

Speed Cones are pulleys having several steps of different diameters on which a belt may run. Fig. 93 shows two speed cones, their object being to secure a velocity ratio which may be varied

between desired limits, and in this manner drive the machine at the particular speed which is necessary for the work it is doing.

Pulleys with Belts connecting Non-Parallel Shafts.—Shafts which are not parallel, and which do not intersect may be connected by an endless belt, provided that the pulleys are correctly placed. Fig. 94 shows the arrangement applied to two shafts at right angles which always rotate in one direction. The only condition that the belt may run properly is that the point at which the belt leaves either pulley must be in the plane of the other pulley, that is, in a plane through the centre of the other pulley. An

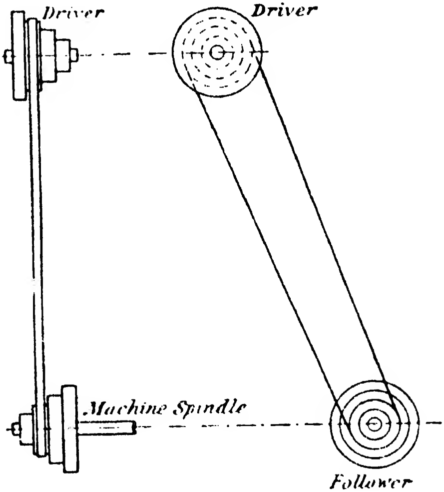


FIG. 93.—Speed cones.

inspection of Fig. 94 shows that this condition holds for the arrangement there shown.

Guide Pulleys.—Shafts which are not parallel, and which may or may not intersect, may be connected by a single endless belt if guide pulleys are used. Fig. 95 shows a plan and elevation of an arrangement of pulleys and guide pulleys, and explains itself. The belt will run equally well in either direction, and the guide pulleys do not affect the velocity ratio between the shafts A and B.

Guide pulleys are also used when the two shafts to be connected are close together, and by properly placing the guide pulleys it is possible to make the belt run equally well in both directions.

Fig. 96 shows two shafts A and B which are perpendicular and very close together, connected by an endless belt passing over the two guide pulleys which run loosely on the spindle C. Whatever the direction of motion, the plane through the centre of each pulley passes through the point of delivery of the pulley from which the belt is received and the condition of proper running is maintained.

Belt pulleys are usually rounded on the face because the tendency of the belt when running is to climb to the highest part

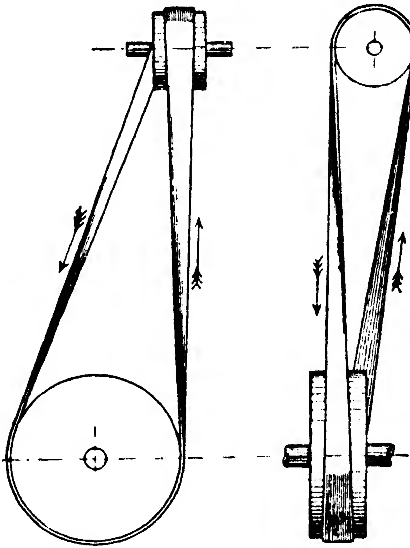


FIG. 94.—Belt drive for perpendicular shafts.

of the rim; the belt will therefore keep its place on the pulley, any chance for it to run off the pulley being reduced to a minimum.

Fast and Loose Pulley.—When a machine has to be frequently started and stopped it can be driven by a belt in the following manner:—The arrangement usually adopted for driving a lathe or other machine tool is shown in Fig. 97. A pulley J on the main shaft which rotates continuously, drives the pulley F, which is keyed on the *countershaft* K. The pulley L rides loosely on the countershaft and is therefore called the *loose* pulley, while F is called the fast pulley. The sliding piece S has fixed to it a fork M, and is moved by pulling the chains H, which are attached to

the ends of a lever carried by the spindle A. The spindle A can turn in bearings suspended by the bracket G, and carries an arm E,

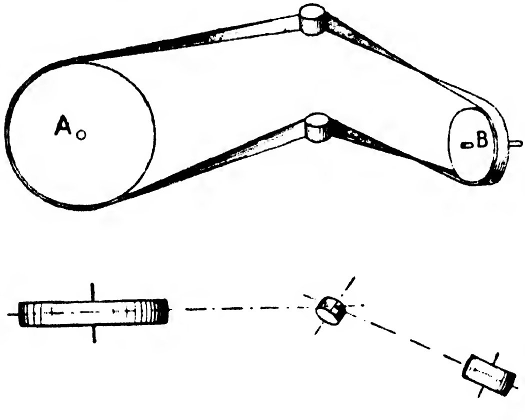


FIG. 95.—Use of guide pulleys.

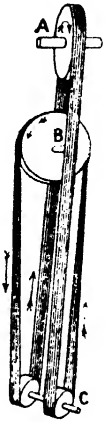


FIG. 96.

which is slotted at the end to receive a pin attached to S. The countershaft rotates in bearings carried by the hangers B fixed to

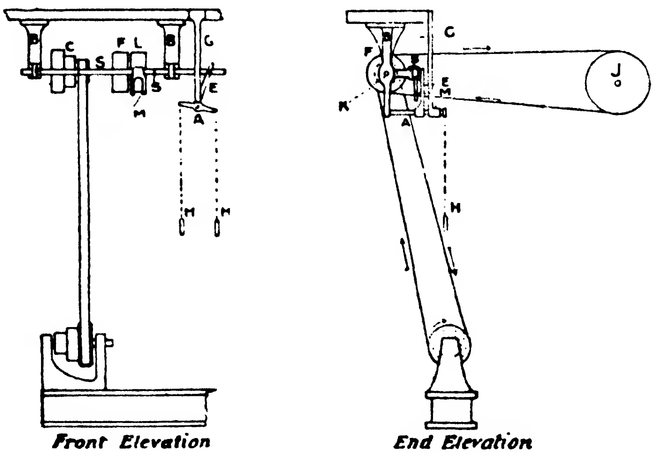


FIG. 97.—Use of fast and loose pulleys.

the ceiling. The breadth of the pulley J on the mainshaft is slightly greater than the sum of the breadths of F and L, and the width of the belt is slightly less than either F or L. The fork M presses on the side of the belt *advancing* to F or L, and by pulling H the belt is moved on to either pulley. If the belt is moved on to L the *countershaft* remains at rest, L rotating loosely on it. If the belt is moved on to F the countershaft rotates since F is keyed to it, and the lathe is driven by the speed cones as shown. The sliding piece S together with the spindle A and arm E is called the *striking gear*.

Reversing Motion by Belting.—A reversing motion may be given to a machine by using an arrangement of open and crossed belts as shown in Fig. 98. The driver A is mounted on the shaft

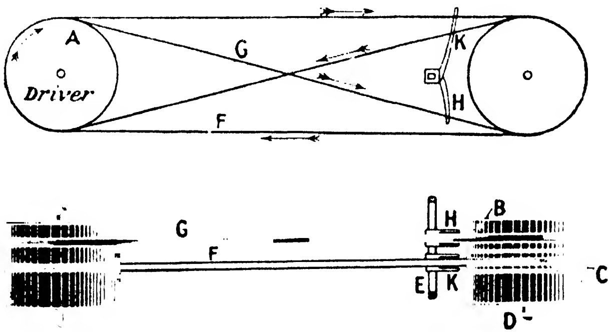


FIG. 98.—Belt reversing motion.

from which the machine is driven, and always rotates in the same direction. The machine shaft carries three pulleys B, C and D, of which B and D are loose and run idly on the shaft, whilst the middle pulley C is keyed to the shaft and therefore rotates with it. F is an open belt, and G a crossed belt. If the belts are in the positions shown in Fig. 98, the crossed belt G being on the *loose* pulley B and the open belt F on the *fast* pulley C, then the machine shaft will be driven from A by the open belt F in a clockwise direction. If now the sliding bar E carrying the belt forks H and K be pulled over so that the open belt is moved on to the *loose* pulley D and the crossed belt on to the *fast* pulley C, then the machine shaft will be driven from A by the crossed belt in a contra-clockwise direction. The fast pulley C is made narrower than B and D in order that when desired, the crossed belt may be on B and the open belt on D, in which case the machine remains

at rest. In an ordinary planing machine the sliding bar E is moved by projections on the table of the machine itself, and the machine is *self-acting* in its reversal.

Ropes and Pulleys.—Ropes are often used instead of belts for transmitting power, especially when the shafts are a long distance apart. Cotton ropes are the best on account of their flexibility, but manila and hemp ropes are also used, although they are less flexible than cotton ones. The use of ropes necessitates special pulleys with grooves cut in the face as shown in Fig. 99. The effect of the groove is to increase the frictional grip of the rope on the pulley and thus reduce the tendency to slip. When a large amount of power is to be transmitted to a great distance, wire ropes are used. The V groove shown in Fig. 99 is not suitable for wire ropes, because the lateral crushing which occurs between the rope and the sides of the V injures the rope; the form of groove

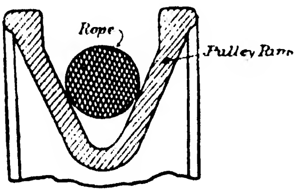


FIG. 99.—Rope driving pulley.

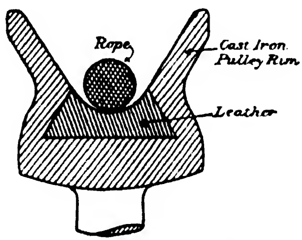


FIG. 100.—Wire rope driving pulley.

used is shown in Fig. 100, the rope resting on the rounded bottom of the groove. The bottom of the groove consists of wood, gutta-percha, or leather, leather being found to answer best in practice.

Driving Chains.—Chains are frequently used instead of belts or ropes for transmitting power in cases where the tension is very great, or when slipping is to be avoided. Fig. 101 shows a very common form of chain and wheel. The cogs or teeth on the wheel fit in the spaces between the links of the chain, and the power is transmitted without any slipping. The chief drawback to the use of this chain is the stretching of the links which takes place after a time, due to wear of the pins and their bearings.

Friction Gearing.—If the outside of one wheel A (Fig. 102) be pressed against the outside of another wheel B on a parallel shaft, the motion may be transmitted from one (the driver A) to the other (the follower B) if the surfaces of the two wheels are sufficiently rough and the resistance of the driven wheel B is not too great. The two wheels have the same speed at their circum-

ferences, and the speed of B would be calculated from that of A exactly as if they were connected by a crossed belt, that is, the velocity ratio—

$$\frac{\text{revolutions per minute of follower B}}{\text{revolutions per minute of driver A}} = \frac{\text{diameter of driver A}}{\text{diameter of follower B}}$$

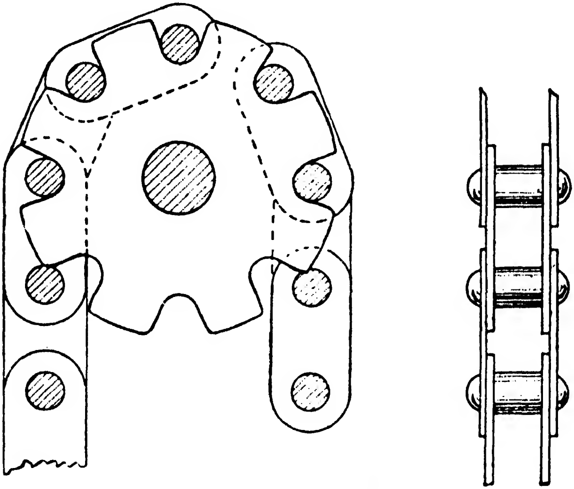


FIG. 101.—Chain driving.

As in a belt drive, slipping may occur if the power to be transmitted is too great for the frictional grip between the surfaces; and friction wheels do not give a positive drive.

Toothed Wheels or Spur Gearing.—If, instead of trusting to the frictional grip between the curved surfaces of two wheels, teeth are cut in each wheel which engage in spaces in the other wheel so that no slipping in the direction of the circumference is possible, the drive becomes *positive* and the velocity ratio is quite definite. Toothed or spur wheel gearing is capable of transmitting a considerable amount of force, and is a very important means of transmission of motion and power.

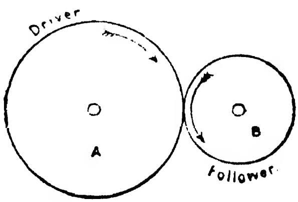


FIG. 102.—Friction gearing.

It is convenient to think of the teeth and spaces as being formed

above and below the curved surfaces of the wheels or cylinders, which, when working together on two parallel shafts, would give the same velocity ratio as is required of the actual toothed wheels. Such cylindrical surfaces are called the *pitch surfaces*, and are shown in Fig. 103 by the dotted *pitch circles*. Pitch circles are, then, circles centred at the spur-wheel centres, and having the same radii as those of corresponding friction wheels to give the same velocity ratio. If a wheel has, say, fifteen teeth, and fifteen spaces between them, a tooth and adjoining space must together occupy one-fifteenth of the circumference of the pitch circle. This amount is called the circular *pitch* of the teeth, that is, the distance measured along the circumference of the pitch circle from a point

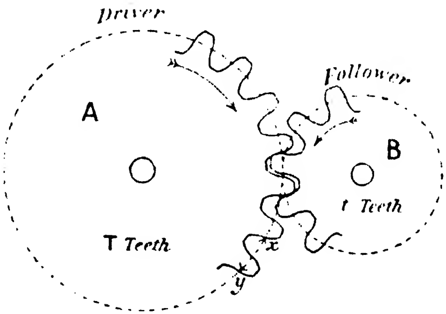


FIG. 103.—Spur gearing.

on one tooth across the intervening space to the corresponding point on the next tooth, or the length on the circumference of the pitch circle occupied by one tooth and one space, as shown at xy, Fig. 103.

If a wheel of diameter d inches has n teeth—

$$\text{circular pitch in inches} = p = \frac{\text{circumference of pitch circle}}{n} = \frac{\pi d}{n}$$

$$\text{or} \qquad \text{number of teeth } n = \frac{\pi d}{p}$$

The number of teeth and spaces must evidently be a whole number, so that the pitch p must divide exactly into the circumference πd, giving a whole number n. Hence we have the following relations :—

$$n = \frac{\pi d}{p} \qquad p = \frac{\pi d}{n} \qquad d = \frac{pn}{\pi}$$

Diametral Pitch.—A length which is the same fraction of the *diameter* of the pitch circle as the circular pitch is of the *circumference* is called the *diametral pitch*, or—

$$\text{diametral pitch in inches } s = \frac{\text{diameter of pitch circle}}{n} = \frac{d}{n}$$

and circular pitch p = diametral pitch $s \times \pi$. It has unfortunately become common to call $1/s$, the number of teeth per inch diameter, the diametral pitch. This is illogical, since pitch is normally a length.

In designing wheels it is often more convenient to use the diametral pitch than the circular pitch, because the diameters of wheels will be rational numbers if a series of simple values are chosen for the diametral pitches instead of for the circular pitches.

Example.—A spur wheel having a diametral pitch of ½ inch (*i.e.* two teeth per inch of diameter, called 2's pitch) has its pitch circle 20 inches diameter. How many teeth will it have, and what will be the circular pitch?

$$\text{Diametral pitch } s = \frac{d}{n} \qquad \text{or } n = \frac{d}{s} = \frac{20}{\frac{1}{2}} = 40 \text{ teeth}$$

$$\text{Circular pitch } p = \frac{\pi d}{n} = \pi \times s = 3{\cdot}1416 \times \tfrac{1}{2} = 1{\cdot}5708 \text{ inch.}$$

Velocity Ratio.—The velocity ratio of a wheel A (Fig. 103) driving a wheel B may be found as for the friction wheels, which might replace the pitch surfaces—

$$\frac{\text{revolutions per minute of follower B}}{\text{revolutions per minute of driver A}} = \frac{\text{diameter of pitch circle of driver A}}{\text{diameter of pitch circle of follower B}}$$

But since the diameters of the pitch circles are proportional to the circumferences, and the wheels A and B, to work together, must have the same pitch, the circumferences are proportional to the number of teeth in the wheels—

$$\frac{\text{revolutions per minute of follower B}}{\text{revolutions per minute of driver A}} = \frac{\text{number of teeth in driver A}}{\text{number of teeth in follower B}}$$

Without reference to the corresponding friction wheels, we might calculate the velocity ratio thus. If the driver A has, say, thirty-six teeth of ¼-inch pitch, and the follower B has twenty-four teeth of

$\frac{1}{4}$-inch pitch, in one revolution of A both wheels turn together through $36 \times \frac{1}{4} = 9$ inches at the pitch circle circumference; but for one revolution of B the distance turned through is $24 \times \frac{1}{4} = 6$ inches, hence the number of turns made by B during one revolution of A is—

$$\frac{36 \times \frac{1}{4}}{24 \times \frac{1}{4}} = \frac{9}{6} = 1 \cdot 5$$

or $$\frac{\text{number of revolutions of B}}{\text{number of revolutions of A}} = \frac{36}{24} = 1 \cdot 5$$

General Rule for Velocity Ratio.

If A has T teeth,

B has t teeth,

N = number of revolutions of A in any given time (say, 1 minute),

n = number of revolutions of B in the same time,

$$\text{Velocity ratio of follower to driver} = \frac{n}{N} = \frac{T}{t}$$

and $$\text{revolutions of follower B, } n = \frac{T}{t} \times N$$

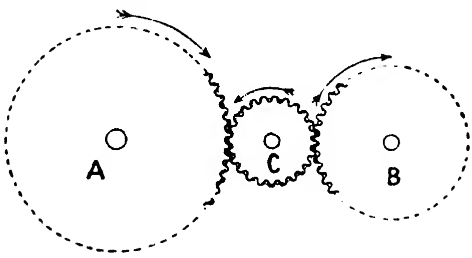


FIG. 104.—Effect of an idle wheel.

Idle Wheel.—If instead of gearing with the follower B the driver A, Fig. 104, drives a wheel C, and C drives B, the only effect of C is to make the wheel B turn in the same direction as A, instead of in the opposite direction which it did in Fig. 103. For, if A makes one turn, all points on the circumference of C turn through a distance equal to the circumference of A, and all points on the circumference of B are driven through the same distance, namely, a length equal to the circumference of A just as before; and B makes the same number of turns as before, but in the opposite direction.

To put it otherwise, if A makes N turns, and C has t' teeth, using the previous rule—

$$\text{C makes N} \times \frac{\text{T}}{t'} \text{ turns}$$

And again applying the rule, B makes $\frac{t'}{t}$ times as many turns as C, that is—

$$\text{revolutions of B} = \text{N} \times \frac{\text{T}}{t'} \times \frac{t'}{t} = \text{N} \times \frac{\text{T}}{t}$$

just as before, when C was not there; the number of teeth (t') in C is immaterial, for t' cancels out.

Example 1.—Three spur wheels, A, B, and C, are on parallel shafts, and are in gear—A with B, and B with C. A has 45 teeth, and runs at 240 revolutions per minute, and C has 105 teeth. What will be the speed of C?

Applying the above rule—

$$\text{Speed of C} = 240 \times \tfrac{45}{105} = \tfrac{720}{7} = 102\tfrac{6}{7} \text{ revolutions per minute}$$

in the same direction as A.

Example 2.—Motion is to be transmitted by means of two spur wheels on parallel shafts with a velocity ratio of driven to driver of 12. The driven shaft has to rotate at 1130 revolutions per minute, and the wheel on it has 20 teeth. How many teeth must the driver have, and with what speed must it rotate?

$$\frac{\text{Number of revolutions of driven wheel}}{\text{Number of revolutions of driver}} = \frac{\text{number of teeth in driver}}{\text{number of teeth in driven}} = \frac{12}{1}$$

$$\begin{aligned} \text{Hence number of teeth in driver} &= 12 \times \text{teeth in driven wheel} \\ &= 12 \times 20 = 240 \\ \text{Speed of driver} &= \tfrac{1130}{12} = 94\tfrac{1}{6} \text{ revolutions per minute.} \end{aligned}$$

Trains of Wheels.—A large velocity ratio cannot be obtained by the use of two spur wheels without one of them being inconveniently large. In such cases a train of wheels is used as shown diagrammatically in Fig. 105, the motion being transmitted from the driver A on shaft 1, through wheels B and C keyed on shaft 2, then through wheels D and E keyed on shaft 3 to the follower F keyed on shaft 4. The wheels B and C, or the wheels D and E, instead of being keyed separately on to the shaft 2 or 3, may be all in one casting, in which case it is called a compound wheel.

Suppose in Fig. 105, A has 40 teeth and makes 60 revolutions per minute, B 25 teeth, C 105 teeth, D 20 teeth, E 150 teeth, and F 30 teeth, then—

revolutions of shaft 2 = revolutions of shaft 1 $\times \frac{40}{25} = 60 \times \frac{40}{25} = 96$

" " 3 = " " $2 \times \frac{105}{20} = 96 \times \frac{105}{20} = 504$

" " 4 = " " $3 \times \frac{150}{30} = 504 \times \frac{150}{30} = 2520$

Hence, we see that by this train of wheels the follower F makes 2520 revolutions per minute, whilst the driver makes only 60 revolutions per minute.

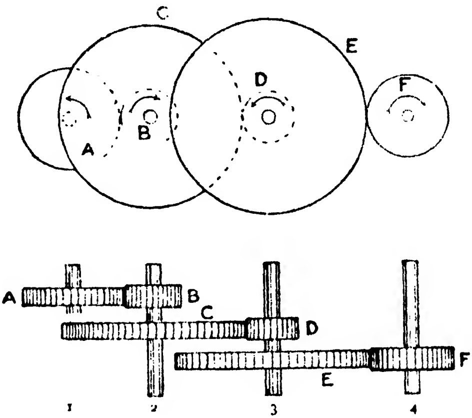


FIG. 105.—Train of wheels.

General Rule for Velocity Ratio of a Train of Wheels.—Referring again to Fig. 105, let A, B, C, D, E, F represent the number of teeth in each wheel, then if shaft 1 makes 1 revolution—

revolutions of shaft 2 = revolutions of shaft 1 $\times \frac{A}{B} = \left(\frac{A}{B}\right)$

" " 3 = " " $2 \times \frac{C}{D} = \left(\frac{A}{B}\right) \times \frac{C}{D}$

" " 4 = " " $3 \times \frac{E}{F} = \left(\frac{A}{B} \times \frac{C}{D}\right) \times \frac{E}{F}$

hence, the velocity ratio or revolutions of last shaft per revolution of the first shaft, or—

$$\frac{\text{revolutions of last shaft}}{\text{revolutions of first shaft}} = \frac{A}{B} \times \frac{C}{D} \times \frac{E}{F} \text{ or } \frac{A \times C \times E}{B \times D \times F}$$

$$= \frac{\text{product of numbers of teeth in drivers}}{\text{product of numbers of teeth in followers}}$$

Example 1.—In a train of wheels the drivers have 40, 60, 80, and

120 teeth; the followers have 25, 50, 30, and 70 teeth. If the first driver makes 80 revolutions per minute, what will be the speed of the last follower?

Applying the above rule—

$$\text{Velocity ratio} = \frac{\text{product of numbers of teeth in drivers}}{\text{product of numbers of teeth in followers}}$$

$$= \frac{40 \times 60 \times 80 \times 120}{25 \times 50 \times 30 \times 70} = 8{\cdot}77$$

Speed of last follower = 80 × 8·77 = 701·6 revolutions per minute

Example 2.—In the train of wheels shown in Fig. 105, A has 20 teeth, B has 30 teeth, C 120 teeth, E 95 teeth, F 25 teeth. A makes 30, and F 456, revolutions per minute: how many teeth must the wheel D have?

$$\text{Velocity ratio} = \tfrac{456}{30} = \tfrac{76}{5}$$

Applying the above rule—

$$\frac{76}{5} = \frac{A \times C \times E}{B \times D \times F} = \frac{20 \times 120 \times 95}{30 \times D \times 25}$$

$$\frac{76}{5} = \frac{4 \times 4 \times 95}{D \times 5}$$

$$5 \times 76D = 5 \times 4 \times 4 \times 95$$

$$D = \frac{5 \times 4 \times 4 \times 95}{5 \times 76} = \frac{4 \times 4 \times 95}{76} = 20 \text{ teeth}$$

Power transmitted by Spur Gearing.—Neglecting friction, the power transmitted may be found by the principle of work as follows:—

The work put into the train by the first driver must be equal to the work obtained from the last follower, since there is no loss by friction, and this must equal the work transmitted.

Let P = tangential pressure in pounds at the pitch circles between the teeth of any two wheels in gear, and d (in feet) the diameter of the pitch circle of one of them (the driver).

Then work transmitted—

= effort applied to driver × distance moved by the effort

= pressure between the teeth of two wheels which gear × distance moved along the pitch circle

= P × πd per revolution

and if the driver of the two wheels makes N revolutions per minute, work transmitted per minute = P × πdN foot-pounds; and

$$\text{Horse-power transmitted H.P.} = \frac{P \times \pi d N}{33000} \quad \text{or}$$

$$P = \frac{33000 \times \text{H.P.}}{\pi d \times N} \quad \text{lbs.}$$

The power transmitted may be expressed in words thus—

$$\text{H.P.} = \frac{\text{pressure between teeth (lbs.)} \times \text{speed of pitch circle (ft. per min.)}}{33{,}000}$$

Example 1.—Two spur wheels in gear are transmitting 5 H.P. The driver has 40 teeth, and the pitch is 1 inch. If the driver makes 200 revolutions per minute, what will be the tangential pressure between the teeth?

$$P = \frac{33{,}000 \times \text{H.P.}}{\pi d \times N}$$

The circumference of the pitch circle of the driver is 40×1 inches or $\frac{40}{12}$ feet.

$$\therefore\ P = \frac{33{,}000 \times 5}{\frac{40}{12} \times 200} = \frac{33{,}000 \times 60}{40 \times 200} = 247{\cdot}5 \text{ lbs.}$$

Example 2.—25 H.P. is to be transmitted with a pressure between the teeth not greater than 500 lbs. What must be the speed of the pitch circle? And if the driven wheel has 100 teeth of $1\frac{1}{2}$ inch pitch, at what speed will it rotate?

$$\text{Speed of pitch circle} = \frac{\text{H.P.} \times 33{,}000}{\text{pressure between teeth}} \text{ feet per minute}$$

$$= \frac{25 \times 33{,}000}{500} = 1650 \text{ feet per minute.}$$

$$\text{Circumference of pitch circle} = \frac{100 \times 1{\cdot}5}{12} \text{ feet}$$

$$\therefore \text{ speed of driven wheel} = \frac{1650}{150} \times 12 = 132 \text{ revolutions per minute.}$$

Example 3.—In the train of wheels shown in Fig. 105, F has 30 teeth of $\frac{3}{4}$ inch pitch. If the train transmits 2 H.P., F making 252 revolutions per minute, what will be the pressure between the teeth of wheels E and F?

$$\text{Circumference of F} = 30 \times \tfrac{3}{4} \text{ inches} = \frac{30 \times 3}{4 \times 12} = \frac{15}{8} \text{ feet}$$

$$P = \frac{2 \times 33{,}000}{\frac{15}{8} \times 252} = \frac{2 \times 33{,}000 \times 8}{15 \times 252} = 139 \text{ lbs.}$$

Rack and Pinion.—A rack is a spur wheel of infinitely large diameter, and the pitch surface is a plane. Fig. 106 shows a rack and spur wheel or pinion in gear. When the pinion is rotated on a fixed shaft it moves the rack along as shown by the arrows in Fig. 106. If the rack is fixed, then the pinion rolls along it.

Example.—A pinion gearing with a rack has 20 teeth of $\frac{1}{2}$ inch pitch, and rotates at 100 revolutions per minute. The rack is fixed to the table of a planing machine. At what speed will the table be

driven? If the pressure between the teeth of the rack and pinion is 500 pounds, what is the horse-power transmitted?

$$\text{Circumference of pitch circle of pinion} = 20 \times \tfrac{1}{2} = 10 \text{ inches} = \tfrac{5}{6} \text{ feet}$$

$$\text{Speed of pitch circle} = \text{speed of rack} = \tfrac{5}{6} \times 100 \text{ feet per minute}$$

$$= 83\tfrac{1}{3} \text{ feet per minute}$$

$$\text{Work transmitted per minute} = \text{pressure between teeth} \times \text{speed}$$

$$= 500 \times \tfrac{500}{6} \text{ foot-pounds}$$

$$\text{H.P. transmitted} = \frac{500 \times 500}{6 \times 33000} = \frac{25}{19{\cdot}8} = 1{\cdot}21 \text{ H.P.}$$

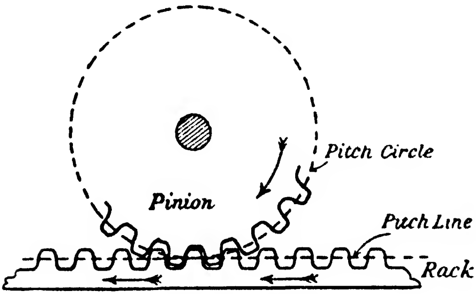


FIG. 106.—Rack and pinion.

EXAMPLES X.

1. An engine drives a line of shafting by means of a belt. The driving pulley on the engine shaft is 6 feet diameter, and that on the shafting is 3 feet 6 inches diameter. If the engine runs at 120 revolutions per minute, what will be the speed of the shafting (*a*) if there is no slip, and (*b*) if the slip of the belt is 3 per cent.?

2. A pulley of 36 inches diameter running at 250 revolutions per minute drives another pulley, A, at 360 revolutions per minute: what is the diameter of A?

3. A pulley, A, 42 inches diameter running at 200 revolutions per minute drives a pulley, B, of 30 inches diameter by means of a belt. On the same shaft as pulley B is keyed another pulley, C, of 36 inches diameter, which drives a machine having a pulley of 15 inches diameter on its shaft. What will be the speed of B, and what will be the speed of the machine spindle?

4. The tensions on the two sides of a belt are 250 lbs. and 100 lbs. respectively. The diameter of one pulley is 2 feet 6 inches, and it runs at 300 revolutions per minute. What horse-power is transmitted by the belt?

5. A belt running at a speed of 2400 feet per minute has a difference of 300 lbs. between the pulls on the tight and slack sides: what horse-power is being transmitted?

6. The speed of a belt is 2000 feet per minute, and it transmits 100 H.P.: find the difference of tensions in the two sides of the belt. Also, if the tension in the tight side is $2\frac{1}{4}$ times that in the slack side, find the two tensions and the width of the belt required if the maximum tension is not to exceed 80 lbs. per inch width of belt.

7. The tension on the tight side of a belt is $2\frac{1}{2}$ times that on the slack side; it runs at 2300 feet per minute, and transmits 120 H.P. The thickness of the belt is $\frac{3}{8}$ inch, and the maximum tension is to be 350 lbs. per square inch of cross-section of the belt. What must be the width of the belt?

8. The width of a belt is 10 inches, and the maximum tension per inch width of belt is not to exceed 80 lbs. The ratio of tensions on the two sides is $2\frac{1}{4}$; the diameter of the follower is 2 feet 6 inches, and it makes 240 revolutions per minute. What horse-power can be transmitted?

9. The diameter of the pitch circle of a spur wheel is 18 inches, and its diametral pitch is $\frac{3}{4}$ inch: how many teeth will it have, and what will be the circular pitch?

10. Two spur wheels, A and B, on parallel shafts are in gear. A has 40 teeth, and rotates at 250 revolutions per minute, whilst B is to make 100 revolutions per minute. How many teeth must B have?

11. Five spur wheels, A, B, C, D, E, are in gear. A has 50 teeth, and runs at 300 revolutions per minute, B has 80 teeth, C has 60 teeth, D 40 teeth, and E 120 teeth. What will be the speed of (*a*) B, (*b*) C, (*c*) E?

12. In the train of wheels shown in Fig. 105, A has 20 teeth, B has 30 teeth, E 95 teeth, D 20 teeth, F 45 teeth. A makes 30 revolutions per minute, and F 456 revolutions per minute. How many teeth must the wheel C have?

13. Two spur wheels in gear are transmitting 10 H.P. The driver runs at 240 revolutions per minute, and has 80 teeth of $\frac{3}{4}$ inch pitch. Neglecting friction, find the pressure between the teeth.

14. A train of wheels transmits 5 H.P. One of the wheels has 60 teeth of 1 inch pitch. How many revolutions per minute must it make if the pressure between the teeth is not to exceed 140 lbs.?

15. A planing machine is driven by a pinion which gears into a rack fixed to the underside of the table of the machine. The pinion has 14 teeth of 1-inch pitch, and makes 20 revolutions per minute. At what speed will the table be driven?

16. The power taken to drive the planing machine in Question 15 is 2·5 H.P. Neglecting friction, what will be the pressure between the teeth of the rack and pinion?

CHAPTER XI

THE INCLINED PLANE AND SCREW

By means of an inclined plane a body may be raised by an effort much smaller than the weight of the body; the less the slope, the smaller the effort required. Suppose the effort P (Fig. 107) is

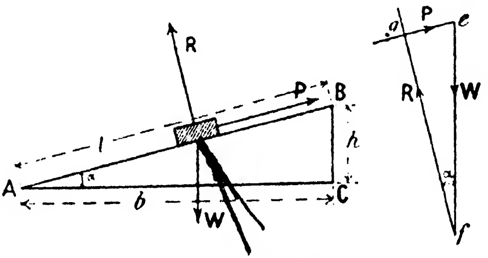


FIG. 107.—Inclined plane.

applied parallel to the slope and we neglect friction. When the body is on the point of moving up the slope, it is in equilibrium under the action of three forces, namely its weight W downwards, the reaction R (perpendicular to the plane if the friction is zero), and the effort P. P may be found by drawing the triangle of forces *efg* for W, R, and P. Set off *ef* to represent W, then draw the line *fg* parallel to R, and *ge* parallel to P; *fg* and *ge* intersect in *g*, and *ge* gives the force P to scale.

P may be found graphically as just described, or it may be calculated. For since *ef* is perpendicular to AC, and *fg* perpendicular to AB, the angle $\widehat{efg}$ between *ef* and *fg* is the same (a) as that between AB and AC. Also *ge* is parallel to P, and therefore perpendicular to R and to *fg*, so that angle $\widehat{fge}$ is a right angle, hence—

$$\frac{\mathrm{P}}{\mathrm{W}} = \frac{ge}{ef} = \sin a$$
$$\text{or } \mathrm{P} = \mathrm{W} \sin a$$

That is, the effort required to draw any load up a slope inclined a to the horizontal is equal to the load multiplied by the sine of the angle of slope.

We might also find the same result by the principle of work, for while P acts through a distance, AB, in its own direction, W is lifted through a distance, BC, in its own direction, hence, if there is no friction,

$$P \times AB = W \times BC$$

$$\text{or } P = W \times \frac{BC}{AB} = W \sin a$$

$$\text{or } P = W \times \frac{h}{l} \text{ or } W \times \frac{\text{height of plane}}{\text{length of plane}}$$

Experiment.—The above may be verified experimentally by the apparatus shown in Fig 65 and described on p 71 The values of W, P, h and l, found by experiment, are here reproduced, and it will be seen how closely the value of P calculated from $P = W \sin a$, or $W \times \frac{h}{l}$ agrees with the value actually observed.

Load W (lbs.)	Actual P (lbs.)	Height (inches)	Length l (inches)	Sine a or $\frac{h}{l}$	Calculated $P = W \times \frac{h}{l}$ (lbs.)
68 5	23 50	9 75	28 5	0 342	23 4
68 5	30 50	12 00	28 5	0 45[illegible]	30 9
68 5	36 25	15·25	28 5	0 5[illegible]	36 5
68 5	40 75	17 [illegible]	28 5	0 60	41 1
68 5	4[illegible] 50	18 75	28 5	0 6[illegible]	45 0
68 5	45 00	20 [illegible]	28 5	0 7[illegible]	48 0

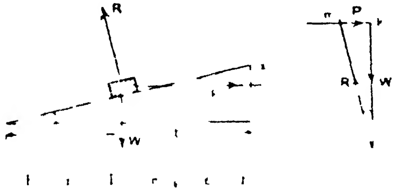


Fig. 108.

Horizontal Force.—If the effort P required to raise the body is applied horizontally (Fig. 108) instead of parallel to the plane we proceed to find P as before by setting off ab' vertically to represent W, drawing from [illegible] a line parallel to R and from [illegible] (parallel to P) meeting the [illegible] from [illegible] in n. The line n [illegible] the triangle

of forces klm then represents P to scale, and evidently in this case—

$$\frac{P}{W} = \frac{mk}{kl} = \tan \alpha$$

$$P = W \tan \alpha, \text{ or } W \times \frac{h}{b}, \text{ or } W \times \frac{\text{height of plane}}{\text{horizontal base of plane}}$$

Experiment.—Using the inclined plane shown in Fig. 65 and applying the effort P horizontally, the following data was obtained :—

Load W (ozs.).	Actual P (ozs.).	Height h (inches).	Base b (inches)	Tan α or $\frac{h}{b}$	Calculated $P = W \times \frac{h}{b}$
68·5	24·66	9·75	27·10	0·36	24·60
68·5	27·45	9·20	25·30	0·40	27·40
68·5	34·75	12·90	25·40	0·507	34·72
68·5	43·55	15·25	24·00	0·635	43·50
68·5	51·40	17·10	22·80	0·75	51·30
68·5	59·65	18·75	21·50	0·87	59·60
68·5	68·55	20	20	1·00	68·50

It will be seen how closely the actual value of P measured experimentally agrees with the calculated value $P = W \tan \alpha$ or $W \times \frac{h}{b}$.

Force at any Angle.—If the effort P is applied at any angle, as in Fig. 109, it can be found graphically, as before, by a triangle of forces, namely, set off np vertically to represent W, draw a line parallel to R (or perpendicular to AB), and a line through n parallel to P to meet the previous line in q. Then qn gives the amount of the effort P to scale.

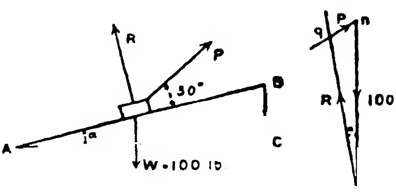


FIG. 109.—Inclined plane with oblique effort.

Example 1.—The slope of an inclined plane is such that it rises 1 foot for each 5 feet of its length. A body weighing 100 lbs. rests on the plane ; find graphically (neglecting friction) the effort required to draw the body up the plane if the effort makes an angle of 30° with the plane.

Set out the plane ABC (Fig. 110) to any convenient scale. For

example, make BC 1 inch long, and AB 5 inches. Draw the line of action of P making 30° with AB as shown. Next draw the triangle of forces npq as described above. Choosing a scale of 1 inch to 20 lbs., np is 5 inches long, representing 100 lbs., and qn is found to measure 1·2 inches,* representing $1·2 \times 20$ or 24 lbs. Hence, the effort required is 24 lbs.

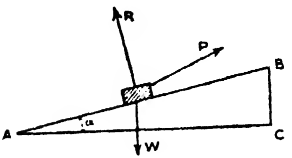


FIG. 110.

Example 2.—An inclined plane rises 1 foot for each 8 feet of its length. Find the effort required (neglecting friction) to draw a body weighing 3 cwt. up the plane, (*a*) when the effort is horizontal; (*b*) when the effort is parallel to the plane.

The length of the base of the plane (AC in Fig. 109 or 110) is calculated from—

$$AB^2 = AC^2 + BC^2$$

hence $\quad AC^2 = AB^2 - BC^2 = 8^2 - 1^2 = 64 - 1 = 63$

and $\quad AC = \sqrt{63} = 7·937$ feet.

$$(a) \text{ Effort } P = W \times \frac{\text{height}}{\text{base}} = 3 \times 112 \times \frac{1}{7·937} = 42·3 \text{ lbs.}$$

$$(b) \text{ Effort } P = W \times \frac{\text{height}}{\text{length}} = 3 \times 112 \times \tfrac{1}{8} = 42·0 \text{ lbs.}$$

From this example it will be seen that for a slope of 1 in 8 (which is greater than most hills met with on roads) it makes very little difference whether we take the force to be horizontal or parallel to the plane. The less the slope of the plane the smaller will be the error made.

Example 3.—A train weighs 250 tons, and the tractive resistance on the level is 12 lbs. per ton. What horse-power will be required to draw the train at a uniform speed of 30 miles an hour (*a*) up an incline of 1 in 200; (*b*) on the level; (*c*) down an incline of 1 in 200, the incline to be reckoned (as is usual) as 1 foot rise for every 100 feet length?

(*a*) The effort required to overcome tractive resistance $= 250 \times 12 = 3000$ lbs.

The effort required to lift the train, $P = W \times \dfrac{\text{height}}{\text{length}} = 250 \times \frac{1}{200}$ tons

$$= \frac{250}{200} \times 2240 = 2800 \text{ lbs.}$$

Total effort required to draw train up the incline
= effort to overcome tractive resistance + effort to lift train up the incline
= 3000 + 2800 = 5800 lbs.

$$\text{Now 30 miles an hour} = \frac{30 \times 5280}{60 \times 60} = 44 \text{ feet per second}$$

$$\text{Horse-power} = \frac{5800 \times 44}{550} = 464 \text{ H.P.}$$

(*b*) Effort required on the level = effort to overcome tractive resistance
$= 3000$ lbs.

$$\text{Horse-power} = \frac{3000 \times 44}{550} = 240 \text{ H.P.}$$

(*c*) In drawing the train *down* the slope, the force of 2800 lbs. *helps* motion, and the force of 3000 *opposes* motion; hence, the force required *down* the plane will be $3000 - 2800 = 200$ lbs., and

$$\text{Horse-power} = \frac{200 \times 44}{550} = 16 \text{ H.P.}$$

An alternative method of solution for (*a*) is as follows:—

Distance moved in one minute $= 44 \times 60 = 2640$ feet.

Distance lifted in one minute $= \dfrac{44 \times 60}{200} = 13{\cdot}2$ feet.

Work done in one minute

$$\begin{aligned} &= \text{tractive resistance} \times \text{distance} + \text{weight} \times \text{lift} \\ &= 3000 \times 2640 + 250 \times 2240 \times 13{\cdot}2 \\ &= 7{,}920{,}000 + 7{,}392{,}000 \\ &= 15{,}312{,}000 \text{ foot-pounds} \end{aligned}$$

$$\text{Horse-power} = \tfrac{15312000}{33000} = 464 \text{ H.P. (as before)}$$

The Screw.—A screw is formed from a solid cylinder of material by cutting a continuous groove in it, successive turns of the groove being separated by the remaining solid material, called the thread of the screw. The path of the thread and the groove is not only round the cylinder, but also along it in the direction of the axis; it is a *helical* path, that is, the shape of a helix. A helix may be drawn on a cylinder by rotating the cylinder at a constant rate and pressing against its curved surface a pencil or a marker of some kind which is moving along parallel to the axis of the cylinder at a steady rate. Suppose the cylinder (Fig. 111) to be covered with paper and a helix traced on the paper by a pencil moving at a steady rate along AB while the cylinder turns about its axis at a constant rate. The left-hand side of Fig. 111 shows the curved line of the helix. If now the paper is unwrapped and straightened out, the helix will be found to be the straight line AB′, shown at the right-hand side of Fig. 111, and if more than the line AB is drawn, the straight lines from A′ and B parallel to AB′, will also be found on the paper, the points A and A′, and the points B and B′, etc., falling together when the paper is wrapped on the cylinder with the joint parallel to the axis. The distance, BB′, is equal to the circumference of the cylinder = diameter $\times \pi = \pi d$.

The distance AB which the pencil moves along parallel to the axis during one turn of the cylinder is called the pitch of the helix $= p$.

If the cylinder be placed with its axis vertical the helix makes

a constant angle with the horizontal; this angle α or $A\hat{B}'B$ is shown on the right-hand side of Fig. 111, and—

$$\tan \alpha = \frac{AB}{BB'} = \frac{p}{\pi d}$$

If instead of merely tracing a line on the cylinder a helical groove of definite shape is cut in it by a tool in the same way the

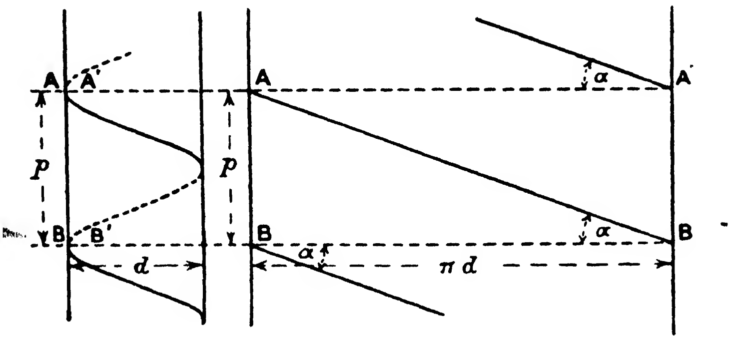


FIG. 111.—Helix.

cylinder becomes a screw. The form of groove differs considerably according to the purpose for which the screw is to be used.

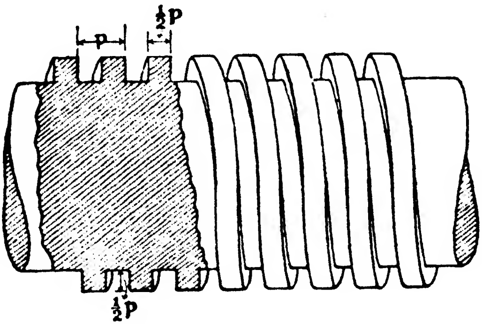


FIG. 112.—Square-threaded screw.

Forms of Screw Threads.

Square Thread.—If a helical groove of square cross section and width equal to half the pitch of the screw is cut in a cylinder.

as shown in Fig. 112, the result is a square-threaded screw. This is the type of thread generally used in the transmission of power, the amount of friction being less than in other forms.

Vee Thread.—A stronger thread (but having more friction) is the **V** thread shown in section in Fig. 113, which illustrates the Whitworth form of **V**, which has been adopted as the standard in engineering work in Great Britain for all sizes from $\frac{1}{8}$-inch diameter

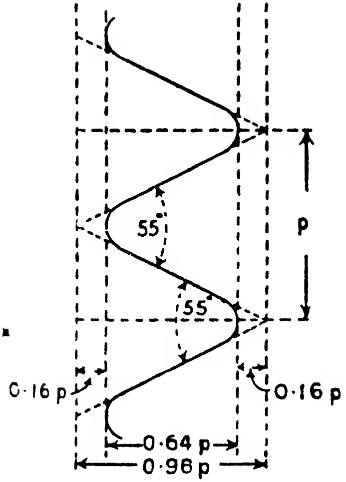


FIG. 113.—Whitworth V thread.

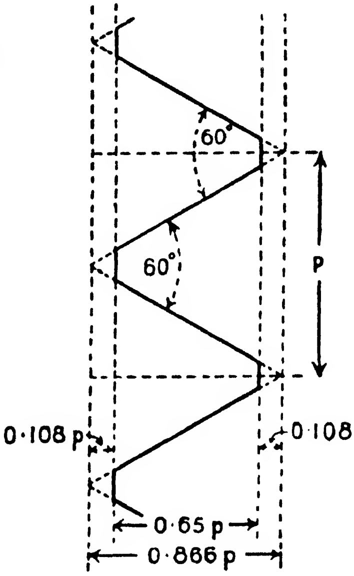


FIG. 114.

upwards. It is easier to make and is stronger than the square thread, but is less efficient and therefore gives a smaller thrust along the axis for a given torque or twisting moment applied to it. It is the form commonly adopted for studs and bolts.

Seller's Thread.—This form of **V** thread is adopted in the United States of America and is shown in Fig. 114.

Nuts.—A nut is formed by cutting a helical groove in a cylindrical hole in a piece of solid material; the groove is of the same cross sectional form as the thread of the screw with which the nut is intended to work. The common hexagonal and square nuts used with **V** thread bolts for holding purposes will be familiar to the reader. If the screw in gear with a nut is held fast and the nut rotated, the nut travels along the screw. If the nut is held fast

and the screw rotated the screw travels in the direction of its own axis. We have an example of this in the simple screw-jack (Fig. 75), in which A is the fixed nut forming part of the frame. If the screw rotates but is prevented from moving along in the direction of its axis, a nut in gear with it and free to travel along will be driven in the direction of the axis of the screw. The student will observe examples of this arrangement in driving the tool of a screw-cutting lathe and in driving planing machines, etc.

Right-hand and Left-hand Threads.—If, looking at the end of a screw, when you turn a screw in a clockwise direction (↻) it advances from you into its nut beyond, it is called a *right*-hand screw. If it moves out of its nut towards you it is called a *left*-hand screw. The two kinds are shown in Fig. 115. If the screws

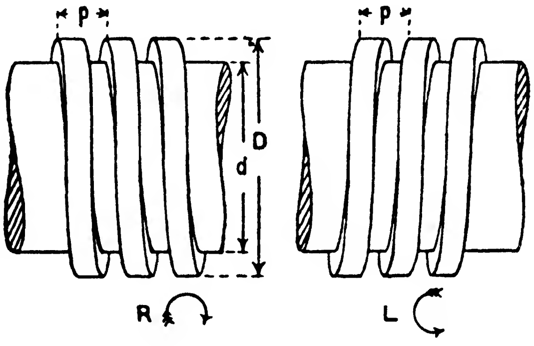


FIG. 115.

are as shown and are rotating so that the upper curved surface is approaching the reader, the right-hand helix R would be traced by a point moving along the surface from right to left, and the left-hand helix L by a point moving from left to right.

Right- and Left-handed Screws.—A common arrangement used for tightening up a rope or tie bar is to make a joint in which one end of the bar has cut on it a right-handed screw thread and the other end a left-handed thread. Each screw thread works in a separate nut, the nuts being fixed together or are part of the same piece of material. Fig. 116 shown a common arrangement in which, when the nut A is rotated in one direction the two ends B and C of the bar are drawn together, and when rotated in the opposite direction they are pushed apart. The reader will notice a similar arrangement used in the couplings for railway carriages.

Multiple Threads.—If a screw of given diameter is required

to have a rather larger pitch in order to give a larger movement to a nut for each turn of the screw, an ordinary single square thread may make the screw too weak, for the width and depth of the thread being half the pitch, the diameter of the screw at the bottom of the thread will be $d = D - \frac{1}{2}p - \frac{1}{2}p = D - p$, so that the diameter d and the strength of the screw diminish with increase of pitch. When the pitch is large a double thread may be used, that is, two threads each of the required pitch running side by side as shown in Fig. 117, where the parts marked A form one thread of pitch p, and those marked B form another separate thread of pitch p. The width and depth of the threads in this case will be $\frac{1}{4}p$ instead of $\frac{1}{2}p$. Similarly three threads each of depth $\frac{1}{6}p$ and pitch p might be used as in a triple-threaded screw, and so on with more threads in one pitch. The pitch of a screw is, then, the distance measured along the screw between any point on a thread and the corresponding point on the next turn of the *same* thread, *or* it is the distance measured along the screw

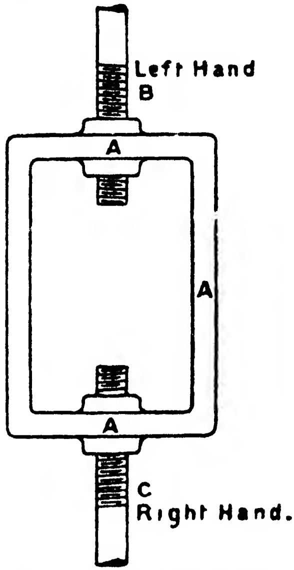


FIG. 116.—Right- and left-handed screw.

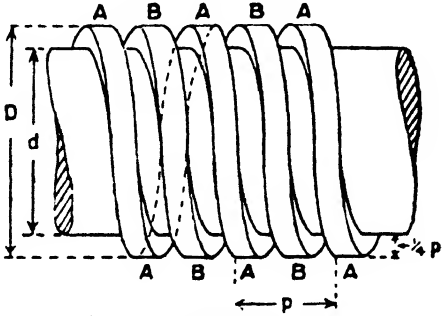


FIG. 117.—Double-threaded screw.

which its nut moves when the nut makes one revolution, the screw being held fixed. This is also called the "lead" of the screw.

Velocity Ratio of Screw and Nut.—If R inches is the radius at which the effort P is applied to a handle or pulley (see Fig. 75), and p inches is the pitch of the screw, during one revolution of the screw P moves $2\pi R$ inches and the screw moves a distance p inches against a resistance such as a load W. Hence the velocity ratio—

$$V = \frac{\text{distance P moves}}{\text{distance W moves}} = \frac{2\pi R}{p}$$

And neglecting friction, the mechanical advantage—

$$\frac{W}{P} = \frac{2\pi R}{p}, \text{ or } W = P \times \frac{2\pi R}{p}, \text{ and } P = W \times \frac{p}{2\pi R}$$

Actually, the load lifted will be considerably less, as we have seen in the results given in Chap. VII. p. 91.

Referring to Fig. 111, we might also regard the vertical screw as an inclined plane corresponding to Fig. 108 with the effort horizontal at the mean radius of the threads. Then the effort—

$$P = W \tan \alpha = W \times \frac{AB}{BB'} \text{ (Fig. 111)}$$
$$= W \times \frac{p}{\pi d}$$

The value of P previously obtained agrees, then, if the effort is applied at a radius $R = \frac{d}{2}$, that is, at, say, the mean radius of the thread. If P is applied at a greater radius, then the effort P is proportionately reduced.

Example 1.—A screw-jack similar to that shown in Fig. 75 has a double thread of $\frac{3}{4}$-inch pitch. The effort is applied at the end of a lever 18 inches long. What effort will be required to lift a load of 5 tons, the efficiency at this load being taken as 40 per cent.?

From the above we have—

$$\text{velocity ratio } V = \frac{2\pi R}{p} = \frac{2\pi \times 18}{\frac{3}{4}}$$

Hence for 1 foot lift of the load—

$$\text{Work done on the load} = 5 \times 2240 \times 1 \text{ foot-pounds,}$$

and this is equal to $\frac{40}{100}$ of that done by the effort.

$$\text{Hence } P \times \frac{2\pi \times 18}{\frac{3}{4}} \times \frac{40}{100} = 5 \times 2240$$

$$P = \frac{5 \times 2240 \times 100 \times \frac{3}{4}}{2\pi \times 18 \times 40} = 185 \text{ lbs.}$$

The same result might also be found by the method explained in Chapter VII., namely—

$$\text{efficiency} = \frac{W}{PV}$$

$$\frac{40}{100} = \frac{5 \times 2240 \times \frac{3}{4}}{P \times 2\pi \times 18}$$

$$\therefore\ P = \frac{5 \times 2240 \times \frac{3}{4} \times 100}{2\pi \times 18 \times 40} = 185 \text{ lbs. as before}$$

Example 2.—A twisting moment of 3000 pound-inches is applied to a screw of ½ inch pitch and produces a thrust on the screw of 6 tons. Find the efficiency of the screw.

Consider one revolution of the screw, then the thrust of 6 tons acts through a distance equal to the pitch of ½ inch, and

$$\text{Work got out or useful work done} = 6 \times 2240 \times \tfrac{1}{2} \text{ pound-inches}$$

$$\text{Work expended by the effort} = \begin{cases} \text{twisting moment} \times \text{angle} \\ \quad \text{turned through in radians} \end{cases}$$

$$= 3000 \times 2\pi \text{ pound-inches}$$

Hence—

$$\text{Efficiency} = \frac{\text{useful work done}}{\text{work expended}} = \frac{3 \times 2240}{3000 \times 2\pi} = 0{\cdot}356 \text{ or } 35{\cdot}6 \text{ per cent.}$$

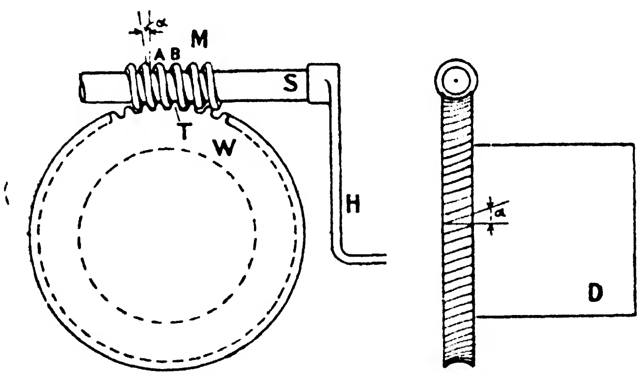


FIG. 118.—Worm and worm wheel.

Worm and Worm Wheel.—In this combination the thrust of a screw is employed directly from its threads to drive the teeth of a wheel without the use of a nut. The threads of the screw or worm M (Fig. 118) have a cross section similar in shape to that of a spur wheel tooth and gear with the teeth of the worm wheel W. The worm is attached to a spindle S, at right angles to the axis of the wheel, and is prevented by its bearings from moving in the

direction of its own axis, and the teeth of the wheel are inclined to the axis of the wheel at the angle α of the worm. When the worm is rotated by a handle or pulley, a whole circumference of the worm thread between A and B comes during the rotation into contact with, say, the right-hand side of the tooth T on the worm wheel and so carries it forward by one pitch of the worm, which is the same as one pitch of the worm wheel if the worm is single threaded. Suppose the worm wheel has 40 teeth; one revolution of the worm carries the wheel round $\frac{1}{40}$ of a revolution, and to carry the wheel round one revolution will require 40 revolutions of the worm, or—

$$\text{Velocity ratio V} = \frac{\text{revolutions of worm}}{\text{revolutions of wheel}} = \frac{40}{1}$$

For any single-threaded worm gearing with a wheel of n teeth the velocity ratio will be

$$\text{V} = \frac{\text{revolutions of worm}}{\text{revolutions of wheel}} = n$$

Thus very large velocity ratios are attained by the use of one worm and wheel.

If the worm is double threaded one revolution of the worm will advance the worm by one pitch of the helix of the screw in which lie two complete threads, and the worm wheel will be advanced by *two* teeth, the pitch of the wheel teeth being *half* the pitch of the worm. In this case, for a wheel of n teeth the velocity ratio will be—

$$\text{V} = \frac{\text{revolutions of worm}}{\text{revolutions of wheel}} = \frac{n}{2}$$

Similarly multiple-threaded worms of large pitch having several threads in each pitch are frequently used, and in this case less than one complete turn or pitch of each thread (and therefore of the worm) may be necessary to drive the wheel. A common example of this is to be found in the gearing which drives the side shaft of a gas or oil engine. If there are m threads per pitch of the screw and n teeth in the wheel, the velocity ratio is $\frac{n}{m}$. Such gears are also used to connect pairs of shafts which are not at right angles to each other by giving the teeth suitable inclinations, and are called skew gears or spiral gears.

Efficiency of Worm Gearing.—In the contact of worm gears there is considerable sliding friction, as well as some where the bearing prevents motion of the worm along its axis, and in these

cases when high velocity ratios are obtained the efficiency is not usually high.

Worm-driven Pulley Blocks.—Fig. 119 illustrates an example of this type of pulley block. The driving chain A passes round the pulley B, on the spindle of which is keyed a worm (not shown) which gears with the worm wheel C. An endless chain, D, is attached to the frame of the machine at E, and passes round a drum which is fixed to the same spindle as C and then round the snatch block G back to the frame at F as shown. The weight to be lifted is slung on the hook under the snatch block G, and the effort applied by a man pulling the chain A.

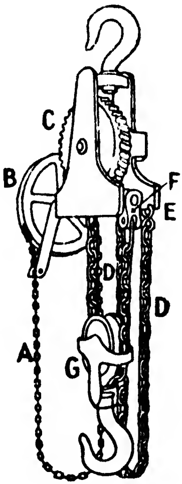


FIG. 119.—Worm-geared pulley blocks.

Compound Screw Jack.—This lifting machine is a compound machine consisting of a worm and wheel and a screw working in a nut. In Fig. 120, which for simplicity shows the machine with all bearings removed, A is a pulley, at the circumference of which the effort is applied, fixed to the spindle B which has the worm C attached to it. The worm C gears with the worm wheel D attached to the screw E by a feather or sliding key. The screw E works in the nut F, which is part of the frame of the machine. On the top of the screw E there is a platform, G, which carries the load to be lifted. The motion is from the pulley (or handle) A through the worm and wheel. The screw E rotates with the wheel D, and in so doing advances through the nut F, lifting the load W.

Efficiency of Combined Machines.—The efficiency of a combined machine is the product of the efficiencies of each part. For example, if 100 foot-pounds of work are done by the effort in a mechanism of 70 per cent. efficiency, the work transmitted is—

$$100 \times \frac{70}{100} = 70 \text{ foot-pounds}$$

If this amount of work is then supplied to another mechanism of 60 per cent. efficiency, the work transmitted by this second mechanism is—

$$70 \times \frac{60}{100} = 42 \text{ foot-pounds.}$$

The combined efficiency is $\frac{42}{100}$ or 42 per cent. or 0·42, that is, the product—

$$\frac{70}{100} \times \frac{60}{100} = 0{\cdot}42, \text{ or } 42 \text{ per cent.}$$

Similarly if there are more than two mechanisms in a compound machine, the efficiencies of each being, E_1, E_2, and E_3, etc., the combined efficiency will be—

$$E = E_1 \times E_2 \times E_3 \times E_4 \times \text{etc.}$$

In the compound screw-jack shown in Fig. 120, the effective circumference of pulley A was 42·5 inches, number of teeth in worm wheel D 38, pitch of screw E $\frac{1}{2}$ inch. For one revolution of A the

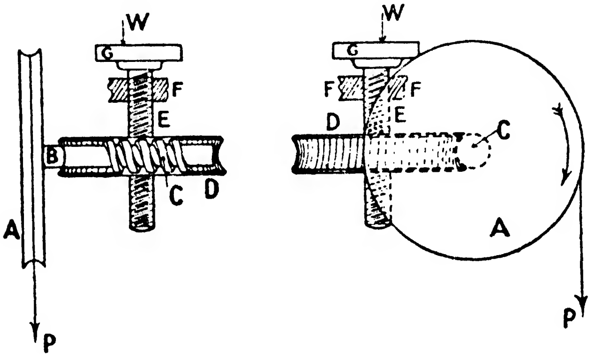


FIG. 120.—Compound screw-jack.

effort moves 42·5 inches, and the worm wheel and screw makes $\frac{1}{38}$ of a revolution. Hence the load is lifted $\frac{1}{38} \times \frac{1}{2} = \frac{1}{76}$ inch, and the velocity ratio V is—

$$\frac{\text{distance moved by P}}{\text{distance moved by W}} = \frac{42 \cdot 5}{\frac{1}{38} \times \frac{1}{2}} = 42 \cdot 5 \times 76 = 3230$$

The following results were obtained in a test of this machine :—

Load W (lbs.).	Effort P (lbs.).	Friction (PV − W) (lbs.).	Efficiency per cent. $\frac{W}{PV} \times 100$
0	0·187	604	0
5	0·203	650	0·76
10	0·218	694	1·42
15	0·234	740	1·98
20	0·249	784	2·48
25	0·265	831	2·92
30	0·281	877	3·30
35	0·295	918	3·67
40	0·313	970	3·96
45	0·329	1017	4·23

These results are shown graphically on a load base in Fig. 121. The maximum load of 45 lbs. is much below the lifting capacity of the machine, and so the efficiency curve does not become nearly horizontal, like those shown in Chap. VII. The reader will find it instructive to produce the effort line, and estimate the probable efficiency for, say, W = 150 lbs.

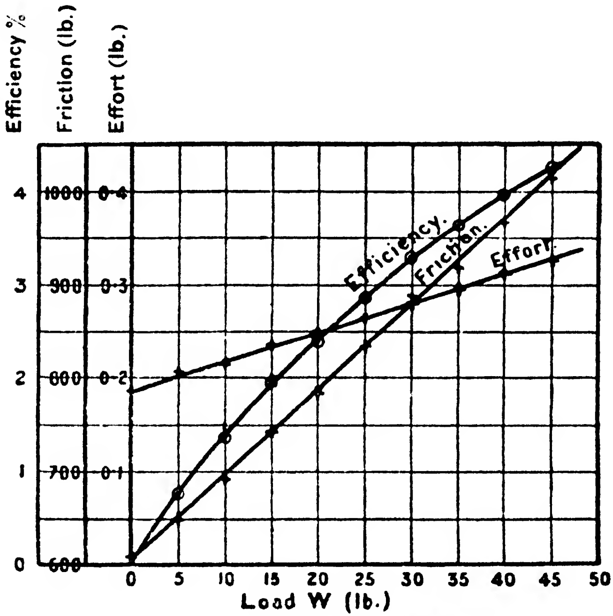


FIG. 121.—Test of compound screw-jack.

EXAMPLES XI.

1. An inclined plane rises 1 foot for each 10 feet of its length. Neglecting friction, find the effort required to draw a body weighing 100 lbs. up the plane, (*a*) when the effort is parallel to the plane, (*b*) when the effort is horizontal.

2. The slope of an inclined plane is 20° to the horizontal. A body weighing 2 cwt. rests on the plane. What force inclined 15° to the plane will be necessary to draw the body up the plane?

3. A tramcar weighs 6 tons, the tractive resistance on the level being 20 lbs. per ton. What horse-power will be required to propel the car at a uniform speed of 12 miles an hour, (*a*) up an incline of 1 in 15, (*b*) on the level. Take the efficiency of the motors and drive as 70 per cent.

4. The indicated horse-power of a locomotive is 786·2. The total weight of engine and train is 200 tons, the tractive resistance being 12 pounds per ton. If the train is ascending a slope at a uniform speed of 30 miles an hour, find the slope, the mechanical efficiency of the engine being 70 per cent.

5. The table of a planing machine is driven by a screw of $\frac{3}{4}$-inch pitch. How many revolutions per minute must the screw make in order that the speed of the table may be 18 feet per minute?

6. A screw-jack has a thread of $\frac{7}{8}$-inch pitch. What effort applied at the end of a handle 15 inches long will be required to lift a load of 2 tons, the efficiency at this load being 45 per cent.?

7. A screw-jack has a double thread, the thickness of the thread being $\frac{1}{4}$ inch. If the lever is 20 inches long, what is the velocity ratio?

8. The table of a planing machine is driven by a screw of 2 inches pitch. The driving pulley on the screw is 20 inches diameter and the difference of tensions of the belt on it is 350 lbs. The coefficient of friction between the table and its guides is 0·20 and the efficiency of the screw is 40 per cent.: find the total weight of the table and the work on it.

9. The efficiency of a screw is 55 per cent. and its pitch 3 inches. What will be the axial thrust of the screw when a twisting moment of 4 tons-inches is applied to the screw?

10. In a right- and left-handed screw coupling for railway carriages the pitch of the two screws is $\frac{3}{8}$ inch. The lever attached to the nut is 18 inches long and a force of 40 lbs. is exerted on the end of it. With what force will the two carriages be drawn together?

11. What must be the speed in revolutions per minute of a single-threaded worm driving a worm wheel of 45 teeth at a speed of 150 revolutions per minute?

12. Solve Question 11 if the worm is double threaded.

13. The worm shown in Fig. 118 is used as a lifting machine. The handle H is 15 inches long, the worm wheel W has 80 teeth, and the drum D is 6 inches diameter. A rope $\frac{3}{4}$ inch diameter is coiled round the drum and carries a load of $\frac{1}{4}$ ton. What effort must be applied at the end of the handle to raise this load if the efficiency is 40 per cent.?

14. In the worm-driven pulley block shown in Fig. 119 the effective diameter of the pulley B is 8 inches. The worm wheel C has 72 teeth and the effective diameter of the drum to which it is fixed is 5 inches. It is found that an effort of 58 pounds is required to lift a load of 3 tons. What is the efficiency of the machine?

15. In a compound screw-jack the handle driving the worm is 13·5 inches long, the number of teeth on the worm wheel is 38 and the pitch of the screw is $\frac{1}{2}$ inch. In order to lift a load of 180 lbs., it is found that an effort of 1·316 lbs. is required. What is the efficiency of the machine?

16. A compound screw-jack has the following dimensions: Length of handle 18 inches, number of teeth on the worm wheel 84, pitch of screw $\frac{3}{8}$ inch. If the worm is single-threaded, what load will be lifted by an effort of 30 lbs. if the efficiency is 10 per cent.?

CHAPTER XII

VARIOUS MACHINES

The Wheel and Differential Axle.—This lifting machine is shown in Fig. 122. It includes a drum or axle, consisting of two

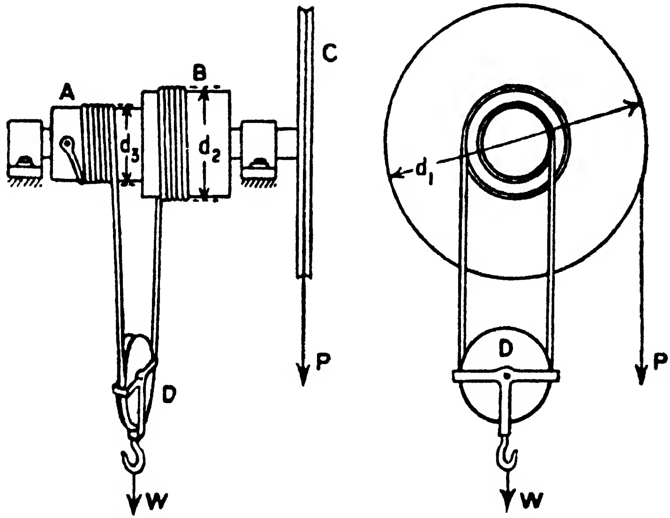


FIG. 122.—Wheel and differential axle.

parts, A and B, of different diameters. A rope or chain is wound on A, as shown, then passes round the snatch block D to the part B. On the two parts A and B the rope is differentially wound; that is, when the axle is turning and winding the rope *on* to the part B, rope is also being unwound from A. The effort is applied by a cord or rope at the circumference of the pulley or wheel C, which is fixed to the compound axle, and therefore rotates with it. The load W to be lifted is carried by the snatch block D.

The velocity ratio is calculated as follows: Consider one

revolution of the wheel C of diameter d_1; the effort P moves a distance πd_1. The axle also makes one revolution, and we have—

Length of rope wound on to B $= \pi d_2$
Length of rope unwound from A $= \pi d_3$
Shortening of rope between A and B $= \pi d_2 - \pi d_3 = \pi(d_2 - d_3)$

Hence, on account of the snatch block, the load is lifted $\frac{\pi(d_2 - d_3)}{2}$, and

$$\text{Velocity ratio } V = \frac{\text{distance P moves}}{\text{distance W moves}} = \frac{\pi d_1}{\frac{\pi(d_2 - d_3)}{2}} = \frac{2d_1}{d_2 - d_3}$$

It should be noticed that d_1, d_2, d_3 are the diameters measured to the centre of the rope, or are equal to the diameter of wheel or axle + diameter of rope.

Experimental Results.—In a test with such a machine, the following results were obtained: Diameter of wheel = 13·7 inches, diameters of axle 5·4 and 4·3 inches respectively. Hence—

$$V = \frac{2 \times 13{\cdot}7}{5{\cdot}4 - 4{\cdot}3} = \frac{27{\cdot}4}{1{\cdot}1} = 24{\cdot}0$$

Load W (lbs.)	Effort P (lbs.)	Friction (PV − W) (lbs.).	Efficiency per cent. $\frac{W}{PV} \times 100$
0	0·094	2·34	0
5	0·45	6·32	44·1
10	0·81	10·31	49·2
15	1·17	14·29	51·2
20	1·53	18·28	52·2
25	1·88	22·26	52·8
30	2·25	26·25	53·3
35	2·61	30·24	53·6
40	2·97	34·21	53·8

These results are shown plotted on a load base in Fig. 123.

Weston Differential Pulley Block.—This machine is shown in ordinary use on the left of Fig. 124, and on the right is shown a diagrammatic sketch. An endless chain passes round the two pulleys A and B and the snatch block C in the following manner: Referring to the right of Fig. 124, the chain passes round A, then round the snatch block C, and then round B. The pulleys A and

B are of different diameters, and both run on the same spindle. Suppose the pulleys make 1 revolution, then an amount of chain equal to the circumference of A is pulled over by the effort, and at the same time an amount equal to the circumference of B passes

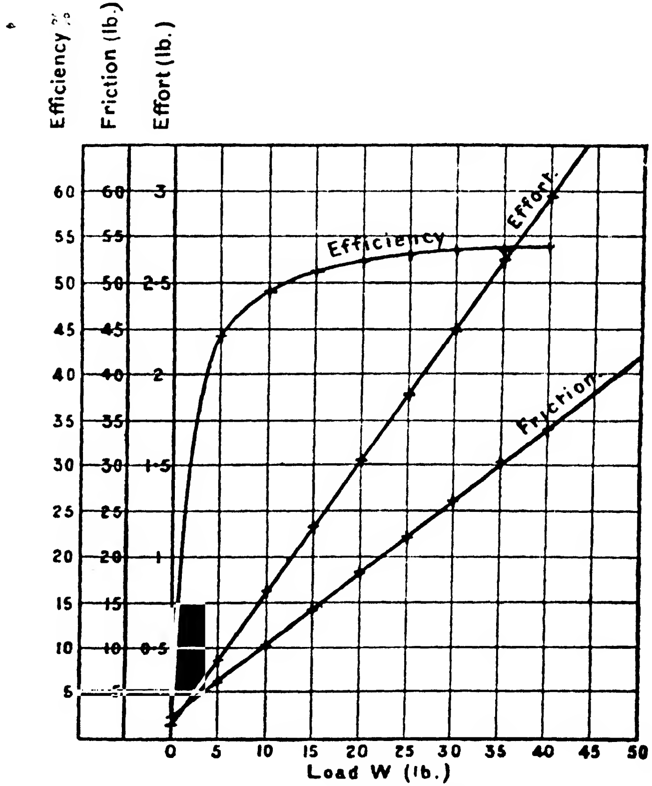


FIG. 123—Test of wheel and differential axle.

over B. Hence, the chain connecting A and B with the snatch block shortens by an amount equal to the difference of the circumferences of the two pulleys, and the load is lifted *half* this amount. The velocity ratio is therefore—

$$V = \frac{\text{Circumference of larger pulley A}}{\frac{1}{2}(\text{difference of circumferences of A and B})}$$

The pulleys have recesses cut in them for the links of the chain to fit into; hence the actual circumferences could not be measured with certainty. The best way is to count the number of flats on

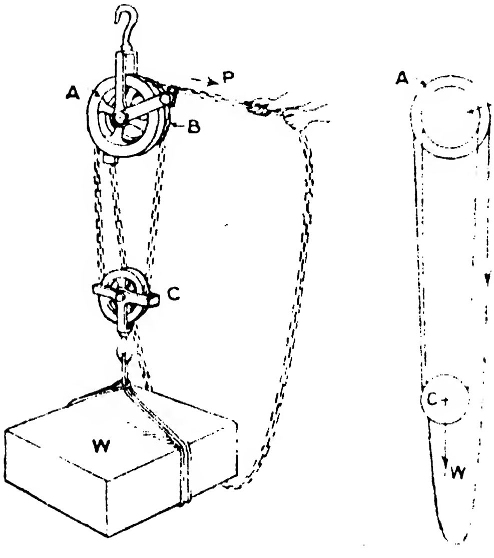


FIG. 124.

each pulley; that number being, of course, a measure of the circumference.

In an actual experiment it was found that the pulleys A and B had 8 and 7 flats respectively; hence—

$$\text{Velocity ratio } V = \frac{8}{\frac{1}{2}(8 - 7)} = 16$$

This value was checked experimentally, when it was found that, in order to lift the load 6 inches, 96 inches of chain had to be pulled off the larger pulley; hence, $V = \frac{96}{6} = 16$ as before.

The results obtained are shown in the following table, and are plotted on a load base in Fig. 125.

Load W (lbs.).	Effort P (lbs.).	Friction (PV − W) (lbs.).	Efficiency per cent. $\frac{W}{PV} \times 100$.
0	1·5	24·0	0
5	2·4	33·4	13·0
10	3·0	38·0	20·8
15	4·2	52·0	22·4
20	5·0	60·0	25·0
25	5·9	69·4	26·5
30	7·0	82·0	26·8
35	7·6	87·0	28·7
40	9·0	104·0	27·8
45	9·6	108·8	29·2
50	10·5	118·0	29·8
55	11·2	124·0	30·7
60	11·5	124·0	32·6

Lifting Crab or Winch.—This machine is used for lifting comparatively heavy loads by means of a small effort such as can be exerted by hand. It is used when a greater velocity ratio and also mechanical advantage is required than can be conveniently obtained by a wheel and axle, in which convenient length of handle is limited by the range of a man's arm, and the diameter of the drum by the consideration of holding all the rope necessary for a given lift. The greater velocity ratio is obtained by the introduction of a pair or pairs of spur wheels between the axles of the handle and drum.

The Single-purchase Crab is shown on the right of Fig. 126 in perspective, and on the left is a diagrammatic end view. It consists of two standards, A_1 and A_2, connected rigidly together by the three stays, B_1, B_2, and B_3, and having bearings for the spindle C and the drum D. On the spindle C is keyed the pinion E, which gears with and drives the large spur wheel F on the drum D. The ends of C are squared to receive handles H, on the ends of which the effort is applied, a man working at each handle if necessary. The load is lifted by a rope which is coiled round the drum D. If the crab is placed directly over the hole through which the load is lifted, the lift is said to be direct. If this is impossible the crab may be placed in any convenient position, and the rope taken from the drum over a pulley placed directly above the hole forming an inclined lift from the crab.

Velocity Ratio.—Let the length of the handle H be 18 inches, the diameter of the drum D 6 inches, the number of teeth in the pinion E 20, and in the spur wheel F 240, then in one revolution of the handle we have—

Motion of effort $= 2\pi \times 18 = 36\pi$ inches.

Motion of pitch circle of pinion $= 1$ revolution

$= 20p$ (p = pitch).

Motion of pitch circle of wheel $= 20p$

Angular motion of wheel $= \dfrac{20p}{240p} = \frac{1}{12}$ revolution

$\therefore$ Motion of load $= \frac{1}{12} \times$ circumference of drum

$= \dfrac{1}{12} \times \pi \times 6 = \dfrac{\pi}{2}$ inches.

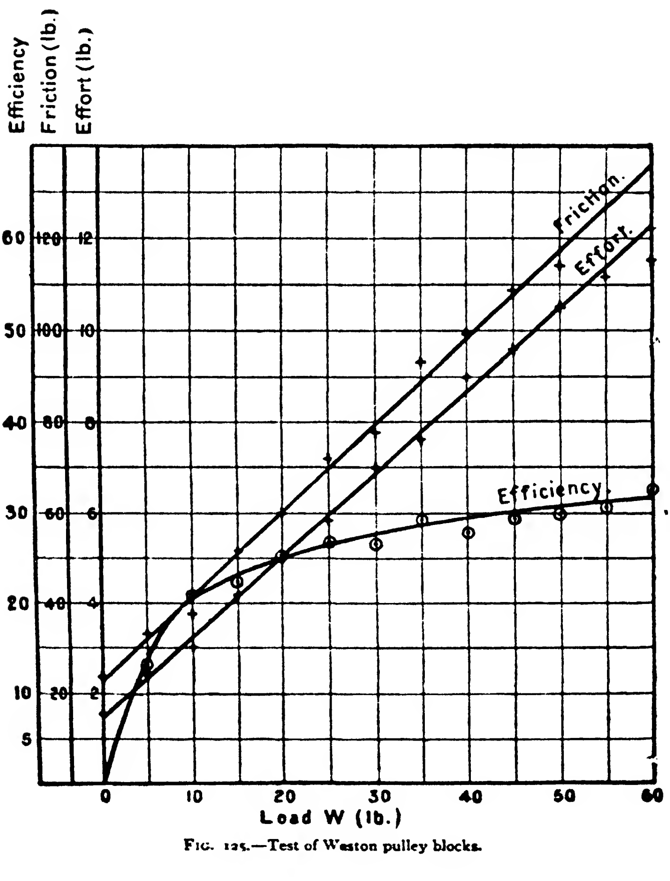


FIG. 125.—Test of Weston pulley blocks.

Hence the velocity ratio—

$$V = \frac{\text{motion of effort}}{\text{motion of load}} = \frac{36\pi \times 2}{\pi} = 72.$$

Or in general terms, if R = radius or length of handle, T_1 = number of teeth in pinion, T_2 = number of teeth in wheel, r = radius of drum, then in one revolution of the handle we have—

$$\text{Motion of effort} = 2\pi R;$$

$$\text{Motion of pitch circle of pinion} = 1 \text{ revolution} = T_1 p;$$

$$\text{Angular motion of pitch circle of wheel} = \frac{T_1 p}{T_2 p} = \frac{T_1}{T_2} \text{ revolution};$$

$$\text{Motion of load} = \frac{T_1}{T_2} \text{ of } 2\pi r.$$

Hence—

$$\text{Velocity ratio } V = \frac{2\pi R}{\frac{T_1}{T_2} \times 2\pi r} = \frac{R}{r} \times \frac{T_2}{T_1}$$

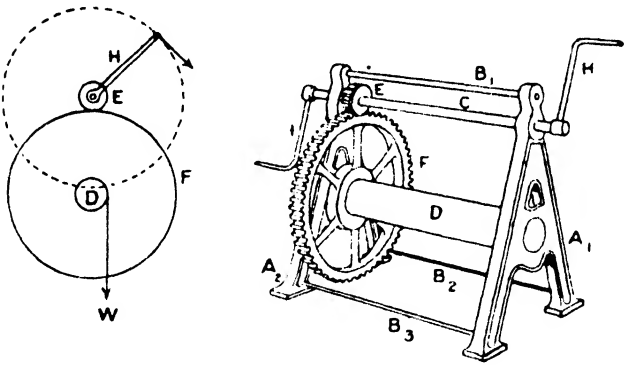


FIG. 126.—Single-purchase crab.

which is the product of the velocity ratios of a wheel and axle of the same sizes as the handle and drum, and a pair of spur wheels of given numbers of teeth.

Pressure between the Teeth.—Consider the crab with the dimensions taken above, then neglecting friction, in 1 revolution of the handle we have—

$$\text{Motion of effort} = 2\pi \times 18 = 36\pi \text{ inches},$$

$$\text{Motion of pitch circle of pinion} = 1 \text{ revolution} = 20p,$$

Hence by the principle of work, neglecting the friction—

$$P \times 36\pi = F \times 20p$$

where P = effort applied at end of handle, and F the pressure between the teeth. Suppose the pitch of the teeth is 1 inch and an effort P of 80 lbs. is applied, then—

$$F = P \times \frac{36\pi}{20p} = 80 \times \frac{36\pi}{20} = 144\pi = 452.5 \text{ lbs.}$$

Or in general terms, in one revolution of the handle—

$$\text{Motion of effort} = 2\pi R$$
$$\text{Motion of pitch circle of pinion} = 1 \text{ circumference} = T_1 p.$$

Hence by the principle of work—

$$P \times 2\pi R = F \times T_1 p$$
$$F = P \times \frac{2\pi R}{T_1 p}$$

Actually, the pressure (F) between the teeth would be less than this amount because of friction.

Again, taking F as the effort at the circumference of the pitch circle of the wheel fixed to the drum—

$$\text{Load lifted } W = F \times \frac{\text{circumference of wheel}}{\text{circumference of drum}} = F \times \frac{T_2 p}{2\pi r}$$

substituting for F its value found above—

$$W = P \times \frac{2\pi R}{T_1 p} \times \frac{T_2 p}{2\pi r} = P \times \frac{R}{r} \times \frac{T_2}{T_1}$$

and the mechanical advantage without friction $\left(\frac{W}{P}\right)$ is $\frac{R}{r} \times \frac{T_2}{T_1}$, which checks the value of V already found.

We might also find the pressure F by the rules for levers taking the handle axle as fulcrum. The radius of the pitch circle of the pinion is $\frac{T_1 p}{2\pi}$, and taking moments about the axle of the handle we get—

$$F \times \frac{T_1 p}{2\pi} = P \times R$$
$$\therefore F = P \times \frac{2\pi R}{T_1 p} \text{ as before.}$$

Example.—In a single-purchase crab the pinion has 25 teeth, the spur wheel 250 teeth, and the radius of the drum is 5 inches. A pulley of 2 feet diameter is on the axle of the pinion (instead of a handle), and

the effort is applied by a cord passing round this pulley. The following results were obtained :—

Load W (lbs.).	Effort P (lbs.).
0	
40	2·8
80	5·0
100	6·0
150	8·45

Find the effect of friction when lifting the load of 100 lbs.

The velocity ratio is—

$$V = \frac{12}{5} \times \frac{250}{25} = 24$$

and the effect of friction is $F = PV - W$

$$= 6 \times 24 - 100 = 144 - 100 = 44 \text{ lbs.}$$

If we wanted to find the effort and friction for loads not given in the above short table (say, for 120 lbs. for example), we should plot the values of W and P from the above table, and W and F as described in Chapter VII. Having plotted these curves, the values of P and F can be read off for any load.

Double-purchase Crab.—This machine is shown in diagrammatic form in Fig. 127. The effort P is applied at the pulley (or handle) A on the same axle of which is a pinion B gearing with the wheel C. On the same axle as C is another pinion D gearing with the wheel E fixed to the drum F. The rope by which the load W is lifted is coiled round this drum.

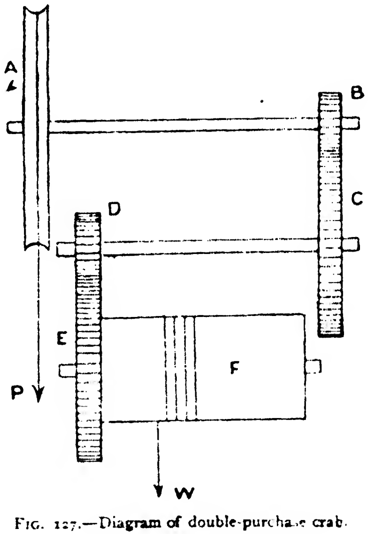


Fig. 127.—Diagram of double-purchase crab.

Velocity Ratio.—Let R be the radius of the wheel A and r the radius of the drum F, and let B, C, D, E denote the numbers of teeth of pitch p in the pinions and wheels respectively, then in *one* revolution of the pulley (or handle) A we have—

$$\text{Motion of effort} = 2\pi R$$

$$\text{Motion of pitch circle of pinion B} = 1 \text{ circumference} = Bp$$

$$\text{Motion of pitch circle of wheel C} = Bp \text{ or } \frac{Bp}{Cp} = \frac{B}{C} \text{ of a revolution}$$

Motion of pitch circle of pinion $D = \frac{B}{C}$ revolution

$$\text{Motion of pitch circle of wheel } E = \frac{Bp}{Cp} \times \frac{Dp}{Ep} = \frac{B}{C} \times \frac{D}{E} \text{ revolution}$$

$$\text{Hence motion of load} = \frac{B}{C} \times \frac{D}{E} \text{ of circumference of drum}$$

$$= \frac{B}{C} \times \frac{D}{E} \times 2\pi r$$

$$\text{and velocity ratio } V = \frac{\text{motion of effort}}{\text{motion of load}} = \frac{2\pi R}{\frac{B}{C} \times \frac{D}{E} \times 2\pi r}$$

$$= \frac{R}{r} \times \frac{C}{B} \times \frac{E}{D}$$

which is the product of the velocity ratios of a wheel and axle $\left(\frac{R}{r}\right)$, the train of wheels B, C, D, E, namely $\left(\frac{C}{B} \times \frac{E}{D}\right)$.

Pressure between the Teeth.—The pressure between the teeth of B and C would be found exactly the same as for the single-purchase crab previously considered. To find the pressure between the teeth of the second pair of wheels D and E we proceed as follows :—

In one revolution of the pulley A (or handle)—

$$\text{Motion of effort} = 2\pi R.$$

Motion of pitch circle of pinion $D = \frac{B}{C}$ revolution or $\frac{B}{C} \times Dp$.

Hence by the principle of work—

$$P \times 2\pi R = F \times \frac{B}{C} \times Dp.$$

$$F = P \times \frac{2\pi R}{\frac{B}{C} \times Dp}$$

Actually the pressure (F) between the teeth of D and E would be less than this amount because of friction.

Experiment.—The particulars of a double-purchase crab, like the one shown diagrammatically in Fig. 127, were : Effective circumference of wheel A = $41\frac{1}{2}$ inches, and of drum F 16 inches. Number of teeth on B = 20, C = 40, D = 30, E = 70. The results obtained in a series of tests are tabulated on p. 170, and are shown plotted on a load base in Fig. 128.

Spur-geared Pulley Blocks.—Fig. 129 snows one form of this type of pulley block designed for lifting 5 cwt., the right-hand side

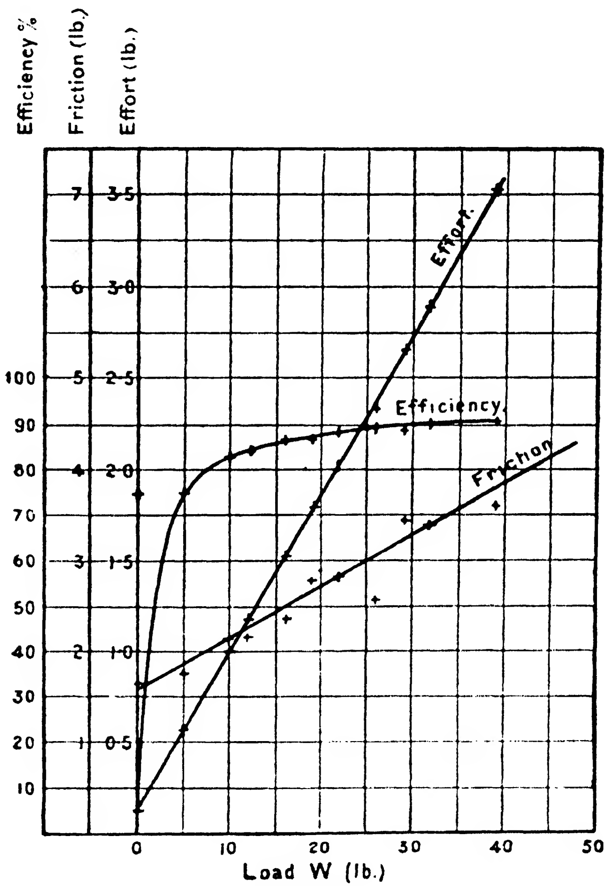


FIG. 128.—Test of double-purchase crab.

being in diagrammatic form for clearness. On the same spindle as the chain wheel A, is the ratchet and clutch E, on which the pawl D is pressed by means of the small helical spring F, and the

pinion B. The pinion B gears with the spur wheel C, on the same spindle of which is keyed the chain drum G. The wheel A and

Load W (lbs.).	Effort P (lbs.).	Friction (PV − W) (lbs.).	Efficiency $\frac{W}{PV} \times 100$
0	0·14	1·69	0
5	0·56	1·78	75·7
10	1·00	2·10	82·6
12	1·18	2·16	84·0
16	1·52	2·39	87·0
19	1·80	2·78	87·2
22	2·05	2·80	88·7
26	2·36	2·56	89·4
29	2·68	3·43	89·6
32	2·92	3·33	90·2
39	3·52	3·59	91·3

the drum G have cogs and recesses in which the links of the chains fit. The effort is applied to an endless chain which passes over

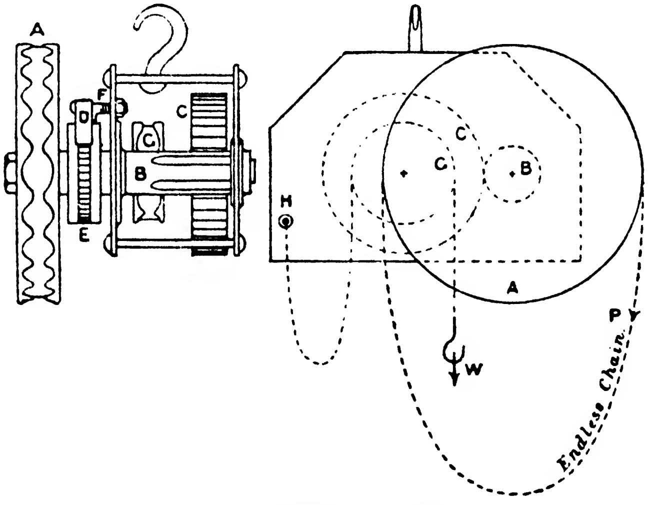


FIG. 129.—Spur-geared pulley block.

the wheel A. The lifting chain passes halfway round the drum G, and is attached at one end to the frame of the machine at H; the

other end has a hook attached from which the load W is hung. When the load is being lifted, the ratchet E passes under the pawl D, B rotating with A and so lifting the load, the drive being through B and C. When the effort is removed the pawl prevents the load running back again, and in this manner a highly efficient gear can be used without overhauling. In the machine shown, the wheel A has 12 cogs, and the drum G 5 cogs, while B has 6 teeth, and C 20 teeth.

The velocity ratio is therefore—

$$V = \frac{12}{5} \times \frac{20}{6} = 8.$$

The results obtained in a series of tests are shown in the following table, and the reader is advised to plot them on a load base as explained in Chap. VII.

Load W (lbs.)	Effort P (lbs.).	Friction PV − W. (lbs.)	Efficiency $\frac{W}{PV} \times 100$
0			0
40	7·0	16	71·4
80	13·0	24	77·0
120	19·5	36	77·0
160	27·0	56	74·0
200	31·0	48	80·7
240	38·0	64	79·0
280	43·5	68	80·5
320	50·0	80	80·0
360	57·0	96	79·0
400	61·5	92	81·2
440	67·0	96	82·0
480	72·5	100	82·8
520	78·5	108	83·0
560	84·5	116	83·0

Screw Cutting.—Fig. 130 shows a simple form of lathe arranged for screw cutting. A is the fast headstock, B is the loose headstock attached to the lathe bed C. Between the lathe spindle and the guide screw K, the *change* wheels F and G are carried on studs fixed to the banjo plate D, which can turn about the axis of the guide screw. The change wheel E is on the lathe spindle, and the change wheel H is on the guide screw. The wheels are put in gear by swinging D over until the wheel F gears with E, in which position D is locked by means of a nut not shown. To put the wheels out of gear, D is swung over in the opposite direction until F disengages with E, and then

locked as before. The saddle M carries a split nut underneath, in which the guide screw K works. When a screw is to be cut in the lathe, the proper change wheels are fitted in gear, and the split nut on M is put into gear with the guide screw K by means of the lever L. When this is done, any rotation of the guide screw causes the saddle M, and therefore the screw-cutting tool N, to travel along the bed, so cutting the screw on J.

When cutting a right-handed screw the saddle moves from the loose headstock B towards the fast headstock, and if the guide screw is right-handed, it must rotate in the same direction as the screw to be cut. If a left-handed screw is to be cut, the saddle

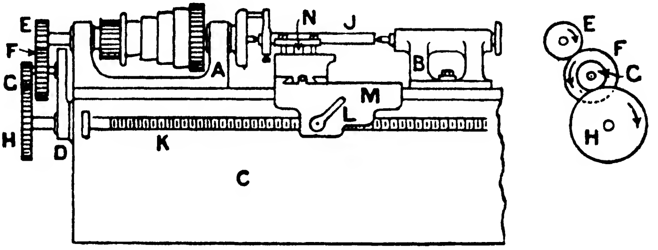


FIG. 130.

moves from the fast headstock to the loose headstock, the guide screw rotating in the opposite direction to the screw to be cut. This reversal of motion is obtained by adding another intermediate change wheel, either between E and F, or between G and H.

Calculation of Change Wheels.—The change wheel on the lathe spindle is the first driver, and that on the guide screw the last follower. Suppose the pitch of the guide screw is $\frac{1}{2}$ inch (right-handed), and it is required to cut a right-handed screw of $\frac{1}{8}$-inch pitch, then the lathe spindle must rotate with 4 times the speed of the guide screw, and in the same direction, the velocity ratio of the lathe spindle to the guide screw being 4. Therefore, a wheel of 30 teeth on the lathe spindle driving a 120 wheel on the guide screw by any *one* intermediate idle wheel will cut the right-handed screw of $\frac{1}{8}$-inch pitch. The introduction of a *second* intermediate wheel in the simple train would cause the guide screw to rotate in the opposite direction, and so cut a *left-handed* screw of $\frac{1}{8}$-inch pitch. In Fig. 131 (*a*) shows the wheels for a right-handed screw, and (*b*) for a left-handed screw. Any other conveniently sized wheels would do, providing that the velocity ratio is the same.

Thus, we might have a 25 wheel on the spindle and 100 wheel on the guide screw, or a 35 wheel on the spindle and a 140 wheel on the guide screw, and so on.

If a simple train with intermediate wheels is required we have the rule—

$$\frac{\text{pitch of screw to be cut}}{\text{pitch of guide screw}} = \frac{\text{number of teeth in driver}}{\text{number of teeth in follower}}$$

When a large velocity ratio between the lathe spindle and guide screw is required, as, for example, when cutting a screw

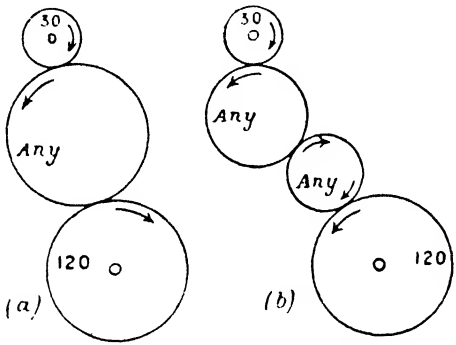


FIG. 131.—Diagram of change wheels for screw cutting.

of fine pitch, a simple train is not practicable; in such cases a compound train is necessary, and we, as explained in Chap. X., have the rule—

$$\frac{\text{pitch of screw to be cut}}{\text{pitch of guide screw}} = \frac{\text{number of teeth in first driver}}{\text{number of teeth in first follower}} \times \frac{\text{number of teeth in second driver}}{\text{number of teeth in second follower}}$$

Example.—Arrange a train of change wheels to cut a right-handed thread of $2\frac{1}{2}$ inches pitch, the guide screw being right-handed and of pitch $\frac{1}{2}$ inch.

Applying the above rule we have—

$$\frac{2\frac{1}{2}}{\frac{1}{2}} = \frac{5}{1} = \frac{2}{1} \times \frac{2\frac{1}{2}}{1}$$

We can arrange any wheels which have the above ratios, for instance—

$$\frac{2}{1} \times \frac{2\frac{1}{2}}{1} = \frac{60}{30} \times \frac{50}{20} \text{ (this train is shown in Fig. 132).}$$

Or–

$$\frac{2}{1} \times \frac{2\frac{1}{4}}{1} = \frac{70}{35} \times \frac{90}{40} \text{ and so on.}$$

Each screw-cutting lathe is supplied with a set of change wheels, the numbers of teeth in which range from 20 to about 140, increasing by 5 teeth from 20 to 80, and then by 10 teeth. There are usually *two* wheels of 40 teeth.

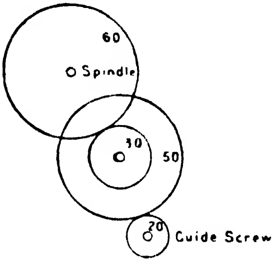


FIG. 132.

Lathe Back Gears.—In addition to the speed cone on the lathe spindle, a further reduction of speed is obtained by using a back gear such as that shown in Fig. 133, where A is the speed cone which rides loosely on the lathe spindle B, and is driven by a belt from a similar cone on the counter-shaft, as in Fig. 97. To A is attached the pinion C, which gears with the wheel D. The wheel D is attached by a hollow sleeve to the pinion E, so that D and E ride loosely on the spindle G, and E gears with the wheel F, which is keyed to the lathe spindle B. When arranged, as in Fig. 133, for each revolution of A the lathe spindle will make—

$$\frac{C}{D} \times \frac{E}{F} \text{ of a revolution}$$

where C, D, E, F represent the numbers of teeth respectively in the wheels.

The wheels, C, D, E, and F, constitute the back gear, and to enable the lathe spindle to be driven at a higher speed without shifting the belt on A, the wheel F is rigidly attached to the cone A by means of a bolt, and D and F thrown out of gear with C and F.

Example.—The speed cones on the countershaft and lathe have diameters 5, $6\frac{1}{2}$, $8\frac{1}{2}$ and 10 inches, and in the back gear C has 15 teeth (Fig. 133), D 45, E 15, F 45 teeth. If the countershaft makes 120 revolutions per minute, at what speeds can the lathe spindle be driven?

With back gear out we have—

Highest speed (belt on 5-inch step on lathe) $= 120 \times \frac{10}{5} = 240$ revs. per minute

With belt on $6\frac{1}{2}$-inch step, speed $= 120 \times \frac{8\cdot5}{6\cdot5} = 157$ revs. per minute

With belt on $8\frac{1}{2}$-inch step, speed $= 120 \times \frac{6\cdot5}{8\cdot5} = 97\cdot5$ revs. per minute

With belt on 10-inch step, speed $= 120 \times \frac{5}{10} = 60$ revs. per minute

With back gear in we have—

With belt on 5-inch step, speed $= 240 \times \frac{15}{45} \times \frac{15}{45} = 26{\cdot}6$ revs. per minute
With belt on $6\frac{1}{2}$-inch step, speed $= 157 \times \frac{15}{45} \times \frac{15}{45} = 17{\cdot}4$ revs. per minute
With belt on $8\frac{3}{4}$-inch step, speed $= 97{\cdot}5 \times \frac{15}{45} \times \frac{15}{45} = 10{\cdot}8$ revs. per minute
With belt on 10-inch step, speed $= 60 \times \frac{15}{45} \times \frac{15}{45} = 6{\cdot}6$ revs. per minute

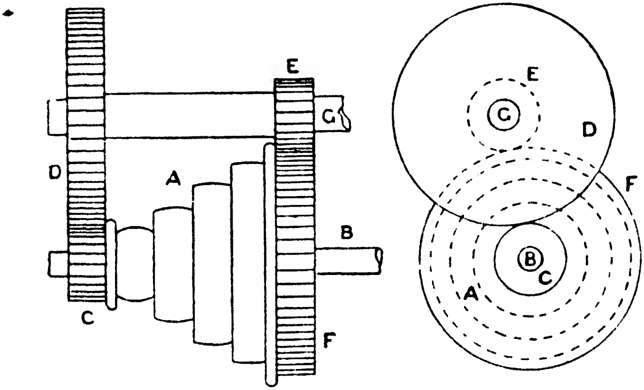


FIG. 133.—Back gear of lathe.

EXAMPLES XII.

1. In a differential wheel and axle, the diameter of the large axle is 8 inches, and of the small axle 6 inches. The effort is applied at the end of a handle 15 inches long. Find the velocity ratio.

2. If, in Question 1, an effort of 30 lbs. lifts a load of 420 lbs., what is the effect of friction, and the efficiency of the machine at this load?

3. In a Weston pulley block there are 10 flats on the larger pulley, and 9 on the smaller pulley. What load will be lifted by an effort of 50 lbs., the efficiency at this load being assumed to be 33 per cent.?

4. A single-purchase crab has handles 15 inches long, drum 6 inches diameter, diameter of lifting rope $\frac{1}{2}$ inch. Number of teeth on pinion 25, on wheel 130. Estimate the velocity ratio.

5. In Question 4, what will be the pressure between the teeth when an effort of 40 lbs. is applied at the handle, (*a*) neglecting friction, (*b*) if the efficiency is 80 per cent.? Take the pitch of the teeth to be $\frac{1}{2}$ inch.

6. A double-purchase crab has the following dimensions: Diameter of drum measured to centre of rope, 7 inches; length of handle, 14 inches; number of teeth in pinions, 12 and 20; number of teeth in spur wheels, 78 and 95. It is found that an effort of 40 lbs. applied at the handle will lift a load of 2 tons. What is the efficiency of the machine?

7. The wheels and pinions in Question 6 have a pitch of 1 inch. What will be the pressure between the teeth of each pair of wheels (neglecting friction), when the effort exerted is 50 lbs.?

8. What change wheels will be required to cut a right-handed screw of 6

inches pitch in a lathe whose guide screw is right-handed and of $\frac{1}{2}$-inch pitch? Of the trains possible, take that in which the wheel on the lathe spindle has 80 teeth, and that on the guide screw 30 teeth.

9. Give a train of change wheels which will cut a screw (left-handed) of $\frac{1}{16}$-inch pitch in a lathe whose guide screw is right-handed and of $\frac{1}{4}$-inch pitch.

10. A back gear has pinions of 20 teeth and 20 teeth, and wheels of 60 an 60 teeth. If the highest speed at which the lathe can be driven is 240 revolutio per minute, what will be the highest speed with the back gear in?

11. The speed cones on the countershaft and lathe have diameters 4, 5, 6, and 8 inches, and in the back gear there are two wheels of 15 teeth, and two of 45 teeth. If the countershaft makes 180 revolutions per minute, at what speeds can the lathe spindle be driven?

CHAPTER XIII

RESULTANT AND COMPONENT FORCES[1]

FORCES may be added or compounded by drawing vectors to scale and finding the length of the resultant of the several vectors. This was the method used in Chap. I. We may, however, also calculate the resultant without actually drawing vectors to scale; in some cases the calculation may be very simple, and in others accurately drawing vectors may be more convenient. We now illustrate the method of calculation by a few simple cases.

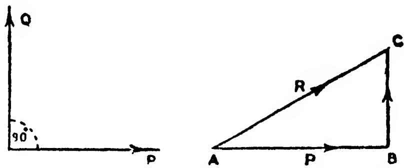


FIG. 134.—Resultant of two forces at right angles.

Resultant of Two Forces at Right Angles.—Let P and Q (Fig. 134) be the forces. If we add these forces vectorally by the method explained in Chapter I., P and Q being at right angles, we have the resultant R represented by AC. This resultant may be calculated as follows:—

$$AC^2 = AB^2 + BC^2 \text{ (see Introduction, p. 6)}$$
$$R^2 = P^2 + Q^2$$
$$R = \sqrt{P^2 + Q^2}$$

Example.—If forces of 8 lbs. and 5 lbs. act on a body at right angles, what force acting through the point of intersection of the two will keep the body at rest?

If Fig. 135 represents a sketch only (not drawn to scale) of the triangle of forces for 8 and 5 lbs. at right angles, we have from the above method, if R is the resultant—

$$R^2 = 8^2 + 5^2 = 64 + 25 = 89$$
$$\therefore R = \sqrt{89} = 9{\cdot}44 \text{ lbs.}$$

[1] An understanding of this Chapter, though desirable, is not absolutely necessary in order to follow the remainder of the book.

The direction of the resultant can be stated by the angle which makes with, say, the 8 lbs. force, for in Fig. 135—

$$\tan \quad = \frac{BC}{AB} = \frac{5}{8} = 0{\cdot}625 \text{ (see Introduction, p. 5).}$$

Looking at the table of tangents we find that 0·652 is the tan of 32° nearly. Hence the resultant is 9·44 lbs., making an angle 32° with the 8 lbs. force, and the force of 9·44 lbs. *opposite* to the

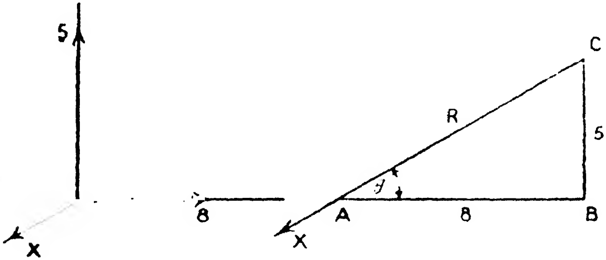


FIG. 135.

sultant, that is, in the direction of the arrow X, is the equilibrant force required to keep the body at rest.

Resultant of Two Forces not at Right Angles.—Let P and Q (Fig. 136) be the forces making an acute angle with each

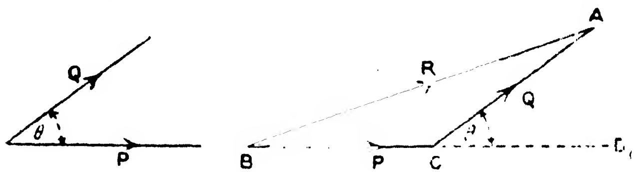


FIG. 136.—Resultant of two oblique forces.

other. If we draw the triangle of forces, we have the resultant represented by BA, which is calculated as follows:—

$$BA^2 = BC^2 + CA^2 + 2 \times BC \times CA \cos A\hat{C}D$$

or $$c^2 = a^2 + b^2 + 2ab \cos A\hat{C}D \text{ (see Introduction, p. 6)}$$

or $$R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

where θ is the angle between the forces P and Q.

If the forces make an *obtuse* angle with each other we see the triangle of forces (Fig. 137) that—

$$R^2 = P^2 + Q^2 - 2PQ \cos B\hat{C}A.$$

Now, BĈA is not θ the angle between P and Q, but is equal to θ, that is, the *supplement* of θ, and we must take care alculating the resultant of two forces making an *obtuse* angle ich other to use the angle $180 - \theta$ (or the supplement of θ angle between the forces) and not θ itself in the above equa- The reader who has studied trigonometry will recognize that is equivalent to using the angle θ in the same formula as that ously used for two forces at an acute angle.

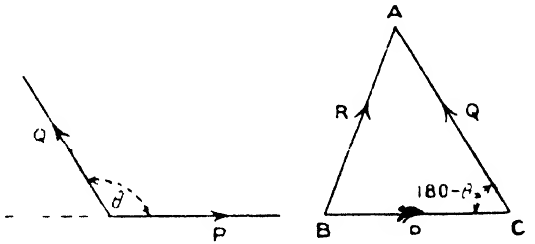


Fig. 137

Example.—If forces of 8 lbs. and 5 lbs. act on a body at an angle what will be their resultant?

$$\begin{aligned} R^2 &= P^2 + Q^2 + 2P.Q \cos 60^\circ \\ &= 8^2 + 5^2 + 2 \times 8 \times 5 \times 0{\cdot}50 \\ &= 64 + 25 + 40 = 129 \\ \therefore R &= \sqrt{129} = 11{\cdot}35 \text{ lbs.} \end{aligned}$$

Example 2.—If forces of 8 and 5 lbs. act on a body at an angle of , what will be their resultant?

$$\begin{aligned} R^2 &= P^2 + Q^2 - 2 + P.Q \cos (180 - 120) \\ &= P^2 + Q^2 - 2.P.Q \cos 60 \\ &= 8^2 + 5^2 - 2 \times 8 \times 5 \times 0{\cdot}50 \\ &= 64 + 25 - 40 = 49 \\ R &= \sqrt{49} = 7 \text{ lbs.} \end{aligned}$$

The reader may check these results graphically.

Resolution of Forces.—Let the vector AB (Fig. 138) repre- a force P in magnitude and direction acting on a body; its ct in any direction such as AC inclined at an angle θ is known he resolved part of P in the direction AC, or as the rectangular *ponent* AB in the direction AC. The magnitude of this com- ent is obtained by drawing a perpendicular BC from B on to direction AC. Then AC represents the component of P in the n direction, and may be calculated as follows:—

$$\frac{AC}{AB} = \cos\theta \text{ (see Introduction, p. 5)}$$
$$\therefore AC = AB \times \cos\theta$$
or component of P in direction AC = $P\cos\theta$.

Hence, to resolve a force in one of any given pair of pe[r]dicular directions making an angle θ with the line of action o[f th]e force we multiply the forc[e by] the cosine of this angle. [To] resolve P (Fig. 138) in th[e di]rection AD inclined 90° t[o AC,] we draw from B the perpen[dicu]lar BD on to AD, then [AD] represents the component o[f P] in that direction. Now, B[C is] equal to AD since ADBC [is a] rectangle, hence BC also r[epre]sents the component of P [in a] direction perpendicular to AC, and we have—

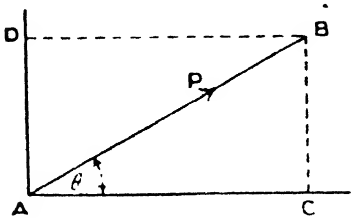


FIG. 138.—Components of a force.

$$\frac{BC}{AB} = \sin\theta \text{ (see Introduction, p. 5)}$$
$$BC = AB\sin\theta = P\sin\theta.$$

We see, then, that there is no need to draw BD but only [BC], and we have—

$$\text{Component of P along AC} = AC = P\cos\theta$$
$$\text{Component of P perpendicular to AC} = BC = P\sin\theta.$$

Example.—If we have a body of weight W resting on an incline[d] plane inclined at an angle α to the horizontal (Fig. 107), the resolve[d] part of the weight W down the plane may be found as follows:—

From the triangle of forces *gef* we have the resolved part of W d[own] the plane P is represented by *eg*, and—

$$\frac{eg}{ef} = \sin\alpha$$

or $\quad \frac{P}{W} = \sin\alpha \quad$ or $P = W\sin\alpha$ (as before, p. 143).

Also the component of W perpendicular to the plane, that is, [the] reaction R (Fig. 107), is given by—

$$R = W\cos\alpha.$$

In Fig. 138 we have resolved the force P into two forces or [com]ponents AD and AC at right angles, which have the same effect a[s the] single force P, in other words, P is the resultant of the two forces [AD] and AC.

Resolution of a Single Force in Two Directions not at ght Angles.—Let it be required to resolve the force P (repre- nted by AB) into two components along AC and AD respectively ee Fig. 139). We draw from B a line parallel to AD to meet , then AC represents the nponent of P along AC and the component of P along).

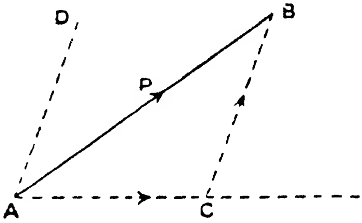


FIG. 139.

Resultant of a Number Forces in the same Plane cting at a Point.—In Chap. we have seen how to find e resultant graphically by eans of the polygon of forces ; now proceed to obtain the sultant by calculation. It will be instructive to take Example 1 22), which has been worked out graphically and compare the ults obtained by the two methods.

Take the four forces ED (which was found to be 9·6 tons tensile), C, CB and BA. The resultant of these forces (equal and opposite the equilibrant) should be 55·6 tons along EA. For conveni- ce the forces are reproduced in Fig. 140.

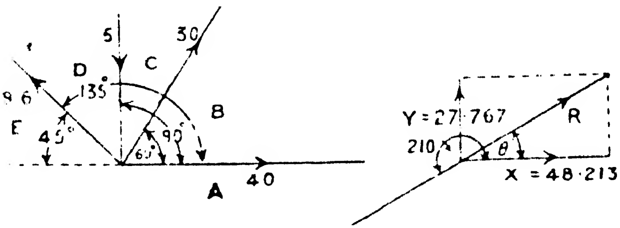


FIG. 140.

We resolve the forces into any convenient two directions at ht angles, say, parallel and perpendicular to the force AB.

Let X be the total component of all the forces in the direction parallel to the force AB reckoned positive from left to right, and Y the total component perpendicular to the force AB reckoned positive if acting upwards.

Then, using the tables on p. 367, we have—

Component of AB in the direction of AB $= + 40$ tons

Component of CB in the direction of AB

$$= - 30 \cos 60^\circ = 30 \times 0·50 = + 15 \text{ tons}$$

Component of DC in the direction of AB

$= 5 \cos 90° = 5 \times 0 = 0$

Component of ED in the direction of AB

$= -9{\cdot}6 \cos 45° = -9{\cdot}6 \times 0{\cdot}707 = -6{\cdot}787$ tons

$\therefore X = +40 + 15 + 0 - 6{\cdot}787 = 48{\cdot}213$ tons.

Component of AB in a direction perpendicular to AB

$= 40 \cos 90 = 0$

Component of CB in a direction perpendicular to AB

$= +30 \sin 60$ or $30 \cos 30 = +30 \times 0{\cdot}866$

$= +25{\cdot}98$ tons

Component of DC in a upward direction perpendicular to AB $= -5$ tons (downwards)

Component of ED in a direction perpendicular to AB

$= +9{\cdot}6 \cos 45 = +9{\cdot}6 \times 0{\cdot}707 = 6{\cdot}787$ tons

$\therefore Y = +25{\cdot}98 - 5 + 6{\cdot}787 = +27{\cdot}767$ tons.

We have now replaced the four forces by two forces X and Y at right angles shown on the right of Fig. 140: their resultant R is given by—

$$\begin{aligned} R^2 &= X^2 + Y^2 \\ &= (48{\cdot}213)^2 + (27{\cdot}767)^2 \\ &= 2324{\cdot}49 + 771{\cdot}01 \\ &= 3095{\cdot}5 \\ R &= \sqrt{3095{\cdot}5} = 55{\cdot}6 \text{ as before.} \end{aligned}$$

Its inclination to X or AB is found as above described.

$$\tan \theta = \frac{Y}{X} = \frac{27{\cdot}767}{48{\cdot}213} = 0{\cdot}576$$

From the tables (p. 367) we see that $\theta = 30°$ nearly, which agrees with Fig. 27, where the angle between EA and BA is 210° or (180 + 30).

Example 2.—Find the resultant of the four forces shown in Fig. 141. Resolving parallel to and perpendicular to the force of 20 lbs., and reckoning forces acting from left to right positive, and forces acting upwards positive, we have—

$$\begin{aligned} X &= 20 + 15 \cos 40° - 30 \cos 60° - 8 \cos 45° \\ &= 20 + 15 \times 0{\cdot}766 - 30 \times 0{\cdot}50 - 8 \times 0{\cdot}707 \\ &= 20 + 11{\cdot}49 - 15 - 5{\cdot}656 = 10{\cdot}834 \text{ lbs.} \\ Y &= 0 + 15 \sin 40 + 30 \sin 60 - 8 \sin 45 \\ &= 0 + 15 \times 0{\cdot}6428 + 30 \times 0{\cdot}866 - 8 \times 0{\cdot}707 \\ &= 0 + 9{\cdot}642 + 25{\cdot}98 - 5{\cdot}656 \\ &= 35{\cdot}622 - 5{\cdot}656 = 29{\cdot}966 \text{ lbs.} \\ \therefore R^2 &= X^2 + Y^2 \\ &= (10{\cdot}834)^2 + (29{\cdot}966)^2 \\ &= 117{\cdot}375 + 897{\cdot}961 \\ &= 1014{\cdot}336 \\ R &= \sqrt{1014{\cdot}336} = 31{\cdot}84 \text{ lbs.} \end{aligned}$$

Its direction is such that $\tan\theta = \frac{Y}{X} = \frac{29{\cdot}97}{10{\cdot}834} = 2{\cdot}77.$

From the tables (p. 367) we see that $\theta = 70{\cdot}2°$ nearly.

Hence the resultant is a force of 31·84 lbs., making an angle of 70·2° with the 20 lbs. force as shown on the right of Fig. 141.

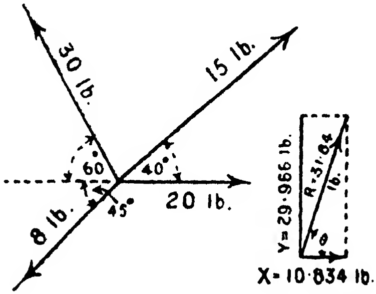


FIG. 141.

EXAMPLES XIII.

1. Calculate the resultant of two forces of 7 and 8 lbs., which are inclined at an angle of 60°.

2. If two forces of 80 and 50 lbs. act at an angle of 135°, what is the magnitude and direction of their resultant?

3. A body of weight 3 cwt. rests on an inclined plane making an angle of 20° to the horizontal: find the component of its weight down the plane, and also perpendicular to the plane.

4. A simple roof truss like the one shown in Fig. 52, has a span PQ of 25 feet, the inclination of the two rafters (angles $R\hat{P}Q$ and $R\hat{Q}P$) being 30°. It carries a single load of 2500 lbs. at the apex R: calculate the stress in each member.

5. In a simple jib crane the jib is inclined at an angle of 60° to the horitzontal, and the tie-rod at an angle of 45°. A load of 5 tons is suspended from the crane head: calculate the forces in the jib and tie-rod respectively.

6. The following four forces act at a point:—a force of 16 lbs in a direction due East, 20 lbs. due North, 30 lbs. in a direction North-West, and 12 lbs. in a direction 30° South of West. Calculate the magnitude and direction of their resultant.

7. Find the magnitude and direction of the resultant of the following forces acting at a point: a force of 80 lbs. due North, one of 20 lbs. North-East, 40 lbs. due East, 60 lbs. in a direction inclined 30° East of South, and 70 lbs. in a direction inclined 60° South of West.

CHAPTER XIV

CENTRE OF GRAVITY

All bodies and every particle of them are attracted by practically parallel forces to the earth, by a force which we call their weight. In any position the resultant force of the weights of all the particles lies in a vertical line through a point called the *centre of gravity* of the body. To support the body, a resultant upward force is required equal to the total weight of the body and passing through the centre of gravity (often written c.g.) of the body. The centre of gravity of many bodies of regular shape is easily found, being at their geometrical centres, and the principle of moments, explained in Chap. II., enables us to find the c.g. of many irregular bodies. The problem of finding the c.g. of different bodies is generally included in the study of theoretical mechanics, but we may notice the general principles employed, and a few simple applications, and state the results for some other causes.

Centre of Gravity experimentally by Suspension. Cube.—If a cube be suspended from one corner by a fine thread, another corner diagonally opposite to it will be found to be vertically below it. The same applies to suspension from any corner of the cube, and as the c.g. must lie vertically below the point of suspension when the cube is supported by the single force of the pull of the suspension thread it must be in each diagonal of the cube. Hence the c.g. is at that point where the four diagonals of the cube intersect.

Sphere.—If a sphere is suspended by a thread from any point on its surface, a diameter of the sphere lies in the vertical line of suspension. Hence the c g. is at the point where all diameters intersect, *i.e.* at the centre of the sphere.

Straight rod.—If a straight rod be placed on a knife-edge there will be found one position at which the rod will balance. The c.g., therefore, will lie in the rod immediately above the knife-edge, or point of balance. If the rod is uniform in cross-section, the c.g. will obviously be at the middle point of its length.

Cylinder.—A cylinder is a uniform rod of constant diameter. The c.g., therefore, will lie at the middle point of the axis of the

cylinder, *i.e.* at the middle of the line joining the centres of the two ends.

Two Bodies.—Fig. 142 shows two spherical pieces of metal, of weights W and w, respectively, the distance between their centres being d. The resultant of the two weights is $W + w$ downwards at G, the c.g. of the two combined, and by the principle of moments, taking moments about A, the c.g. of weight W, we have—

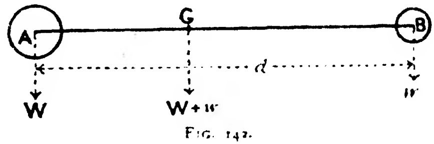


FIG. 142.

$$(W + w) \times AG = w \times d,$$

$$\therefore AG = \frac{w}{W + w} \times d.$$

Similarly—

$$BG = \frac{W}{W + w} \times d.$$

Hence the c.g. G divides the distance between the centres of gravity of the two weights inversely as the magnitudes of the weights, or—

$$\frac{AG}{BG} = \frac{w}{W}$$

Solid Pyramids and Cones have their centres of gravity $\frac{1}{4}$ the way along a line from the centre of the base to the apex (Fig. 143).

A Solid Hemisphere has its centre of gravity on a line drawn from the centre of the base circle and perpendicular to it distant $\frac{3}{8}$ of the radius from the base (Fig. 143).

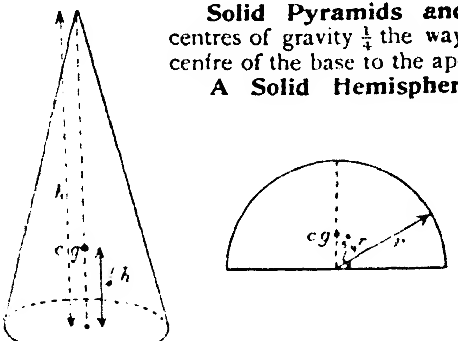


FIG. 143.—Centres of gravity of cone and hemisphere.

Example 1. — A body consists of a solid cylinder 8 inches diameter and 12 inches long. At one end is a solid cone 8 inches high. Find the position of the c.g. of the body.

The centre of gravity of the cylinder is at A (Fig. 144), 6 inches from

the base of the cone. The c.g. of the cone is at B, $\frac{1}{4}$ of 8 or 2 inches from the base of the cone; and since the weight of each part is proportional to its volume, we have—

Weight of cylinder acting at A is proportional to $\pi \times (4)^2 \times 12 = 192\pi$

Weight of cone acting at B is proportional to $\pi \times 4^2 \times \frac{8}{3} = \frac{16 \times 8\pi}{3} = \frac{128\pi}{3}$

Hence, taking moments about A, we have—

$$\left(192\pi + \frac{128\pi}{3}\right) \times AG = \frac{128\pi}{3} \times 8$$

$$\left(192 + \frac{128}{3}\right) AG = \frac{128 \times 8}{3}$$

$$\frac{704}{3} AG = \frac{128 \times 8}{3}$$

$$AG = \frac{128 \times 8}{704} = 1{\cdot}45 \text{ inches}$$

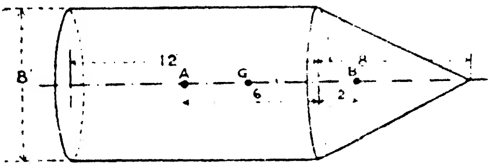


FIG. 144.

Hence the c.g. is on the axis of the body, and distant $6 + 1{\cdot}45$ or $7{\cdot}45$ inches from the end of the cylinder, or $20 - 7{\cdot}45 = 12{\cdot}55$ inches from the apex of the cone.

Example 2.—If the body in the previous example is made of cast iron (the weight of 1 cubic inch being 0·26 lb.), and rests with its base on a rough horizontal surface, what horizontal force applied at the apex of the cone will just turn it over?

The volume of the solid body (see Example 1) is—

$$192\pi + \frac{128\pi}{3} = \frac{704\pi}{3} \text{ cubic inches}$$

$$\text{Hence its weight} = \frac{704\pi}{3} \times 0{\cdot}26 = 192 \text{ lbs.}$$

Let P be the required force (Fig. 145). When the body is just on the point of overturning, the "righting moment" due to its own weight will just be equal to the "overturning moment" due to the force P.

Hence, equating these moments (about C in Fig. 145), we have—

$$\text{Overturning moment} = \text{righting moment}$$

$$P \times 20 = 192 \times 4$$

$$\therefore P = \frac{192 \times 4}{20} = 38{\cdot}4 \text{ lbs.}$$

Centre of Gravity of a Lamina by Suspension.—Suspend a thin plate or sheet of metal of irregular area by means of a thin cord from any point A, say, Fig. 146. The c.g. of the plate must lie vertically below A. Produce and mark the line of the cord below A on the lamina. Now suspend the plate from any other point, B, say. The c.g. must lie vertically below B. Hence the c.g. must lie in point of G, where the two suspension lines cross.

A *surface* is without thickness or weight, hence the above definition of the c.g. strictly has no application to it. The position of the c.g., or "centroid," as it is called, for an area may be found

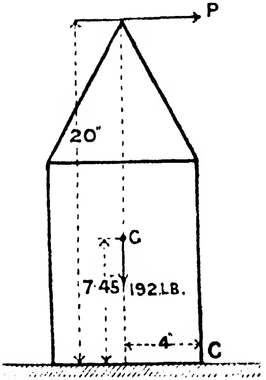


FIG. 145.

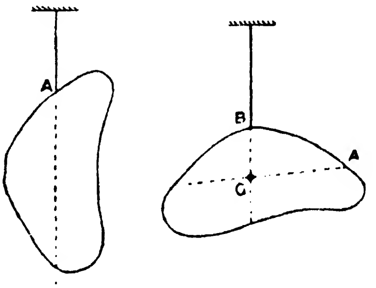


FIG. 146.—Centre of gravity of lamina by suspension.

by cutting out its shape from a piece of stiff cardboard, and then proceeding as above (Fig. 146).

Square and Rectangle.—The centroid lies at the intersection of the diagonals.

Circle.—The centroid lies at the intersection of any two diameters, *i.e.* at the centre of the circle.

Triangle.—The centroid will be found to be one-third the way along a line drawn from the centre of any side to the angle opposite (Fig. 147). The position of the c.g. of many areas may be easily calculated by the principle of moments as illustrated in the following examples :—

Example 1.—Find the position of the c.g. of the area shown in Fig. 148. The area is symmetrical about the line AB ; hence the c.g. lies somewhere in this line. Let G be the c.g., distant x inches from A. Dividing the area into the two rectangles *abcd* and *efgh*, and taking moments about the point A, we can find x as follows :—

Area of *abcd* $= 6 \times 2 = 12$ sq. ins.; its c.g. is 1 in. from A
Area of *efgh* $= 8 \times 2\frac{1}{2} = 20$ sq ins.; its c.g. is $4 + 2 = 6$ ins. from A
Area of complete figure $= 12 + 20 = 32$ sq. ins.

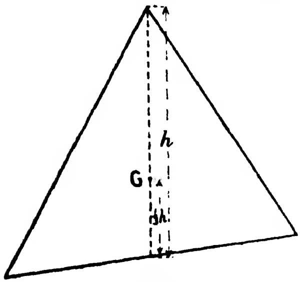


FIG. 147.—Centre of gravity of a triangle.

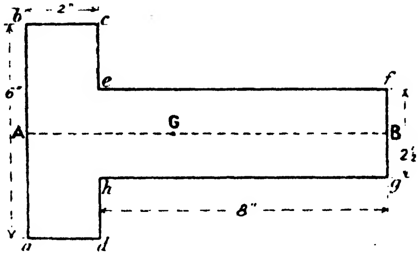


FIG. 148.

Taking moments of the areas about the line *ab*, we have—

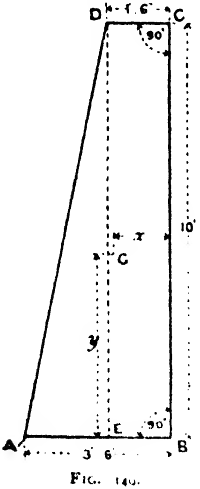


FIG. 149.

$$32 \times x = 12 \times 1 + 20 \times 6$$
$$32x = 12 + 120 = 132$$
$$x = \frac{132}{32} = 4{\cdot}125 \text{ inches.}$$

Hence the c.g. is $4\frac{1}{8}$ inches from A on the line AB.

The student should check this by suspension.

Example 2.—Find the c.g. of the area shown in Fig. 149. Dividing the area into the triangle ADE and the rectangle DCBE, we proceed as follows:—

To find the distance of the c.g. G from the side CB, we take moments about CB thus—

Area of triangle ADE $= \frac{1}{2} \times \text{AE} \times \text{DE} = \frac{1}{2} \times 2 \times 10$
$= 10$ square feet;

and its c.g. is—

$\frac{1}{3} \times \text{AE}$, or $\frac{1}{3} \times 2 = \frac{2}{3}$ foot from DE,
or $\frac{2}{3} + \frac{3}{2} = \frac{13}{6}$ feet from CB

Area of rectangle DCBE $= \text{EB} \times \text{DE} = 1{\cdot}5 \times 10$
$= 15$ square feet;

and its c.g. is—

$$\frac{\text{EB}}{2} = \frac{1{\cdot}5}{2} = 0{\cdot}75 \text{ foot from CB}$$

Total area $= 10 + 15 = 25$ square feet

Taking moments about CB, we have—

$$25 \times x = 10 \times \frac{13}{6} + 15 \times 0{\cdot}75$$
$$= 21{\cdot}66 + 11{\cdot}25 = 32{\cdot}91$$
$$x = \frac{32{\cdot}91}{25} = 1{\cdot}31 \text{ feet.}$$

Similarly, taking moments about AB, we get the distance y of G from AB.

$$25 \times y = 10 \times \frac{DE}{3} + 15 \times \frac{DE}{2}$$

$$25y = 10 \times \tfrac{10}{3} + 15 \times \tfrac{10}{2} = 33{\cdot}33 + 75 = 108{\cdot}33$$

$$y = \frac{108{\cdot}33}{25} = 4{\cdot}43 \text{ feet}$$

Hence the c.g. is 1·31 feet from CB, and 4·43 feet from AB, which the student should check by suspension.

Centre of Gravity of a Quadrilateral Graphically.—Divide the area *abcd* (Fig. 150) into two triangles, *abc* and *acd*, by drawing the diagonal *ac*. Find G_1, the c.g. of triangle *abc*, and G_2, the c.g. of triangle *acd*, by construction. This is done by bisecting *ac* at *e*; then join *eb* and *ed*, and make eG_1 equal to one-third of *eb*,

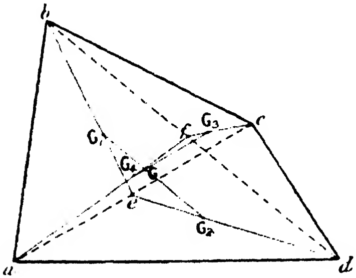


FIG. 150.

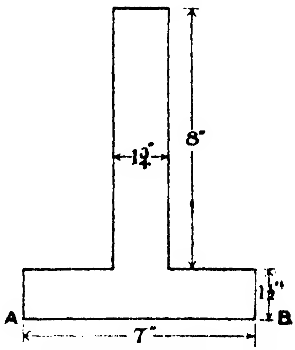


Fig for Question 3 Examples XIV.

and eG_2 one-third of *ed*. The c.g. of the quadrilateral must lie somewhere in the line G_1G_2.

Now divide the figure into the triangles *abd* and *bcd* by drawing the diagonal *bd*, and find the c.g. of each of these triangles in the same way, *i.e.* bisect *bd* in *f*, join *fc* and *fa*, and make fG_3 equal to one-third of *fc*, and fG_4 equal to one-third of *fa*. Then the c.g. of the quadrilateral lies in the line G_3G_4. Hence the c.g. lies in the point of intersection of G_1G_2 and G_3G_4, namely, point G.

EXAMPLES XIV.

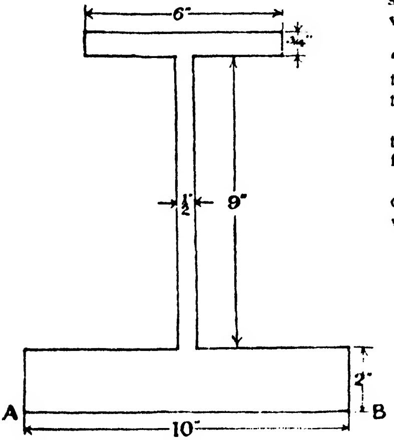
1. A solid body is made up of a cylinder 4 inches diameter and 6 inches long, with a cone on one end 4 inches high. Find the position of its centre of gravity, the material being the same throughout.

2. A uniform steel bar weighs 15 lbs. and is 3 feet long. A cast-iron ball 7 inches diameter is fixed on the bar, with its centre 4 inches from one end; and another ball of cast iron 4 inches diameter is fixed on the bar, with its centre 2 feet from the same end. Find the position of the centre of gravity of the system (1 cubic inch of cast iron weighs 0·26 lb.).

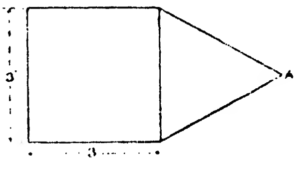
3. Find the position of the "centroid" of the area given in the figure. State its distance from the line AB.

4. Find the distance of the "centroid" of the I section shown in the figure from the line AB.

5. A lamina consists of a square of 3 inches side, on one side of which is an equilateral triangle (see figure). Find the position of the centroid, stating its distance from point A.



Question 4



Question 5.

6. A solid body is made up of a cylinder of 5 inches diameter and 8 inches long, and a hemisphere 2½ inches radius fitting one end of the cylinder. Find the position of its centre of gravity.

CHAPTER XV

THE ELASTIC LAW

Elastic Stretching.—When a piece of indiarubber cord is stretched, say, by carrying a suspended weight, it may stretch a very visible amount. When the tension is removed from the cord by removing the weight, the cord returns to its original length, provided that it has not been stretched too much. This property of a material returning to its original dimensions, after being deformed by the action of force, is called *elasticity*. That a rubber cord possesses this elasticity may be verified by the simple apparatus shown in Fig. 151, in which A and B are two needles stuck into a rubber cord, and move over a steel rule or scale S, when the cord is stretched by a weight W placed in a scale pan. By measuring the distance apart of B and A before the weight W is put in the scale pan, and after it has been removed, we find that the cord recovers its original length between B and A, that is, the cord is perfectly elastic, provided that it has not been stretched more than a certain amount.

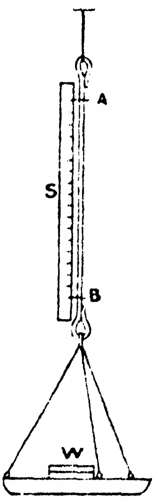


FIG. 151.—Elastic stretch of rubber cord.

Stretch of Wire.—We may also verify that a steel wire is elastic by the apparatus shown in Fig. 152. A steel wire will not stretch so much for a given length as rubber, so we require a more delicate means of measuring the stretch; this consists of a vernier clipped to the lower end of the wire, and moving over a scale. We also use as long a length of wire as is convenient, in order to get a large amount of elongation or stretch with a moderate load.

The following table gives the reading of the vernier in a particular experiment when different loads were carried by the wire (which was 0·036 inch diameter).

Load.	Reading of Vernier.	Stretch (by difference).
0	0·010	—
5	0·050	0·04
10	0·090	0·08
15	0·125	0·115
20	0·165	0·155
25	0·210	0·200
30	0·250	0·240
35	0·285	0·275
0	0·010	0·000

After a total weight of 35 lbs. had been suspended, the whole load was removed, and the reading of the vernier was 0·01 inch, the same as at the start, showing the wire had resumed its original length, or, in other words, was perfectly elastic.

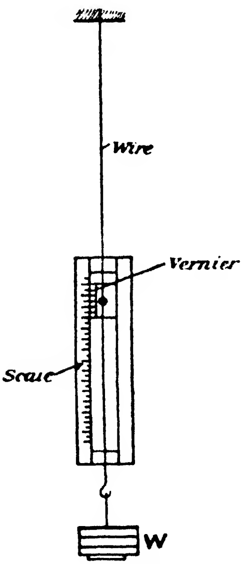


FIG. 152.—Elastic stretch of wire.

Relation between Stretch and Load.—If the stretch in the previous table is plotted to a large scale on a base of loads, as in Fig. 153, the result is a straight line passing through the origin or intersection of the axis. This means, as we have seen in Chap. VIII., that the stretch is proportional to the load. Thus, reading from the curve, the stretch with a load of 30 lbs. is 0·24 inch, or at the rate of 0·008 inch per lb. of load. Thus, at 7 lbs. load the stretch is 7 × 0·008 = 0·056 inch, which agrees with the amount read from the curve at 7 lbs. load.

Stretch of a Helical Spring.—As another example of elastic stretching, we may take a helical spring with its axis vertical, as shown in Fig. 154, and loaded by weights placed on a suitable hanger. The following results were obtained from a spring of $\frac{1}{4}$-inch diameter round steel wire, having 10 coils each 4 inches outside diameter.

Load (lbs.).	Reading of Vernier.	Stretch (by difference) (inches).
0	0·0	—
1	0·10	0·10
2	0·20	0·20
3	0·30	0·30
4	0·40	0·40
5	0·50	0·50
6	0·60	0·60
7	0·70	0·70
10	1·00	1·00
12	1·20	1·20
15	1·50	1·50
10	1·00	1·00
5	0·50	0·50
0	0·0	0·0

In this case the spring, as a whole, stretches, but the ¼-inch wire does not stretch; it twists about its helical axis.

The removal of part or whole of the load shows amounts of stretch which are the same as before loading to 15 lbs., indicating that the material is quite elastic up to 15 lbs., or 1·5 inches stretch. Fig. 155 shows the relation

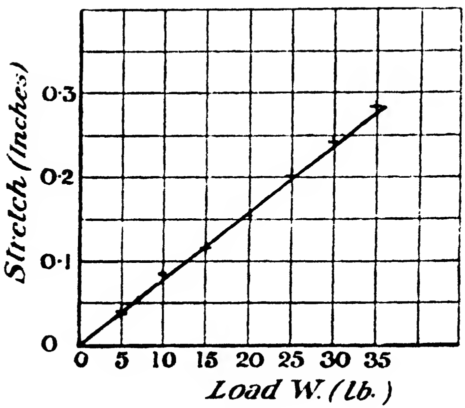


FIG. 153

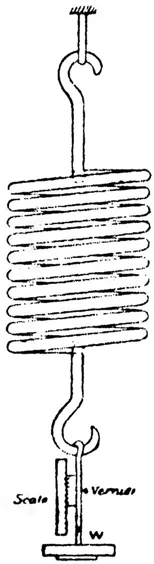


FIG. 154.—Stretch of a helical spring.

between the stretch of the spring and the load, and again it is

evident that the stretch is proportional to the stretching force, being at the rate of 0·1 inch per lb. of load.

Compression of a Helical Spring.—As another example we may take a helical spring with its axis vertical, arranged to be put in compression, as shown in Fig. 156. The results shown in the table on the following page were obtained from a spring made of $\frac{1}{4}$-inch square steel, having 12 coils, each $2\frac{1}{4}$ inches outside diameter.

The removal of part or whole of the load shows amounts of shortening which are the same before loading to 24 lbs., showing that the material is quite elastic up to 24 lbs. or 0·36 inch compression.

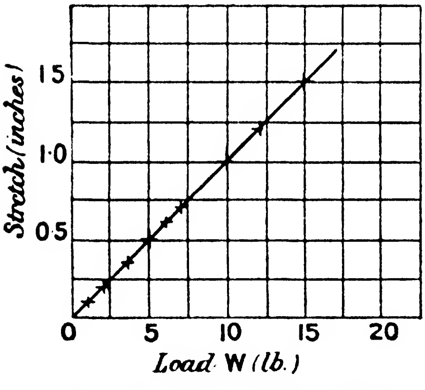


FIG. 155.—Elastic stretch of a helical spring.

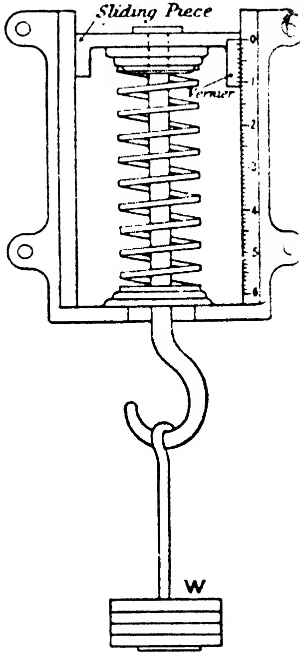


FIG. 156.—Compression of a helical spring

Fig. 157 shows the relation between the shortening of the spring and the load, and it is evident that the shortening is proportional to the compressive force, being at the rate of 0·015 inch per lb. of load.

Elastic Bending.—The elasticity of a material may also be shown in bending a rod or stick, called a *beam*, placed on supports at its ends, and loading it with weights midway between the supports. Fig. 158 shows a beam so bent, in a simple apparatus consisting of a frame, in which the beam rests with its ends on inverted

Load W (lbs.).	Reading of Vernier.	Shortening (by difference.)
0	0·73	0
4	0·79	0·06
8	0·85	0·12
12	0·91	0·18
16	0·96	0·23
20	1·03	0·30
24	1·09	0·36
16	0·96	0·23
8	0·85	0·12
0	0·73	0

V supports (called *knife edges*). The lowering or deflection of the beam at the load below the knife edges is read on the vernier. It is easy to verify with the apparatus that the beam returns to its

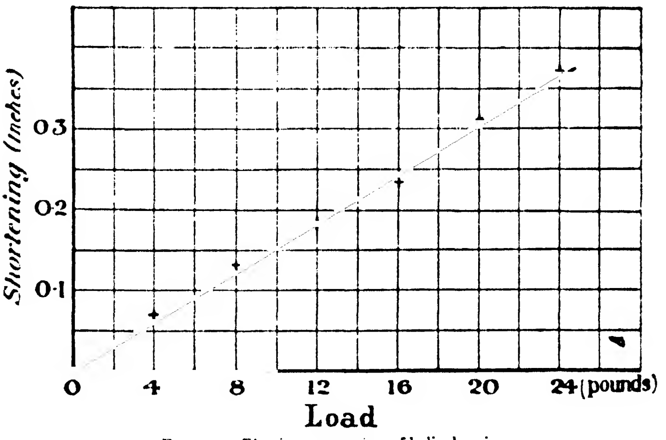


FIG. 157.—Elastic compression of helical spring.

original height when the load is removed, if too great a load is not used. The readings in the table on page 196 were taken for a beam of red deal of rectangular section 1 inch broad and ½ inch deep and 30 inches between the supports.

After loading the beam with 19·5 lbs., and then removing the load, the deflection entirely disappeared. Fig. 159 shows the plotting of the above deflections on a base of loads; and it is evident

that here again the deflection is proportional to the load, being at the rate of 0·0265 inch per lb. of load.

Load.	Reading of vernier (inches).	Deflection (inches).
0	1·330	0
2	1·380	0·050
6	1·495	0·165
11	1·620	0·290
16	1·745	0·415
19·5	1·835	0·505
0	1·330	0

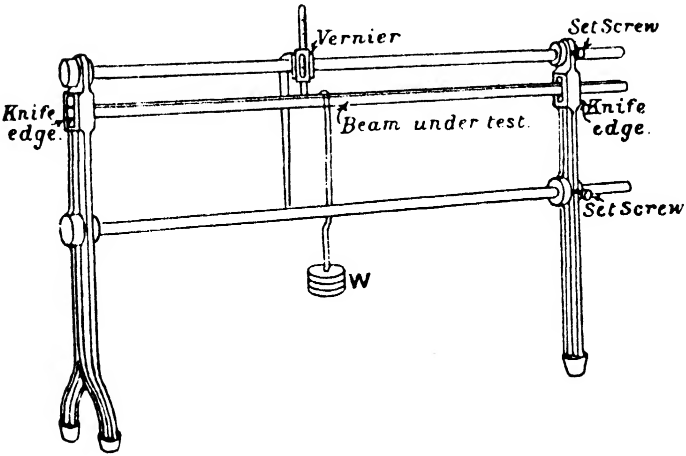


FIG. 158.—Elastic bending of a beam.

A further experiment may be made by firmly clamping one end of a rod horizontally (Fig. 160), loading the free end and measuring its vertical deflection. Such a type of beam is called a cantilever. It is easy to show that with certain moderate loads the rod is elastic, *i.e.* that it returns to its original position when the load on the free end is removed, and also that up to such loads the deflection is proportional to the load.

Twisting.—The elasticity of a material may also be shown by twisting a rod or wire. An arrangment applicable to long thin rods

is shown diagrammatically in Fig. 161. The upper end of the rod or wire is firmly clamped in a vertical position, and the lower end is clamped to a pulley, to which a couple, having the wire as axis,

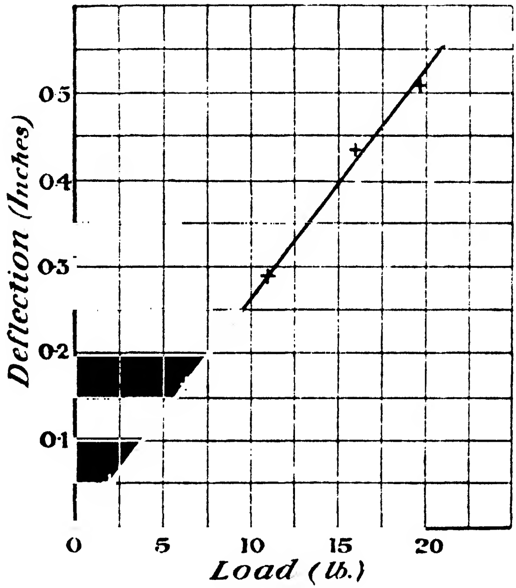


FIG. 159.—Elastic bending deflection of a beam.

is applied by horizontal cords, passing over pulleys and carrying equal weights in scale pans. The twist of the lower end of the wire may be measured by the movement past a fixed pointer of a graduated dial attached to the pulley, and to avoid the effects of

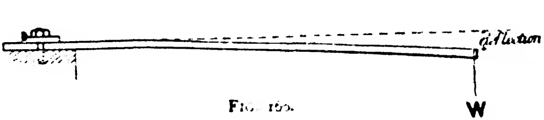


FIG. 160.

possible slipping in the top clamp a horizontal pointer may be attached to the wire near the top. The movement of this pointer over a fixed dial, subtracted from the angular movement of the lower end, gives the angle of twist between the two points.

The following readings were taken with a steel wire $\frac{1}{4}$ inch

diameter and 9 feet long, the diameter of the pulley being 10 inches:—

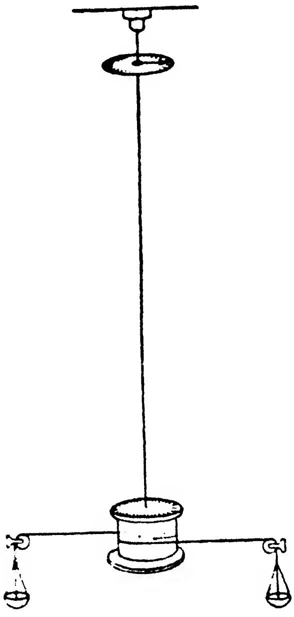


FIG. 161.—Twist of a rod or wire.

Load (lbs.)	Angle of Twist (degrees).
0	0
1	6
2	12
3	18
4	24
5	29
6	36
2	12
0	0

After loading with 6 lbs., and then removing the load, the twist entirely disappeared. Fig. 162 shows the plotting of the above angles of twist on a base of loads; and it is evident that the twist is proportional to the load, and therefore the torque (since the arm of the couple is constant and equal to the diameter of the pulley, 10 inches), being at the rate of 6° per lb. of load, or 6° for a torque of 10 lbs.-inches, *i.e.* 0·6° per 1 lb.-inch of torque.

Helical Spring.—Another example is taken in the form of a helical spring, subjected to twisting as shown diagrammatically in Fig. 163. In this case the free end of the spring is subjected to an axial twist, in the same way as the wire in Fig. 160. In this case the spring as a whole twists. But the wire of which it is made does not twist; it *bends*.

It is easy to show that, up to a certain load, the material of the spring is elastic, *i.e.* it returns to its original condition when the twisting moment on it is removed.

The Elastic Law.—We have now described various simple experiments in which material has been strained by stretching, compressing, bending, and twisting, and has remained perfectly elastic that is, it has returned to its original dimensions and shape when the straining force has been removed. For all loads for which this perfect elasticity remains we have seen that the deflection or deformation has been always *proportional to the load.* This is the law of elasticity called *Hooke's Law*, and is applicable to every kind of

elastic straining, including stretching, compression, bending, and

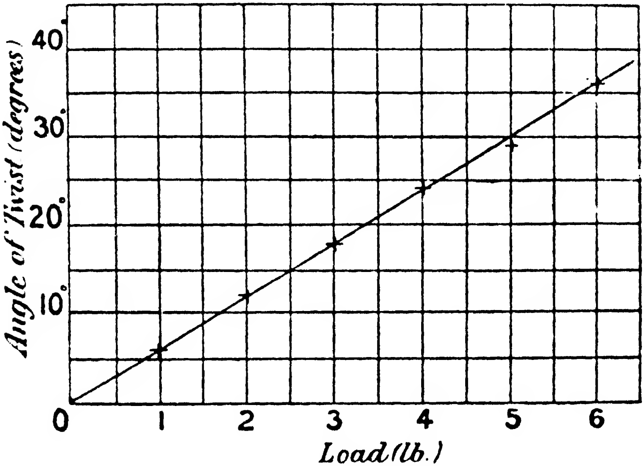


FIG. 162.—Elastic twist of a steel wire.

twisting. It is to be remembered that it applies for loads which only strain the materials in such a way that they return to their original size and shape when freed from the load. Such loads for which the material is perfectly elastic are said to be within the *elastic limit*. The proportional deflections to which the elastic law refers are for loads within the elastic limit; for greater loads the elastic law does not hold.

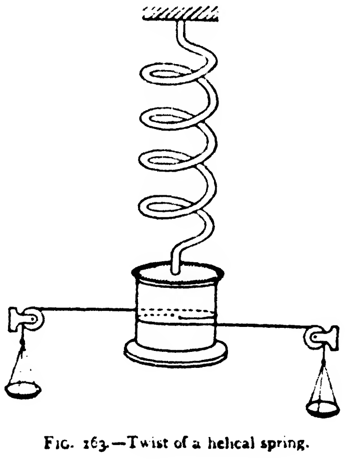


FIG. 163.—Twist of a helical spring.

CHAPTER XVI

STRENGTH AND STIFFNESS OF MATERIALS

Stress.—When a piece of material is acted upon by a force it is said to be stressed or in a state of stress.

Strain.—When material is put in a state of stress it changes its size or shape. This change is called strain. Thus, when a rope is pulled it is stressed, and it stretches; the stretch and *not* the pull is the strain of the rope.

Tension.—A direct pull produces a very simple form of stress, and is called a tensile stress or a tension.

If a bar AB (Fig. 164) called a tie-bar, or simply a tie, is

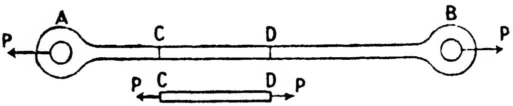


FIG. 164.—Tension on a tie-bar.

pulled with forces P along its axis at its ends, the whole length is under a tension or pull P. Thus the piece CD is subjected to equal and opposite forces P at its ends. The pull at C is exerted on CD by the adjoining length AC, and the pull at D on CD is exerted by the piece DB; across every imaginary section such as C equal and opposite pulls P are exerted by the two parts separated by the section. The total stress across any section of the tie-bar is P reckoned in pounds or tons. Stress is usually reckoned per square inch of area, and is then called *intensity of stress* or sometimes the *unit stress*. If a tie-bar carries a total load of 6 tons, and its cross-section has an area $1\frac{1}{4}$ square inches, the average intensity of stress is—

$$\frac{6 \text{ tons}}{1{\cdot}25 \text{ square inches}} = 4{\cdot}8 \text{ tons per square inch.}$$

This stress may or may not be uniformly distributed across the area of cross section. Thus one-half of the section may withstand

more pull than the other half, in which case the stress is unevenly distributed. If every part of the area of section carries the same pull as any equal portion of the area the stress is evenly distributed, and the stress is said to be a uniformly distributed one, or a *uniform stress*. In this case the pull at every cross section such as C or D (Fig. 164) may be looked upon as a number of parallel forces which have a resultant equal to the total pull acting along the axis. When the pull is not exactly along the axis of the bar the stress is not uniformly distributed.

Tensile Strain.—When a piece of material such as a tie-bar, wire or rope is pulled, it stretches. The total stretch may be called the strain, but tensile strain is usually reckoned as a fraction of the length stretched, or the stretch per unit of length. Thus, if a bar 2 feet long stretches $\frac{1}{8}$ of an inch the stretch per unit length, or the fractional strain is—

$$\text{tensile strain} = \frac{\frac{1}{8}\text{ inch}}{24\text{ inches}} = \frac{1}{192}\text{ or } 0{\cdot}0052$$

and for any stretch

$$\text{tensile strain} = \frac{\text{increase of length}}{\text{original length}}$$

the increase and original length being stated in the same units. The strain is then simply a fraction or ratio, and will be the same whether foot or inch units are employed to measure the stretch and, whether reckoned on a long or a short length.

Tensile Test of a Wire.—To examine the tensile strain of a wire for various stresses we may use the apparatus already referred to and shown in Fig. 152. The load is steadily increased from zero upwards until the wire breaks, by adding weights in a scale pan or on a hanger. The vernier is read to obtain the stretch after each addition to the load. The result of such a test on an iron wire 0·055 inch diameter (0·00237 square inches cross sectional area), and 90·7 inches long from the fixed

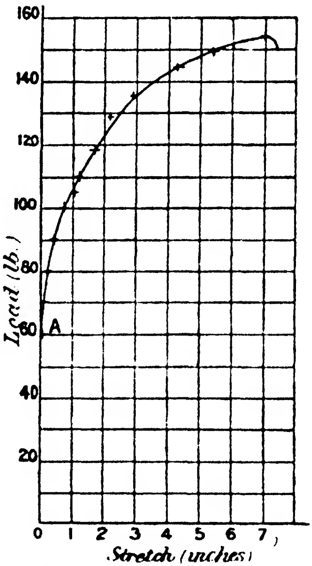


FIG. 165.—Tensile test of a long iron wire.

end to the attachment of the vernier is shown in the following table :—

Load in lbs.	Reading of vernier.	Stretch in inches (by difference).
0	0·10	0
10	0·11	0·01
20	0·122	0·c22
30	0·135	0·035
40	0·15	0·05
50	0·162	0·062
60	0·175	0·075
70	0·20	0·10
80	0·30	0·20
90	0·50	0·40
100	0·85	0·75
105	1·14	1·04
110	1·39	1·29
118	1·88	1·78
128	2·28	2·18
135	3·01	2·91
144	4·37	4·27
149	5·50	5·4
154	7·10	7·0 (broke)

The load and stretch are shown plotted in Fig. 165.

The stretch up to a load of 60 lbs. is so small in comparison with the final stretch, that it is advisable to plot the first part of the table up to 60 lbs. on a larger scale of extensions as shown in Fig. 166. This shows by the straight line BA the stretch proportional to the load, according to the elastic law up to a load of 60 lbs. A careful test with another piece of the same wire will show that up to this point, which is the elastic limit, the stretch disappears if the load is removed. This is practically true for iron and steel and most common ductile metal, *i.e.* metals which can be drawn out by tension to a considerable extent. At a load a little above 60 lbs. the table and Fig. 165 show a great increase in the stretch for little increase in load, and with further increase in the load much greater stretching occurs than before the elastic limit (A) was reached ; but the stretch is not proportional to the load, for this part of the curve is not a straight line. If the load is now removed very little of the considerable stretch will disappear ; the wire is permanently strained, and this kind of inelastic stretching is called *plastic* strain.

Modulus of Elasticity.—We have seen that within the elastic limit the strain is proportional to the load ; if the area of section of

the wire were doubled and the load were also doubled (as if two wires and loads were used), the stretch would be the same as before; that is, the strain depends upon and is proportional to the intensity of stress, and not merely upon the total pull. As the stress and strain are proportional to each other, we may write—

$$\text{stress} = \text{strain} \times \text{stress per unit of strain}$$
$$\text{stress} = \text{strain} \times \text{a constant},$$

although the greatest strain is much less than one unit. This constant is called the direct or stretch modulus of elasticity, or

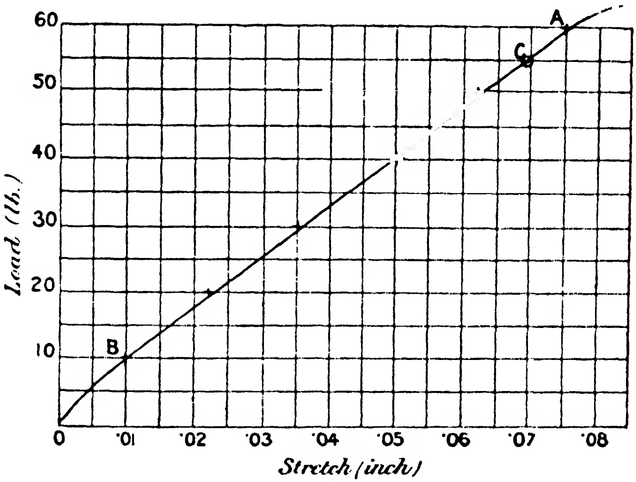


FIG. 166.

sometimes Young's modulus of elasticity for the material of the wire. Thus—

Intensity of stress = strain × E, or stretch per unit length × E, and

$$E = \frac{\text{stress intensity}}{\text{stretch per unit length}} \text{ or } \frac{\text{pull} \div \text{area}}{\text{extension} \div \text{original length}}$$

The modulus of elasticity (E) is a measure of the resistance which a material offers to stretching; it may also be stated as the stress which would produce unit strain (or double the length of the wire), if it continued to stretch proportionally to the stress instead of reaching an elastic limit.

We may best calculate E from an experiment such as that

described for a wire by selecting two points such as B and C (Fig. 166) on the straight line a long distance apart, and writing—

$$\begin{aligned} \text{load at C} &= 55 \text{ lbs.} \\ \text{load at B} &= \underline{10 \text{ lbs.}} \\ \text{increase in load} &= 45 \text{ lbs.} \\ \text{stretch at C} &= 0·069 \text{ inch} \\ \text{stretch at B} &= \underline{0·010 \text{ inch}} \\ \text{increase in stretch} &\ \ 0·059 \text{ inch} \end{aligned}$$

$$E = \frac{\text{increase of stress intensity}}{\text{increase of stretch per inch}} = \frac{45 \div 0·00237}{0·059 \div 90·7}$$
$$= 29{,}189{,}000 \text{ lbs. per square inch.}$$

If the plotted curve passes in a straight line through O, one point may be chosen at O instead of B, and then—

$$E = \frac{\text{stress intensity}}{\text{stretch per inch}}$$

Errors due to a variety of causes frequently arise in the early part of the test, that is, with small loads ; in such cases, points such as B and C (Fig. 166) should be chosen where the plotted results give a good straight line so that any deviation near O does not affect the accuracy of the result.

In order to show the properties of the material without reference to the size of wire, it is advisable to restate the table on p. 202 thus—

Load (lbs.).	Intensity of stress lbs. per square inch $= \frac{\text{load}}{0·00237}$	Stretch (inches).	Strain $= \frac{\text{stretch}}{90·7''}$
0	0	0	0
10	4220	0·01	0·00011
20	8440	0·022	0·00024
30	12660	0·035	0·00038
40	16880	0·05	0·00055
50	21100	0·062	0·00068
60	25320	0·075	0·00082
70	29540	0·10	0·0011
80	33760	0·20	0·0022
90	37980	0·40	0·0044
100	42200	0·75	0·0082
105	44310	1·04	0·0114
110	46420	1·29	0·0142
118	49796	1·78	0·0195
128	54016	2·18	0·0240
135	56970	2·91	0·0320
144	60766	4·27	0·0470
149	62878	5·40	0·0595
154	64990	7·0	0·0771

The intensity of stress and the fractional strain may now be plotted as in Fig. 167, which is practically Fig. 165 plotted to a different scale.

The reader should make tests himself on several different kinds of wire, plot for each the curves shown in Figs. 165, 166, and 167, calculate the value of E, and also calculate the following quantities now given for the above test :—

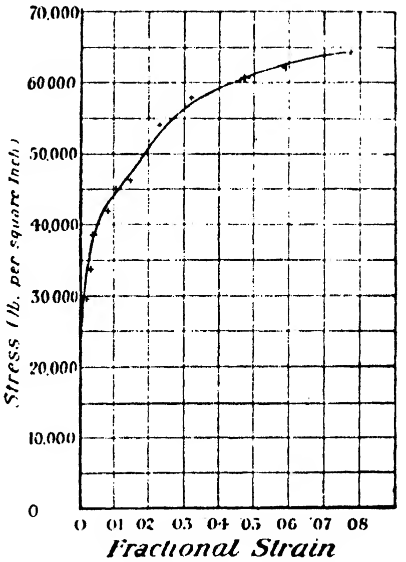


FIG. 167.—Stress and strain in test, shown in Fig. 165.

Stress at elastic limit (see point A, Fig. 166)

$$= \frac{60 \text{ lbs.}}{0{\cdot}00237 \text{ square inch}}$$

$= 25320$ lbs. per sq. inch.

Tenacity or ultimate strength—

$$= \frac{\text{breaking load}}{\text{area}}$$

$$= \frac{154}{0{\cdot}00237}$$

$= 64990$ lbs. per sq. inch.

These being reckoned on the *original* area of section and not on the area after straining, which is continually becoming smaller as the length increases.

$$\text{Final elongation} = \frac{7}{90{\cdot}7} \times 100 = 7{\cdot}7 \text{ per cent.}$$

This elongation of 7 inches may be measured by putting the broken parts of the wire together. It will be noted that a wire which stretches considerably also contracts to a smaller diameter at the place where fracture takes place ; a material which behaves in this way is said to be ductile.

Example.—A mild steel tie-bar $1\frac{3}{8}$ inch diameter and 5 feet long carries a load of 8 tons. Find the stress and also the stretch if E = 13,000 tons per square inch.

$$\text{Area of cross section} = \frac{\pi}{4} \times (1\tfrac{3}{8})^2 = 1{\cdot}48 \text{ square inches}$$

$$\text{stress} = \frac{\text{total load}}{\text{area}} = \frac{8}{1{\cdot}48} = 5{\cdot}4 \text{ tons per square inch.}$$

$$\text{Now } E = \frac{\text{intensity of stress}}{\text{stretch per inch}}$$

$$\text{stretch per inch } (\textit{i.e.} \text{ strain}) = \frac{\text{intensity of stress}}{E} = \frac{5{\cdot}4}{13{,}000}$$

$$\text{total stretch} = \frac{5{\cdot}4}{13{,}000} \times 5 \times 12 = 0{\cdot}0249 \text{ inch.}$$

Pressure or Thrust and Compression.—A short thick bar AB (Fig. 168) of material subject to thrusting or pushing forces P at its ends is said to be in compression or under compressive stress. Across every section, such as C or D, adjoining pieces of the bar exert equal and opposite thrusts P on each other. The average compressive stress across any section is equal to the total force P, divided by the area of cross-section, and may be stated in pounds per square inch, or tons per square inch. As with tension, the stress may be uniformly distributed, or it may be of varying intensity.

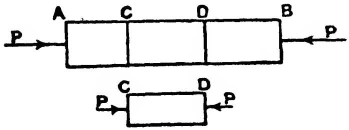


FIG. 168.—Thrust and compression.

Compressive strain or shortening is measured as a fraction of the original length, so that—

$$\text{compressive strain} = \frac{\text{decrease in length}}{\text{original length}}$$

Struts.—Any bar or part of a framework subjected to compressive stress is called a strut There is one striking difference between struts and tie rods. While a pull tends to straighten out any kinks or curvature in a long rod, a thrust tends to develop such imperfections and to buckle up the rod (Fig. 169). For this

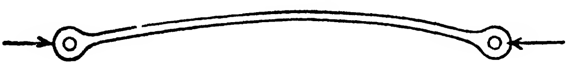


FIG. 169.—Thrust on a strut

reason the intensity of compressive stress, allowable in any but a very short rod, is much less than the tension which may be safely allowed. The calculation of the strength of long struts and columns or pillars involves various difficulties, but the reader may observe in any structural steelwork that the struts are not usually solid round bars, but are of some hollow or spread-out type of section such as shown in Fig. 170. For a given amount of

material and a given length such a strut is stiffer to resist buckling or bending than one of compact section such as a circle or square.

Strength of a Thin Cylindrical Shell.—When a thin circular cylinder or pipe contains fluid under pressure it is subjected to uniform pressure normal or perpendicular to the walls, and this

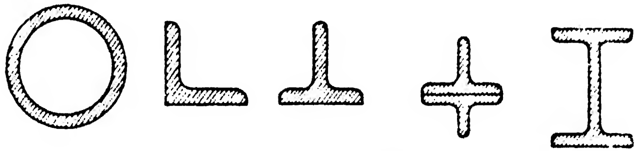


FIG. 170.—Some strut sections.

causes two unequal tensile stresses in the material, one in a direction tangential to the perimeter of a cross-section, usually called the circumferential or hoop tension, and the other in a direction perpendicular to the cross-section, called the longitudinal stress.

Circumferential or Hoop Tension.

Let d = internal diameter of the thin shell in inches
t = thickness of plate
p = internal pressure in pounds per square inch
f_1 = circumferential or hoop stress in pounds per square inch.

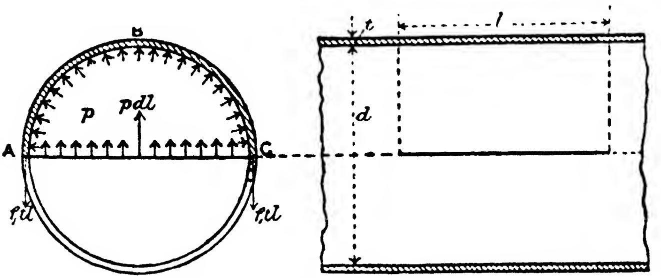


FIG. 171.—Hoop tension in a thin cylinder.

The shell is acted upon by forces tending to pull it asunder along its length into two half cylinders. Consider the half cylinder ABC (Fig. 171). Across the diametral plane AC the total force due to the internal pressure tending to burst the shell would be—

Pressure per square inch × area on which this pressure acts
$= p \times (d \times l)$ pounds.

The total resisting force offered by the material of the shell to bursting will be—

Intensity of stress in material × area of material exposed to bursting
$$= f_1 \times 2t \times l.$$

there being two strips of material each of thickness or width t and length l. The shell being in equilibrium under the action of these two forces, the bursting force must be equal to the resisting force. Equating the two we have—

$$\text{Bursting force} = \text{resisting force}$$
$$p \times d \times l = 2f_1 \times t \times l$$
$$pd = 2f_1t, \text{ or } f_1 = \frac{pd}{2t}$$

Longitudinal Tension.—Across any cross-section AB (Fig. 172) the bursting force due to the internal pressure is—

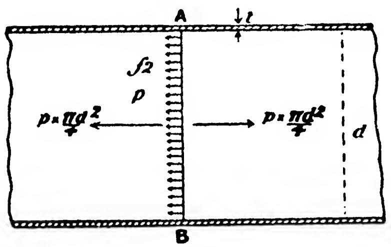


FIG. 172.—Longitudinal tension in a cylinder.

Pressure per square inch × area on which this pressure acts
$$= p \times \frac{\pi d^2}{4} \text{ lbs.}$$

The total resisting force offered by the material of the shell to bursting is—

Intensity of stress in material × area of material exposed to bursting.

Let f_2 = longitudinal stress in pounds per square inch on the ring-shaped section, the area of which is $\pi d \times t$ square inches; then the resisting force is—

$$f_2 \times \pi d \times t \text{ lbs.}$$

Equating the bursting and resisting forces, we have—

$$\text{Bursting force} = \text{resisting force}$$
$$p \times \frac{\pi d^2}{4} = f_2 \times \pi d \times t$$
$$pd = 4f_2t, \text{ or } f_2 = \frac{pd}{4t}$$

Hence we see that the longitudinal stress is only half the circumferential or hoop stress; and in designing a shell to withstand internal pressure, we must use the expression $pd = 2f_1t$.

Example 1.—A cylindrical seamless shell 7 feet internal diameter has to stand an internal pressure of 200 lbs. per square inch, the plates being $\frac{7}{8}$ inch thick. Find the hoop stress.

$$f_1 = \frac{pd}{2t} = \frac{200 \times 7 \times 12}{2 \times \frac{7}{8}} = 9600 \text{ lbs. per square inch.}$$

Example 2.—What thickness of cast-iron pipe 10 inches internal diameter will be required to stand an internal pressure of 50 lbs. per square inch, if the stress in the pipe is not to exceed 1000 lbs. per square inch?

$$pd = 2ft$$
$$\therefore 50 \times 10 = 2 \times 1000 \times t$$
$$t = \frac{500}{2000} = \frac{1}{4} \text{ inch}$$

Shearing.—Material is said to be under the action of shear stress when there is a force tending to make one portion slide past the adjoining portion. Thus a bolt or rivet (Fig. 173) connecting

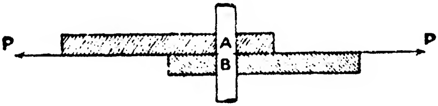


FIG. 173.—Shearing stress.

two pieces of metal plate and opposing a force tending to separate the two pieces, as shown, is subjected to shear stress at the section shown by the dotted line. The tendency of the stress is to *shear* the bolt into the two parts A and B. The average shear stress in Fig. 173 would be—

$$\frac{\text{total shearing force (P)}}{\text{area of cross-section between A and B}}$$

but the stress would not necessarily be at all uniformly distributed across the section between A and B.

The following table gives the average ultimate strength and modulus of elasticity of a few common materials.

Material.	Ultimate strength in tons per square inch.	Shearing strength in tons per square inch.	Young's Modulus in tons per square inch.
Mild steel . . .	27 to 32	21 to 24	13,000
Wrought iron . .	18 to 24	15 to 18	12,500
Cast iron	7 to 10	9 to 11	6,000
Brass	8	8	5,000
Copper (hard) . .	18	—	5,000

Riveted Joints.—*Lap and Butt Joints.*—In a *lap joint* one plate overlaps the other, the two plates being connected by one or more rows of rivets. If there is only one row of rivets, as in Fig. 174, the joint is called a single-riveted lap joint. When two rows of rivets are used, the joint is called a double-riveted lap joint (see Fig. 175). In *chain riveting* the rivets in the two rows are opposite one another; in *zigzag* riveting the rivets in one row are opposite the spaces in the next row, as shown on the right in Fig. 175.

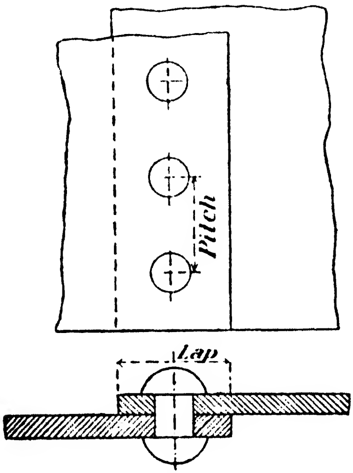


FIG. 174.—Single-riveted lap joint.

In a *butt joint* the plates are kept in the same plane, and the joint is covered on one or both sides by a *cover-plate* which is riveted to the plates. Fig. 176 shows a single-riveted butt joint, and Fig. 177 a double-riveted butt joint.

The *pitch* is the distance from centre to centre of the rivets in one row.

In a *lap joint* the *lap* is the distance, at right angles to the joint, between the edges of the two overlapping plates; in a *butt joint* the *lap* is the distance between the joint and the end of the cover-plate.

A rivet is in *single shear* when shearing can take place only on *one* cross-section of the rivet, as in lap joints and in butt joints with one cover-plate (Figs. 174, 175, 176).

A rivet is in *double shear* when shearing must take place on *two* cross-sections of the rivet to sever the joint, as in butt joints with two cover-plates (Fig. 177).

Strength of Riveted Joints.—

Let t = thickness of plates in inches.
d = diameter of rivets in inches.
p = pitch of rivets in inches.
f_t = tensile resistance of plates in tons per square inch.
f_s = shearing resistance of rivets in tons per square inch.

To fix the diameter of rivets, Professor Unwin gives the simple rule—

$$d = 1{\cdot}2\sqrt{t}$$

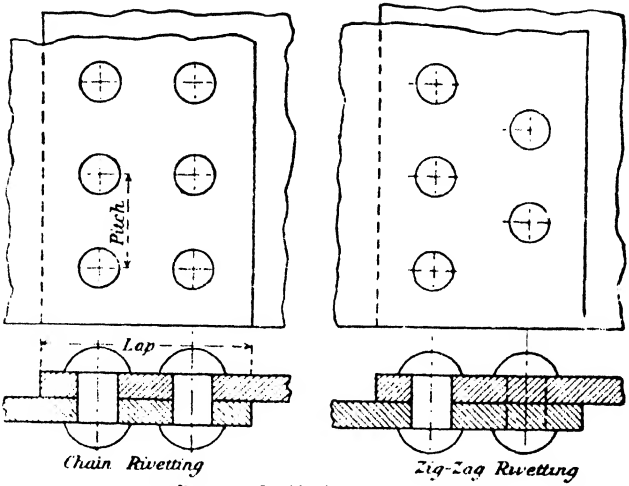
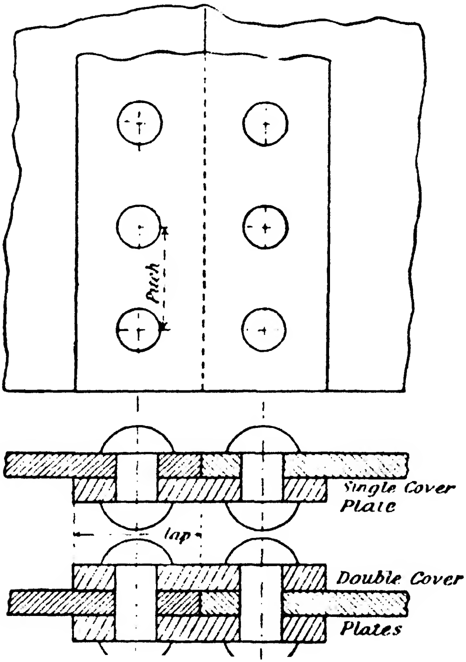


FIG. 175.—Double-rivetted lap joints.



In girder work the rivets are usually $\frac{3}{4}$ inch and $\frac{7}{8}$ inch diameter. Rivets which have to be riveted up by hand when the girder is in position should never exceed $\frac{3}{4}$ inch diameter, on account of the difficulty of driving tight rivets of larger size by hand.

Consider a strip of a single-riveted lap joint of width equal to the pitch (Fig. 178). Such a joint may fail in *four* ways.

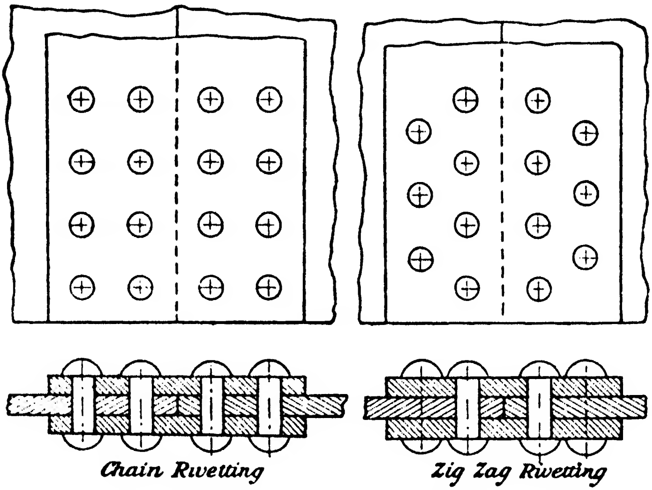


FIG. 177.—Double-rivetted butt joints.

(*a*) The rivet may shear (Fig. 178 (*a*)). The area-resisting shear is $\frac{\pi d^2}{4}$.

The resistance to shear is $f_s \times \frac{\pi d^2}{4}$ tons.

(*b*) The plate may tear along the line of minimum section (Fig. 178 (*b*)).

The area of either plate along this line is $(p - d)t$.

The resistance to tension is $f_t \times (p - d)t$ tons.

(*c*) The plates and rivet may be crushed (Fig. 178(*c*)), and this would make the joint loose. The above rule ($d = 1{\cdot}2\sqrt{t}$) usually gives a diameter of rivet large enough to prevent crushing in this manner.

(*d*) The plate may break away in front of the rivet (Fig. 178(*d*)); this would be due to the rivets being too close to the edge of the

plate. The minimum distance allowed in practice from the centre of the rivet hole to the edge of the plate is $1\frac{1}{2}d + \frac{1}{16}$ inch, and in girder work this is usually increased to $2d$.

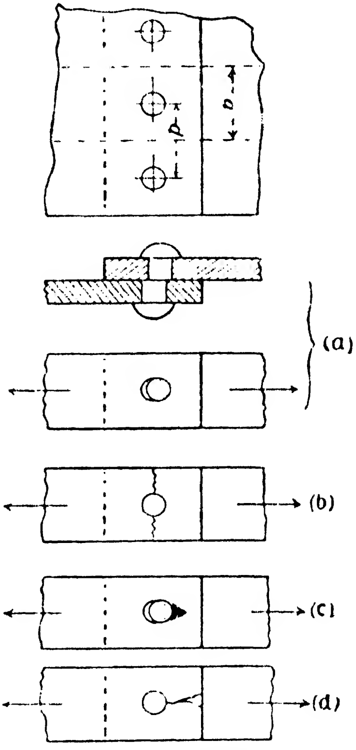


FIG. 178.—Possible failures of lap joints.

In designing the joint it should be made so that the tendencies to rupture in each of the four ways are equal. The commonest method is to fix the diameter of the rivet first (this is rarely made less than $\frac{1}{2}$ inch) then decide on the overlap making it $1\frac{1}{2}d + \frac{1}{16}$; the pitch is then obtained by equating the shearing resistance (Fig. 178(*a*)), and the tensile resistance (Fig. 178(*b*)). This gives—

$$\left.\begin{matrix}\text{Shearing}\\ \text{resistance}\end{matrix}\right\} = \left\{\begin{matrix}\text{Tensile}\\ \text{resistance}\end{matrix}\right.$$

$$f_s \times \frac{\pi d^2}{4} = f_t(p - d)t$$

$$p = \frac{\pi}{4} \cdot \frac{d^2}{t} \cdot \frac{f_s}{f_t} + d$$

In practice the minimum pitch allowed is $2d$ for boiler work, but in girder work the pitch is very rarely made less than $3d$.

Example.—Calculate the diameter and the pitch of the rivets for a single riveted lap joint, the thickness of the plates being $\frac{3}{4}$ inch. Take $f_t = 6$ and $f_s = 5$ tons per square inch.

$$d = 1{\cdot}2\sqrt{t}$$
$$= 1{\cdot}2\sqrt{0{\cdot}75} = 1{\cdot}039 \text{ say 1 inch.}$$

Shearing resistance = Tensile resistance

$$5 \times \frac{\pi}{4} = 6(p - 1)\tfrac{3}{4}$$

$$p = \frac{\pi}{4} \times \tfrac{4}{3} \times \tfrac{5}{6} + 1$$
$$= 0{\cdot}87 + 1 = 1{\cdot}87 \text{ inches.}$$

The pitch would be made 2 inches (the minimum value $2d$).

Percentage Strength of Joint.—The strength of the joint is less than the original plate, because the sectional area of plate along the centre line of the row of rivets has been reduced by drilling the rivet holes. Taking a width of joint equal to the pitch, as in Fig. 178, we see that before drilling the sectional area of the plate is $p \times t$, and after drilling it is $(p - d)t$, hence—

$$\frac{\text{strength of joint}}{\text{strength of seamless plate}} = \frac{(p - d)t}{pt} = \frac{p - d}{p}$$

In the previous example this will be—

$$\frac{2 - 1}{2} = 0{\cdot}5$$

or the strength of the joint is 50 per cent. of the strength of the seamless plate, and its efficiency is 50 per cent.

Thin Riveted Shells.—Steam boilers are not seamless tubes, but are made up of plates riveted together.

Let $e = \dfrac{\text{strength of joint}}{\text{strength of seamless plate}}$ or e = efficiency of joint.

The hoop tension will be found as follows:—

$$\text{bursting force} = p \times d \times l$$
$$\text{resisting force} = 2ftl \times e$$

Equating the two we have—

$$pdl = 2ftle$$
$$pd = 2ft \times e$$
$$f = \frac{pd}{2t \times e}$$

Example.—A boiler 8 feet diameter has to work at a pressure of 160 lbs. per square inch. Calculate the thickness of plate required if the maximum tensile stress is not to exceed 5 tons per square inch, assuming the efficiency of the joint to be 70 per cent.

$$pd = 2fte$$
$$t = \frac{pd}{2fe} = \frac{160 \times 96}{10 \times 2240 \times 0{\cdot}7} = \frac{48}{49} \text{ inch, say, 1 inch.}$$

EXAMPLES XVI.

1. A tie-bar, 1½ inches in diameter, is under a tension of 6·5 tons; what is the intensity of tensile stress?

2. A round bar of mild steel 1 inch diameter and 10 inches long is subjected

to a tension of 10 tons. Find the total stretch, and the fractional strain if $E = 13120$ tons per square inch.

3. A steel wire $\frac{1}{8}$ inch diameter and 10 feet long stretches 0·08 inch under a tension of 245·4 lbs. Calculate the stress and the modulus of elasticity.

4. A copper wire of 0·05 inch in diameter and 90·7 inches long is subjected to increasing loads, the extension for each load being measured. The following results were obtained :—

Load (lbs.) .	0	5	10	15	20	25	30	34	36·5	39	41·5	44	46·5	50	55
Scale (reading) inch .	0·01	0·025	0·04	0·055	0·07	0·15	0·35	1·13	1·81	2·70	3·65	4·75	6·54	8·03	10·22

Calculate the value of Young's modulus for this sample of copper wire.

5. What load in pounds must be hung to an iron wire 15 feet long and 0·1 inch diameter to make it stretch $\frac{1}{8}$ inch ($E = 29{,}000{,}000$ lbs. per square inch)?

6. A hollow cast iron strut is 10 inches external and 8 inches internal diameter and 10 feet long. How much will it shorten under a load of 60 tons? Take E as 8000 tons per square inch.

7. A round tie-bar of mild steel, 18 feet long and $1\frac{1}{4}$ inch diameter lengthens $\frac{1}{16}$ inch under a pull of 7 tons. Find the intensity of tensile stress in the bar, and the value of the modulus of elasticity.

8. A cylindrical steam boiler is 6 feet internal diameter, and is made up of plates $\frac{5}{8}$ inch thick. If the internal steam pressure is 150 lbs. per square inch, what is the intensity of hoop stress in the plates?

9. What thickness of cast iron pipe 16 inches internal diameter will be required to stand an internal pressure of 80 lbs. per square inch, if the intensity of stress in the material is not to exceed 1000 lbs. per square inch?

10. Taking the shearing strength of wrought iron to be 16 tons per square inch, calculate the force necessary to punch a hole $\frac{3}{4}$ inch diameter in a $\frac{5}{8}$-inch plate.

11. Calculate the diameter and the pitch of the rivets for a single-riveted lap joint for plates $\frac{5}{8}$ inch thick. Take $f_t = 6$ and $f_s = 4\frac{1}{2}$ tons per square inch.

12. What will be the percentage strength of the joint in Question 11?

13. A boiler is 6 feet 6 inches in diameter and has to work under a steam pressure of 180 lbs. per square inch. If the maximum stress in the plates is not to exceed 5 tons per square inch, what thickness of plate will be required? (Assume efficiency of joint to be 75 per cent.)

CHAPTER XVII

BENDING

Two very simple forms of bending are illustrated in Fig. 179; (*a*) is a horizontal beam simply supported at its ends and carrying

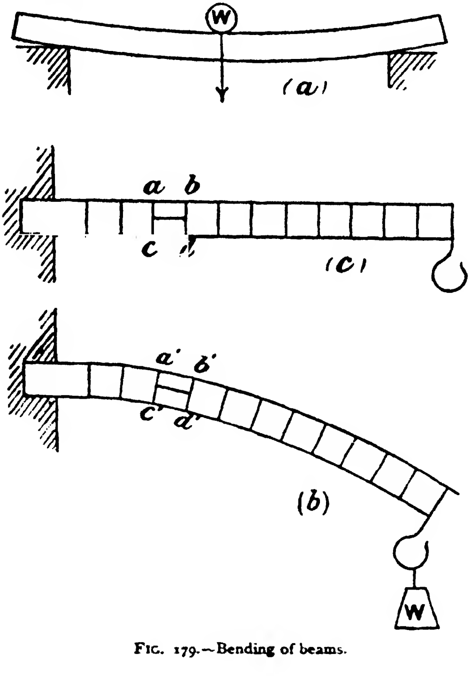


FIG. 179.—Bending of beams.

a load W midway between them; (*b*) is a form of beam called a *cantilever* which is fixed in a wall at one end and not supported elsewhere; it is shown carrying a load W at its free end. All parts

of structures which are acted upon by forces at right angles to their length are called *beams*; sometimes a beam may be simply a piece of timber, in other cases a piece of rolled steel of I section, or, again, a complex body built up of various pieces of steel riveted together. The latter forms are generally called girders. Actual steel beams used by engineers bend very slightly, and to make clear to the student the kinds of strain in a bent beam it is advisable to use a beam of some more flexible substance such as indiarubber. Such a beam is illustrated at (*c*) Fig. 179, and has a number of equidistant vertical lines ruled upon one side. After the beam is bent by hanging a weight W on the end the lines are no longer parallel to one another; the upper ends are further apart and the lower ends closer together than previously. Careful measurement will show that the distances marked *ab* have stretched to a greater length *a'b'* and the lengths *cd* have shortened to *c'd'*. A line midway between *ab* and *cd* will remain of the same length after bending. From this it is evident that for the bending illustrated at (*b*) Fig. 179—

1. The upper layers of the beam are stretched by tension.
2. The lower layers are shortened by compression.
3. Towards the middle layer the strain (and stress) diminishes.

The layer of material which is neither stretched nor shortened is called the neutral surface of the beam.

It is also possible to show by measurement that the strains at a given level are greater at the fixed or wall end, and diminish to the free or loaded end.

For the loading shown at (*a*) Fig. 179, the upper layers are in compression and the lower ones in tension, and in any cases of bending a straight beam there will be a tension on the side which becomes convex and compression on the side which becomes concave.

Bending Stresses.—The stress produced in a beam by the forces which act upon it are not as in a tie-bar simply proportional to those loads or forces, but to the moment of those forces.

The forces on such a portion of a beam as ABCD, Fig. 180, are—

(1) The weight W at its free end.

(2) The forces exerted by the material of the fixed end of the beam on the piece ABCD across the section AB.

We may consider these internal forces as being divided into horizontal and vertical forces (components as in Chap. XIII.) The horizontal forces consist of the resultant T of the tensions in the upper layers and the resultant thrust T' of the lower layers. If we imagine the vectors *ab* and *bc* to represent T and W, and the

complete polygon of forces for the piece ABCD has to be closed (in accordance with Chap. I.) by another horizontal and a vertical side, it is quite evident that the sides must be $c'd'$ and $d'a'$ as shown at $a'b'c'd'$ and that—

(1) The horizontal thrust T′ must be equal to the horizontal tension T.

(2) The internal vertical force F, at AB, must be upward and equal to W.

This vertical force (W) resisted by the internal force is called the *shearing force* at AB since it is the force tending to shear the beam vertically at AB.

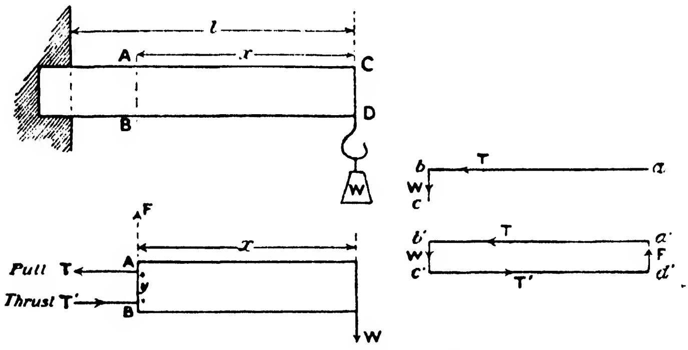


FIG. 160 — Forces in a strained beam.

Bending Moment.—Since the piece ABCD is in equilibrium under the action of the four forces W, F, T and T′, from the principle of moments (Chap. II.) we see that the contra-clockwise moment $T \times y$ exerted by the pull and thrust (T and T′) must be of equal magnitude to the clockwise moment exerted by W and F. The moment exerted by W and F is called the *bending moment* on the beam at AB, and the equal and opposite moment Ty or $T'y$ exerted by T and T′ is called the *moment of resistance of the beam* at this section AB. The magnitude of the moment at AB is evidently—

$$T \times y = W \times x - F \times 0 = Wx$$

and for the piece ABCD it would be the same whatever point we select from which to calculate the moment of the two forces W and F distant x apart; but for the section at the fixed end, the bending moment would be $W \times$ length of cantilever or $W \times l$. For any beam we may state that *bending moment* at any section is

the resultant moment of all the external forces to either side of the section. (It is the same magnitude to *either* side, but of opposite kind, since by the principle of moments the total moment of all the forces is zero, *i.e.* the contra-clockwise moment is equal to the clockwise moment.)

Moment of Resistance at any section is the moment of the longitudinal tension and thrust induced by the bending moment, and is of magnitude equal to the bending moment.

Shearing Force at any section is the resultant external force perpendicular to the beam on *either* side of the section.

Bending Moments and Shearing Forces in Simple Cases.—The table on pp. 222 and 223 shows the six standard cases of beams and explains itself. In the first four cases the student should work out for himself the bending moment and shearing force at different points along the length of the beam, and from his calculations draw the bending moment and shearing-force diagrams. In the last two cases, where the beam is built in horizontally at both ends, the bending moment at the fixed end is opposite in sign to that in the middle of the beam. The effect of the bending moments at the ends is to make the beam *convex upwards*, whilst the opposite kind of bending moment in the middle makes the beam *concave upwards*. At each side of the middle of the span there is one point at which the beam is straight, and at which the bending moment is zero: these points are called points of *contra-flexure*. Further consideration of the bending moments and shearing forces for built in beams is too difficult for this stage of the subject.

Example 1.—A cantilever 15 feet long carries three loads as follows:—5 tons at the free end, 4 tons 10 feet from the wall, and 8 tons 4 feet from the wall. Calculate the bending moment and shearing force at the wall and at each of the loads, and draw the bending moment and shearing-force diagrams.

Fig. 181 shows the beam loaded as in the question.

Bending moment at the Wall (A).

$$\text{B.M.} = 5 \times 15 + 4 \times 10 + 8 \times 4$$
$$\text{B.M.} = 75 + 40 + 32 = 147 \text{ tons-feet.}$$

Shearing force at A $= 5 + 4 + 8 = 17$ tons.

Bending moment at B $= 5 \times \text{BD} + 4 \times \text{BC}$
$= 5 \times 11 + 4 \times 6 = 55 + 24 = 79$ tons-feet.

Shearing force at B $= 4 + 5 = 9$ tons.

Bending moment at C $= 5 \times \text{CD} = 5 \times 5 = 25$ tons-feet.

Shearing force at C $= 5$ tons.

The bending moment and shearing-force diagrams are shown drawn to scale in Fig. 181.

Example 2.—Fig. 182 shows a beam simply supported at its ends

and carrying three loads. Calculate the bending moment at the middle of the span and at each load.

The first thing to do is to find the reactions R_1 and R_2 at the points of support.

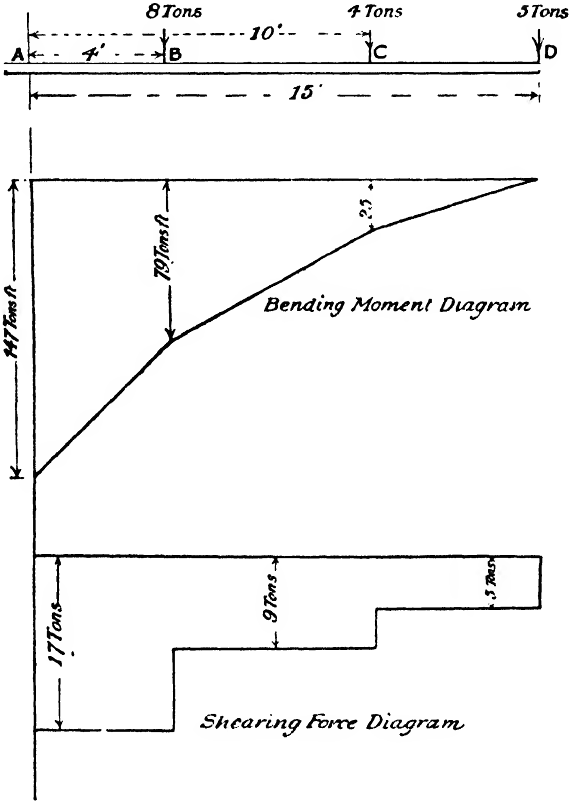


FIG. 181.

Taking moments about the left-hand support D (contra-clockwise) we get—

$$R_2 \times 50 = 8 \times CD + 12 \times BD + 4 \times AD$$
$$50R_2 = 8 \times 38 + 12 \times 23 + 4 \times 10$$
$$= 304 + 276 + 40$$
$$= 620$$
$$R_2 = \frac{620}{50} = 12{\cdot}4 \text{ tons.}$$

The total load on the beam is $8 + 12 + 4 = 24$ tons

$$\therefore R_1 = 24 - 12{\cdot}4 = 11{\cdot}6 \text{ tons.}$$

At mid-span (point X 25 feet from either support). Taking the forces on the right of X we have—

$$\begin{aligned} \text{Bending moment at mid-span} &= R_2 \times EX - 8 \times CX \\ &= 12{\cdot}4 \times 25 - 8 \times 13 \\ &= 310 - 104 = 208 \text{ tons-feet.} \end{aligned}$$

At C. Taking the forces on the right of C—

$$\text{Bending moment} = R_2 \times CE = 12{\cdot}4 \times 12 = 148{\cdot}8 \text{ tons-feet.}$$

At B. Taking the forces on the right of B—

$$\begin{aligned} \text{Bending moment} &= R_2 \times EB - 8 \times CB \\ &= 12{\cdot}4 \times 27 - 8 \times 15 \\ &= 334{\cdot}8 - 120 = 214{\cdot}8 \text{ tons-feet.} \end{aligned}$$

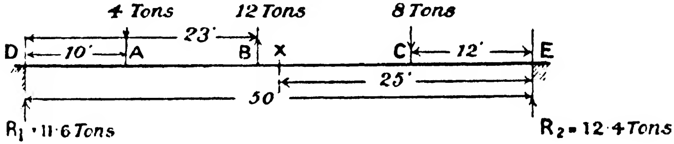


FIG. 182.

We may check this figure by taking the forces to the *left* of B—

$$\begin{aligned} \text{Bending moment} &= R_1 \times DB - 4 \times AB \\ &= 11{\cdot}6 \times 23 - 4 \times 13 \\ &= 266{\cdot}8 - 52 = 214{\cdot}8 \text{ tons feet as before.} \end{aligned}$$

At A. Taking the forces to the left of A—

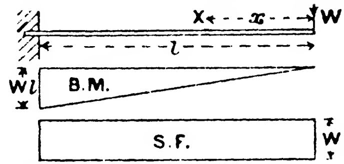
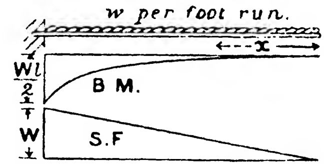
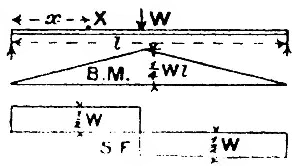
$$\text{Bending moment} = R_1 \times AD = 11{\cdot}6 \times 10 = 116 \text{ tons-feet.}$$

We might also check this value by considering the forces to the *right* of A.

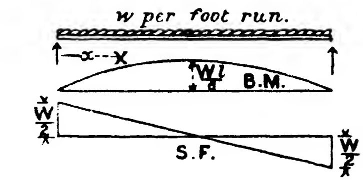
$$\begin{aligned} \text{Bending moment} &= R_2 \times AE - 8 \times CA - 12 \times BA \\ &= 12{\cdot}4 \times 40 - 8 \times 28 - 12 \times 13 \\ &= 496 - 224 - 156 \\ &= 496 - 380 = 116 \text{ tons-feet as before.} \end{aligned}$$

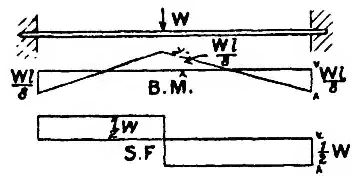
Intensity of Bending Stress.—In almost all simple forms of beams which are long in comparison with their width or depth, the most important stresses are the longitudinal tension on the convex side and the longitudinal compressive stress on the concave side, which give the resultant pull and thrust T and T′ and exert the moment of resistance to bending Fig. 180. We will take the beam in Fig. 180 to be of rectangular cross-section of breadth b inches and depth d inches, as shown at KLMN in Fig. 183.

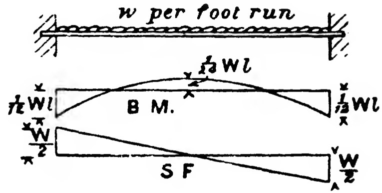
SIX STANDARD CASES OF BEAMS.

Description of Beam.	Bending Moment (B.M.)	Shearing Force (S.F.).	Diagrams of Bending Moment (B.M.) and Shearing Force (S.F.).
I.—*Cantilever.* Load W at free end.	B.M. at X = $W \cdot x$ Maximum B.M. = $W \cdot l$ and occurs at the fixed end.	S.F. = W Shearing Force is constant all along the beam.	
II.—*Cantilever.* Load W uniformly distributed at the rate of w per foot run.	B.M. at X = $w \cdot x \times \frac{x}{2} = \frac{wx^2}{2}$ Maximum B.M. at the fixed end $= \frac{wl^2}{2} = \frac{Wl}{2}$	S.F. at X = $w \cdot x$ Maximum S.F. at the fixed end $= wl = W$	
III.—*Beam supported at the ends.* Load W at middle of span.	B.M. at X = $\frac{W}{2} \times x$ Maximum B.M. at middle of span = $\frac{1}{4} Wl$.	S.F. at X = $\frac{1}{2}$W and is uniform over each half of the span, being upwards on one half and downwards on the other half.	

IV.—*Beam supported at the ends.* Load W uniformly distributed at the rate of w per foot run.	B.M. at X $= \frac{wl}{2} \times x - wx \times \frac{x}{2}$ $= \frac{1}{2}wlx - \frac{1}{2}wx^2$ Maximum B.M. at middle of span $\left(\text{i.e. } x = \frac{l}{2}\right)$ $= \frac{1}{2}w \cdot l \cdot \frac{l}{2} - \frac{1}{2}w\left(\frac{l}{2}\right)^2$ $= \frac{wl^2}{8} = \frac{1}{8}Wl$	S.F. at X $= \frac{wl}{2} - wx$ Maximum S.F. at the supports $= \frac{1}{2}wl = \frac{1}{2}W$. S.F. is upwards on one half, downwards on the other half, and zero at mid-span.
V.—*Beam fixed at the ends.* Load W at middle of span.	B.M. at fixed ends } = B.M. at middle $= \frac{Wl}{8}$ B.M. at middle is opposite in sign to B.M. at the fixed ends.	S.F. $= \frac{1}{2}W$ as in Case III.
VI.—*Beam fixed at the ends.* Load W uniformly distributed at the rate of w per foot run.	Maximum B.M. at fixed ends } $= \frac{wl^2}{12} = \frac{Wl}{12}$ B.M. at middle of span } $= \frac{wl^2}{24} = \frac{Wl}{24}$ B M. at middle is opposite in sign to B.M. at the fixed ends.	S.F. same as Case IV.







Within the limits of elasticity, the tensile stress on the section AB (shown at KLMN) varies from a maximum intensity, KL, at the outside to zero at the dotted neutral plane through O. The intensity of tensile stress is represented by the lengths of the horizontal lines in the triangle KLO, and the intensity of compressive stress by the lengths of the horizontal lines in the triangle NMO. Consequently, the mean of the intensity of tensile stress, on the upper half of the rectangular section is half the maximum intensity KL at the outside, and if f is the maximum intensity of stress at the edges KL and MN (tensile at KL and compressive at MN) of the section in pounds per square inch—

$$T = T' = \text{area}\left(b \times \frac{d}{2}\right) \times \frac{1}{2}\text{ (stress intensity at outside)}$$

$$\therefore T = \tfrac{1}{4}bd \times f \text{ pounds.}$$

The distance of T (and T′) from O is $\frac{2}{3}$ of the distance of the edge KL from O. (The reader may note that the stress on the

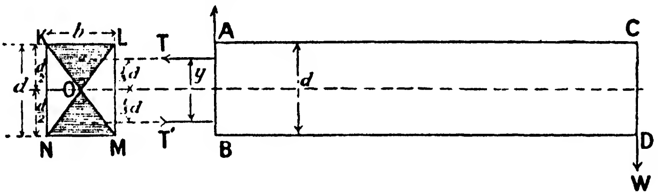


FIG. 183.—Moment of resistance of a rectangular beam.

upper half, varying uniformly in intensity from O to the outside, is equivalent to a uniform or unvarying stress of the maximum intensity (f), but acting only on the area KLO, so that $T = f \times$ area KLO $= \frac{1}{4}fbd$, and that the position of the resultant T is similar to the position of the centroid of a triangular area KLO, which is evidently $\frac{1}{3}$ of $\frac{d}{2}$ from KL (see Chapter XIV.), or $\frac{2}{3}$ of $\frac{d}{2} = \frac{1}{3}d$ from O.) Hence, the distance y of T from T′ is $\frac{2}{3}d$, and the

$$\begin{aligned}\text{Moment of resistance} &= T \times y = T \times \tfrac{2}{3}d \text{ pound-inches}\\ &= \tfrac{1}{4}fbd \times \tfrac{2}{3}d\\ &= f \times \tfrac{1}{6}bd^2 \text{ pound-inches.}\end{aligned}$$

Hence, since the moment of resistance balances the bending moment, we may write for a beam of rectangular section—

$$\text{Bending moment} = f\tfrac{1}{6}bd^2 \text{ pound-inches}$$

$$\text{or } f = \frac{\text{Bending moment in pound-inches}}{\frac{1}{6}bd^2} \text{ pounds per square inch.}$$

The intensity of stress caused by a given bending moment is then *inversely proportional to the breadth and to the square of the depth* of the beam. Or, to put it in another way, for a given safe intensity of stress, the *bending moment* which a beam will safely resist *is proportional to the breadth and to the square of the depth.* Thus we say, that *the strength of a rectangular beam is proportional to the breadth and to the square of the depth.* We have seen that, for any particular kind of support, the bending moment produced by a given load is proportional to the length of the beam; hence the strength of any given type of beam is *inversely proportional to its length.*

Example.—A timber beam of rectangular cross-section has a breadth of 6 inches, and depth 10 inches. It is 15 feet long and simply supported at the ends, and carries a load of 2 tons at the middle of the span. Find the greatest stress in the beam.

The reaction at each end = 1 ton.

The greatest bending movement is under the load at mid-span: it is equal to reaction × half the span.

$$= 1 \times \frac{15}{2} \times 12 = 90 \text{ tons-inches}$$

$$\text{Now, } f = \frac{\text{bending moment in tons-inches}}{\frac{1}{6}bd^2}$$

$$= \frac{90}{\frac{1}{6} \times 6 \times 100} = 0{\cdot}9 \text{ tons per square inch,}$$

Modulus of Section.—The moment of resistance $f \times \frac{1}{6}bd^2$ for a rectangular section may be written—

$$\text{moment of resistance} = f \times \text{modulus of section} = f \times Z$$

the quantity $\frac{1}{6}bd^2$ being the *modulus of section* Z for a rectangular beam section. For other sections the modulus of section will have other values.

For example, for a solid round rod of diameter d inches

$$Z = \frac{\pi}{32}d^3 \text{ and}$$

$$\text{moment of resistance} = \frac{\pi}{32}d^3 f$$

being the maximum intensity of stress.

It is to be noted that the relation—

$$\text{moment of resistance} = f \cdot Z$$

is only true for loads which do not strain the beam beyond the elastic limit.

An Important Beam Section.—We have seen that the important longitudinal stresses of tension and compression induced by bending reach their maximum intensity at the outer skin of the beam, and that the material inside bears a much smaller stress, and is therefore not very economically employed in resisting bending moments. In rolled steel beams, to use the material to the best advantage, an I section is used, as shown in Fig. 184, which approximates in shape to two thin horizontal rectangular flanges, connected by a still thinner vertical rectangular web. In this way nearly all the material is at or near the outer edges of the section, and so gets almost the full maximum bending stress upon it. It is easy to estimate approximately the moment of resistance of such a section.

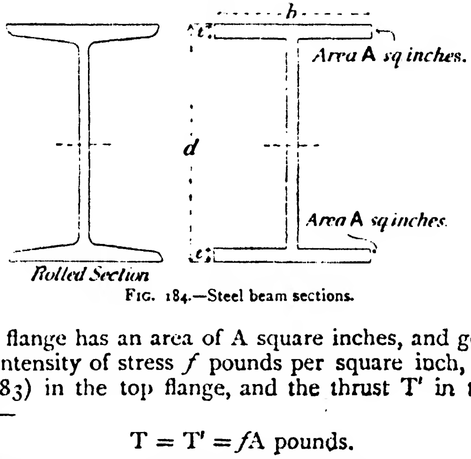


FIG. 184.—Steel beam sections.

For if each flange has an area of A square inches, and gets the full maximum intensity of stress f pounds per square inch, the pull T (see Fig. 183) in the top flange, and the thrust T′ in the bottom flange, are—

$$T = T' = fA \text{ pounds.}$$

And the distance (y) apart is practically equal to d, the full depth of the section; the resisting moment—

$$T \cdot y = T \cdot d = f \cdot A \cdot d \text{ pounds-inches, or—}$$

$$\text{resisting moment} = \begin{cases} \text{maximum intensity of stress} \times \text{area of one} \\ \quad \text{flange} \times \text{depth}; \end{cases}$$

and the modulus of section is $A \times d$; and if each flange is b inches wide and t inches thick $A = b \cdot t$, or

$$\text{modulus of section} = b \times t \times d \text{ and}$$
$$\text{resisting moment} = fbtd \text{ pound-inches.}$$

We have supposed that the flanges bear all the pull and thrust T and T′. Actually the web carries a little, but its moment of

resistance is small compared to that of the flanges. The web carries much of the shearing force.

Example 1.—A rolled steel joist 10 inches deep has flanges 6 inches wide by $\frac{3}{4}$-inch thick. Find the maximum intensity of stress produced in it by a load of 5 tons at the middle of the span, which is 14 feet.

The reaction of each support is $\frac{5}{2} = 2{\cdot}5$ tons

Maximum bending moment (at mid-span) = reaction × $\frac{1}{2}$ length of beam

$= 2{\cdot}5 \times 7 \times 12$

$= 210$ tons-inches

Moment of resistance = bending moment

$$fbtd = 210$$

$$f \times 6 \times \tfrac{3}{4} \times 10 = 210$$

$$f = \frac{210 \times 4}{6 \times 3 \times 10} = 4{\cdot}66 \text{ tons per square inch.}$$

Example 2.—Find the maximum span which may be adopted with a beam of the same section as that in Example I., if the maximum intensity of stress is not to exceed 6 tons per square inch, and the beam carries a uniformly distributed load of $1\frac{1}{2}$ tons per foot of its lengths.

Let l = span required in feet and w = load per foot run.

The maximum bending moment is at mid-span C (Fig. 185).

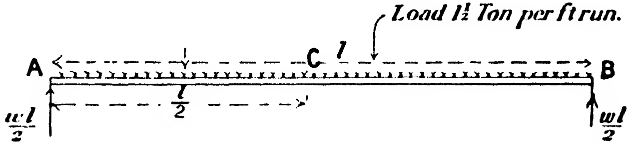


FIG. 185.

Each reaction is half the total load on the beam $= \frac{wl}{2} = \frac{3}{4}l$ tons.

Maximum bending moment (at C)

$$= \text{reaction} \times \tfrac{1}{2}\text{ span} - \text{load on AC} \times \tfrac{1}{2}\text{ AC}$$

$$= \tfrac{3}{4}l \times \frac{l}{2} - \tfrac{3}{4}l \times \frac{l}{4}$$

$$= \tfrac{3}{16}l^2 \text{ tons-feet} = \tfrac{3}{16} \times 12l^2 \text{ tons-inches}$$

Moment of resistance = bending moment

$$fbtd = \tfrac{3}{16} \times 12l^2$$

$$6 \times 6 \times \tfrac{3}{4} \times 10 = \frac{3 \times 12l^2}{16}$$

$$l^2 = \frac{6 \times 6 \times 3 \times 10 \times 16}{4 \times 3 \times 12} = 120$$

$$l = \sqrt{120} = 11 \text{ feet.}$$

Braced Framed or Trussed Girders.—Large girders of bridges and roofs are often made up of bars jointed together to form a frame-work. We may illustrate the principle by a model such as that shown in Fig 186, which consists of a cantilever, loaded at its free end cut into two parts and joined by a horizontal string AB at the top and a horizontal strut, DC at the lower edge. If the strut DC

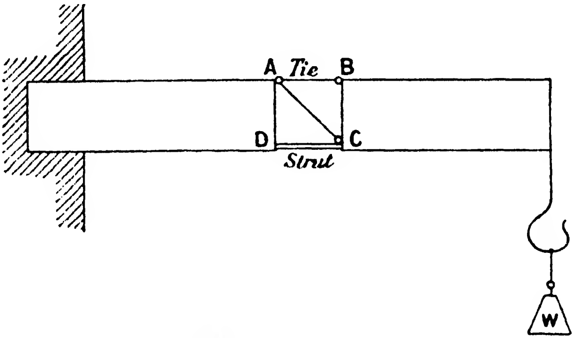


FIG. 186.—Braced girder model.

has smooth, well-rounded ends with little friction, AB and CD will not support the load W, and the free end would *shear* off. A diagonal string tie from A to C, however, will enable a weight, W, to be supported on the free end. This arrangement illustrates the uses of the various parts of a cantilever truss. The top flange is a tie, and carries the pull T similar to that in a solid beam (Fig. 183);

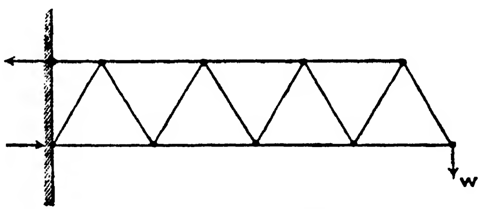


FIG. 187.—Braced cantilever.

the lower flange carries the thrust T′, and the diagonal carries the shearing force, being itself in tension, and so pulling upwards at C. A complete cantilever truss is shown in Fig. 187, while Fig. 188 shows two forms of horizontal truss for spanning two supports. In these cases (Fig. 188) the lower flanges or booms

will be in tension, and the upper booms in compression, while the diagonals carry the shearing force.

Stiffness and Deflection of Beams.—By stiffness in a beam we understand, resistance to deflection, so that the relative stiffness of two beams is inversely proportional to their deflections under a given load. For all rectangular beams, the deflections are proportional to—

$$\frac{Wl^3}{Ebd^3}$$

where W is the total load on the beam, l the length, E Young's Modulus, b the breadth, and d the depth. The actual deflection is

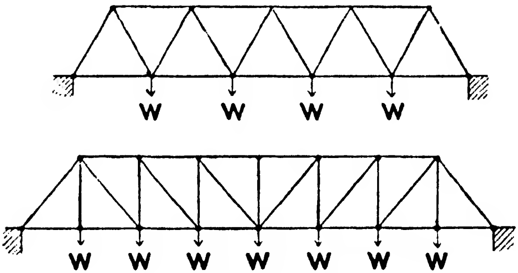


FIG. 188.—Braced girders.

equal to the same quantity multiplied by some constant depending upon the method of loading and supporting the beam. The stiffness is then—

(1) Proportional to the breadth (b);
(2) Proportional to the *cube* of the depth (d^3);
(3) Inversely proportional to the cube of the length (l^3).

For the cantilever loaded at the free end (line 1 in table on p. 222), the deflection is $\frac{4Wl^3}{Ebd^3}$. For the beam simply supported at its end and loaded in the middle the deflection is $\frac{Wl^3}{4Ebd^3}$, or $\frac{1}{16}$ as much as in the previous case.

Example 1.—A beam of fir 2 inches broad, 3 inches deep and 4 feet between supports deflects 0·08 inch under a load of 250 lbs. placed midway between the supports. Find the deflection of a baulk of the same timber 10 inches wide and 15 inches deep on supports 16 feet apart under a load of 5 tons placed midway between the supports.

The deflection is proportional to the load W, to l^3, and inversely proportional to b and to d^3

$$\therefore \text{ Deflection} = 0{\cdot}08 \times \frac{5 \times 2240}{250} \times \left(\frac{16}{4}\right)^3 \times \frac{2}{10} \times \left(\frac{3}{15}\right)^3$$

$$= 0{\cdot}08 \times \frac{224}{5} \times (4)^3 \times \frac{1}{5} \times \left(\frac{1}{5}\right)^3$$

$$= 0{\cdot}08 \times \frac{224 \times 64}{5 \times 5 \times 5 \times 25} = 0{\cdot}366 \text{ inch.}$$

Example 2.—A beam of fir 16 inches square in section is carried by two supports with a span of 18 feet, and carries a load at the middle of the span of 15 tons. Find (1) The maximum intensity of stress produced, (2) The deflection at the middle of the span. Take E as 700 tons per square inch.

(1) Modulus of section $= \frac{1}{6}bd^2 = \frac{1}{6} \times 16 \times 16 \times 16 = \frac{2048}{3}$

Maximum bending moment at mid-span $\Big\}$ = reaction × ½ span

$$= \frac{15}{2} \times \frac{18 \times 12}{2} = 810 \text{ tons-inches}$$

$$\text{Stress } f = \frac{810 \times 3}{2048} = 1{\cdot}184 \text{ tons per square inch.}$$

(2)
$$\text{Deflection} = \frac{Wl^3}{4Ebd^3}$$

$$= \frac{15 \times (18 \times 12)^3}{4 \times 700 \times 16 \times (16)^3} = 0{\cdot}823 \text{ inch.}$$

Experiments on Beams.—The above laws may be proved experimentally by means of the simple apparatus shown in Fig. 189

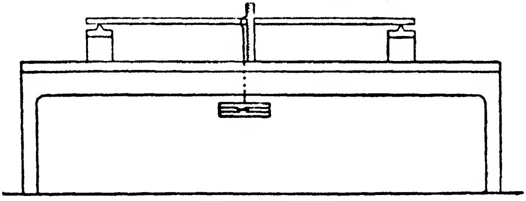


FIG. 189.—Strength and deflection of beams.

The beam is carried on **V** supports near its ends, the supports resting on a rigid cast iron bed. Loads are hung on the beam at mid-span by means of the hanger shown. The deflection is read off on the scale for different loads, the loads being increased till fracture occurs. For experiments within the elastic limit only the more accurate apparatus already explained in Chap. XV, Fig. 158, may be used.

The following are the results of experiments made on beams of *red deal*, with the apparatus shown in Fig. 189 :—

Beam A. 12 inches span; section 1 inch board, ¼ inch deep.		*Beam B.* 24 inches span; section ¼ inch broad, 1 inch deep.		*Beam C.* 12 inches span, 1 inch broad, 1 inch deep.		*Beam D.* 12 inches span, ¼ inch broad, 1 inch deep.		*Beam E.* 12 inches span, 1 inch broad, ¼ inch deep.	
Load W (cwts.).	Deflection at mid-span (inches).	Load W (cwts.).	Deflection at mid span (inches).	Load W (cwts.).	Deflection at mid-span (inches).	Load W (cwts.).	Deflection at mid-span (inches).	Load W (cwts.)	Deflection at mid-span (inches)
0	0	0	0	0	0	0	0	0	0
¼	0·05	¼	0·1	¼	0·015	¼	0·03	¼	0·03
½	0·10	½	0·2	½	0·04	½	0·05	½	0·06
¾	0·15	¾	0·3	¾	0·055	¾	0·066	¾	0·085
1	0·20	1	0·44	1	0·065	1	0·084	1	0·12
1¼	0·26	1¼	0·60	1¼	0·075	1¼	0·10	1¼	0·15
1½	0·355	1⅝	1·20 (broke)	1½	0·09	1½	0·12	1½	0·18
1⅝	0·50 (broke)			1¾	0·105	1¾	0·136	1¾	0·22
				2	0·13	2	0·155	2	0·38 (broke)
				2¼	0·16	2¼	0·24		
				2½	0·20	2½	0·32		
				2⅞	0·26	2¾	0·54 (broke)		
				3	0·31				
				3·67	0·36 (broke)				

The above results are shown plotted in Fig. 190.

Consider the Three Beams C, D, E.—We have stated that the deflection is for a given load proportional to l^3, and inversely proportional to b and to d^3. From the curves shown in Fig. 190,

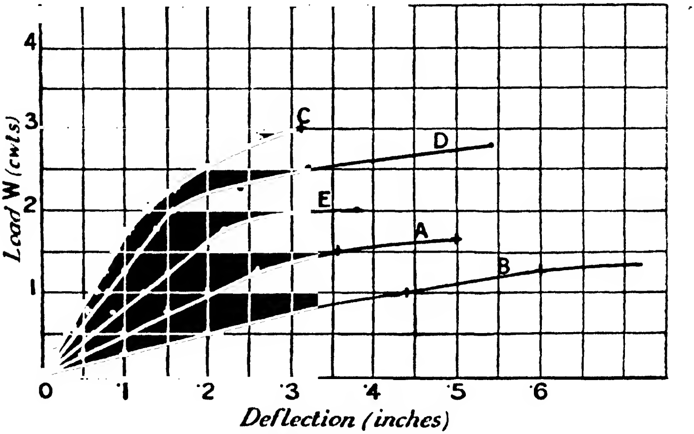


FIG. 190.—Strength and deflection tests of beams.

within the elastic limit we see that for a load of $1\frac{1}{2}$ cwts. the deflections of these beams are—

	C.	D.	E.
Deflection	0·09	0·12	0·185

Applying the above laws, we can calculate what the deflection should be for the beams D and E, from the deflection 0·09 inch of C.

Deflection of D should be $0·09 \times \frac{4}{3} = 0·12$ inch, which agrees with the observed deflection.

Deflection of E should be $0·09 \times (\frac{4}{3})^3 = 0·09 \times \frac{64}{27} = 0·21$ inch, which agrees fairly well with the observed deflection 0·185 inch.

These results approximately illustrate experimentally that the deflection is inversely proportional to the breadth (D), and inversely proportional to the cube of the depth (E).

Consider next the Beams A and B.—Within the elastic limit for a load of 1 cwt. we see, from the curves in Fig. 190, that the deflection of A is 0·21 inch, and of B is 0·44 inch.

Now—

span of A is 12 inches, breadth of A is 1 inch, depth of A is $\frac{1}{2}$ inch,
span of B is 24 inches, breadth of B is $\frac{1}{2}$ inch, depth of B is 1 inch.

Applying the above laws, we can calculate what the deflection should be for the beam B, from the deflection 0·21 inch of A.

Deflection of B should be $0{\cdot}21 = (\frac{24}{12})^3 \times \frac{2}{1} \times (\frac{1}{2})^3 = 0{\cdot}21 \times 2 = 0{\cdot}42$ inch, which agrees fairly closely with the observed value 0·44 inch.

Comparison of the Strengths of the Beams.—We have seen that the strength of a beam is proportional to the breadth and to the square of the depth, and inversely proportional to the length. Applying these rules, we can estimate the breaking load for the beams D and E by calculation from the breaking load of beam C.

For beam C. Span = 12 inches, breadth = 1 inch, depth = 1 inch, breaking load = 3·67 cwt.

For beam D. Span = 12 inches, breadth = $\frac{3}{4}$ inch, depth = 1 inch.

For beam E. Span = 12 inches, breadth = 1 inch, depth = $\frac{3}{4}$ inch.

Breaking load of D should be $3{\cdot}67 \times \frac{3}{4} = 2{\cdot}75$ cwts.

Breaking load of E should be $3{\cdot}67 \times (\frac{3}{4})^2 = 2{\cdot}07$ cwts.

which agree with the actual breaking loads (see Fig. 190 and Tables).

Similarly, we can calculate the breaking load of beam B from that of A.

Beam A. Span = 12 inches, breadth = 1 inch, depth = $\frac{1}{2}$ inch, breaking load $1\frac{5}{8}$ cwts.

Beam B. Span = 24 inches, breadth = $\frac{1}{2}$ inch, depth = 1 inch.

Breaking load of B should be $1\frac{5}{8} \times \frac{1}{2} \times (\frac{2}{1})^2 \times \frac{1}{2} = 1\frac{5}{8}$ cwt., which agrees with the actual breaking load.

Modulus of Rupture.—It should be noted that, although the above experiments show that the breaking load is proportional to the breadth, square of the depth, and inversely proportional to the length, or, in other words, that the ultimate bending moment is proportional to the breadth and square of the depth, the formula—

$$\text{moment of resistance} = f \times \tfrac{1}{6}bd^2, \quad \text{or } f = \frac{\text{bending moment}}{\tfrac{1}{6}bd^2}$$

is only true within the elastic limit. The quantity—

$$\frac{\text{ultimate bending moment}}{\tfrac{1}{6}bd^2}$$

is called the *Modulus of Rupture*, and is a useful constant to give

the ultimate load of beams of cast iron and timber. For a breaking load W at the middle of the supports the ultimate bending moment is $\frac{1}{4}Wl$, hence—

$$\text{modulus of rupture} = \frac{\frac{1}{4}Wl}{\frac{1}{6}bd^2} = \frac{3}{2}\,\frac{Wl}{bd^2}$$

For example, the experiments given above for the beam E of red deal, of which the span is 12 inches, breadth 1 inch, depth $\frac{3}{4}$ inch, and the breaking load 2 cwt., give—

$$\text{modulus of rupture} = \frac{3}{2} \times \frac{2 \times 112 \times 12}{1 \times \frac{3}{4} \times \frac{3}{4}}$$
$$= 7168 \text{ lbs. per square inch,}$$
or 3·2 tons per square inch.

Example 1.—*If a cast iron beam 1 inch by 1 inch in section and 1 foot long, fixed at one end will just bear 330 lbs. at its other end before breaking, what load at the free end would be required to break a cast iron cantilever 2 inches wide and 3 inches deep and 3 feet long. Also, what would be the load at mid-span if the beam were supported at each end?*

Bending moment in first case = 330 × 12 = 3960 pound-inches.

For the 2 inch width this bending moment will be multiplied by 2, and for the 3 inch depth it will be multiplied by 3^2 or 9; hence the bending moment required to break the beam is—

3960 × 2 × 9 = 71280 pound-inches

And since the length of the cantilever is 36 inches, the bending moment (Wl) is—

Load in pounds × 36 = 71280
and load = $\frac{71280}{36}$ = 1980 lbs.

If the beam were supported at each end the bending moment ($\frac{1}{4}Wl$) would be—

Load in pounds × $\frac{36}{4}$ = 71280
Load = $\frac{71280}{9}$ = 7920 lbs.

Example 2.—*A beam of red deal $\frac{1}{2}$ inch broad and 1 inch deep is carried on two supports 2 feet apart and just bears a load of 180 lbs. midway between the supports before breaking. What load could be safely carried at mid-span on a joist of the same timber, 12 inches broad, 16 inches deep, span 20 feet. Take the safe load to be $\frac{1}{5}$ of breaking load.*

Bending moment in first case = $\frac{1}{4}Wl$ = 180 × $\frac{24}{4}$ = 1080 pound-inches.

For the 12 inches width this bending moment will be multiplied by

$\frac{12}{.}$ or 24 and for the 16 inches depth it will be multiplied by 16^2 or 256; hence the bending moment required to break the large beam is—

$$1080 \times 24 \times 256 \text{ pound-inches}$$

and since the span of the beam is 20 feet or 240 inches the bending moment ($\frac{1}{4}Wl$) will be—

$$\text{Load in pounds} \times \tfrac{240}{4} = 1080 \times 24 \times 256$$

$$\text{load} = \frac{1080 \times 24 \times 256}{60} = 110{,}592 \text{ lbs.}$$

and the safe load will be—

$$\tfrac{110592}{8} = 13{,}824 \text{ lbs.}$$

Check.—This result might also be obtained more directly, since the breaking load is proportional to the breadth, to the square of the depth, and inversely proportional to the length, hence—

$$\text{breaking load} = 180 \times 24 \times 256 \times \tfrac{2}{20} = 110592 \text{ lbs.}$$

and the safe load will be—

$$\tfrac{110592}{8} = 13823 \text{ lbs., as before.}$$

EXAMPLES XVII.

1. A cantilever 20 feet long carries three loads as follows: 2 tons at the free end, 6 tons 12 feet from the wall, 5 tons 6 feet from the wall. Calculate the bending moment and shearing force at the wall and at each of the loads, and draw the bending moment and shearing force diagrams.

2. A beam 24 feet long is simply supported at its ends and carries the following loads: 7 tons, 4 feet from the left-hand end, 2 tons 11 feet from the left-hand end and 10 tons 18 feet from the left-hand end Calculate the bending moment at the middle of the span and at each load.

3. A cantilever 15 feet long carries a uniformly distributed load of $1\frac{1}{2}$ tons per foot run over its entire length. Calculate the bending moment at a section of the beam 10 feet from the free end.

4. A timber beam of rectangular cross-section has a breadth of 5 inches and depth 12 inches. It is 20 feet long and is simply supported at its ends, and carries a load of 3 tons at the middle of the span. Find the greatest stress in the beam.

5. A beam of fir, breadth 16 inches, depth 16 inches, is carried by two supports with a span of 18 feet. It carries a load of 15 tons at the middle of the span. Find the maximum intensity of stress produced.

6. A rolled steel joist 12 inches deep with flanges $\frac{7}{8}$ inch thick and 6 inches wide carries on a span of 10 feet a load of 3 tons at the middle of the span. Find the maximum intensity of stress produced.

7. If the joist in Question 6 carries a uniformly distributed load of 2 tons per foot run, what will be the maximum intensity of stress produced in it?

8. Find the maximum span which may be adopted with a rolled joist 10 inches deep with flanges 6 inches wide by $\frac{3}{4}$ inch thick, if the maximun intensity of stress is not to exceed 6 tons per square inch and the beam carries a uniformly distributed load of 1 ton per foot run.

9. A timber beam of rectangular cross-section has a breadth of 4 inches and a depth of 9 inches. It is 12 feet long and is simply supported at the ends. What load hung at the middle of the span will produce a deflection there of $\frac{1}{2}$ inch? $E = 700$ tons per square inch.

10. A beam of fir 16 inches square in section is carried by two supports with a span of 18 feet and deflects 0·823 inch under a load of 15 tons placed midway between the supports. Find the deflection of a beam of the same timber 4 inches broad and 8 inches deep on supports 6 feet apart under a load of 4 tons placed midway between the supports.

11. A cantilever of breadth 5 inches and depth 10 inches is 8 feet long and carries a load of 1 ton at the free end. Calculate the deflection at the free end, assuming $E = 700$ tons per square inch.

12. A cast iron beam 1 inch broad and 2 inches deep will just break when a load of 6 cwt. is applied at the middle of a span of 3 feet. Calculate the modulus of rupture.

13. If a cast iron beam 1 inch by 1 inch in cross-section and 2 feet long and fixed at one end will just bear 165 lbs. at its free end without breaking, what load at the free end would be required to break a cast iron cantilever $1\frac{1}{2}$ inches wide and 3 inches deep and $4\frac{1}{2}$ feet long? Also what would be the load at mid-span if the beam were simply supported at each end?

14. A timber beam 1 inch broad and 1 inch deep is carried on two supports 1 foot apart, and just bears a load of 720 lbs. midway between the supports before breaking. What load could be safely carried at mid-span on a joist of the same timber, 10 inches broad, 20 inches deep, and 24 feet between supports? Take the safe load to be $\frac{1}{4}$ of the breaking load.

CHAPTER XVIII

TORSION

If a round rod of metal is firmly fixed at one end to resist twisting and a twisting moment or torque $T = F \times x$ (Fig. 191) is supplied by means of a pulley or a handle to the other end, the rod twists, whether visibly or not. At any circular cross-section such as AB, every little bit of material is under shear stress, which tends to drag the neighbouring material on the other side of the section AB tangentially to a circle centred at the axis of the rod. The material to the right of AB (Fig. 191) tends to turn the fixed end in a clockwise direction, and that to the left AB exerts equal and opposite forces on the free end giving a contra-clockwise moment, the total effect being a resisting torque or moment equal and opposite to T. The torsional shear stress exerting this resisting torque varies in intensity from a maximum at the circumference to zero at the axis of the rod. If f_s is the maximum intensity of stress in pounds per square inch at the outer skin, and d is the diameter of the rod or shaft in inches, the total resisting torque is—

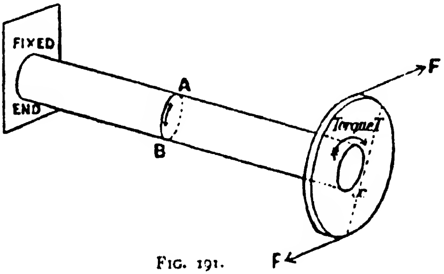


FIG. 191.

$$\frac{\pi}{16}d^3 f_s = T \text{ pound-inches},$$

and the maximum intensity of shear stress $f_s = \dfrac{T}{\frac{\pi}{16}d^3}$ pounds per square inch.

The torque T on a shaft transmitting a given amount of power

has been explained in Chap. IX., and the above formula enables us to calculate the stress produced.

Example.—What torque may be transmitted in a shaft 2 inches diameter with a maximum sheer stress of 7000 lbs. per square inch; and if the shaft makes 80 revolutions per minute, what horse-power may be transmitted?

$$T = \frac{\pi}{16}d^3f_s = \frac{\pi}{16} \times 2 \times 2 \times 2 \times 7000$$
$$= 10995 \text{ pound-inches},$$
$$= \frac{10,995}{12} = 916 \text{ pound-feet}.$$

$$\text{Now, horse-power} = \frac{\text{Torque (pound-feet)} \times \text{radians per minute}}{33,000}$$
$$= \frac{916 \times 2\pi \times 80}{33,000} = 13{\cdot}9 \text{ H.P.}$$

Flanged Shaft Couplings are largely used for connecting lengths of shafting together, particularly when the diameter of the shafting is not less than 3 inches. Fig. 192 shows one simple

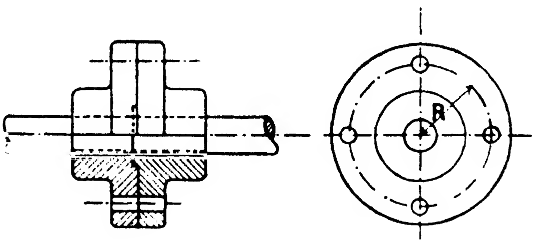


FIG. 192.

form of this coupling. If the line of shafting is to be in perfect alignment, great care is necessary in fitting the couplings. In the best practice they are either shrunk on or forced on to the shaft by hydraulic pressure, about $\frac{3}{8}$ to $\frac{1}{4}$ inch length of shaft being arranged to project from one face to enter the other and keep the two in position when the shafts are the same diameter, keys are also fitted as shown in the figure. When the couplings are shrunk on, the keys need not fit tightly, but if, as is often the case, the couplings are not shrunk on, these keys must fit well. In cases where the shafts to be connected are of different diameters, a projection is turned on the face of one coupling which fits into a recess turned in the face of the other coupling. The two halves of the coupling are held firmly together by bolts which are subjected to

shear stress, and transmit the torque from one piece of shafting to the other.

If the radius of the bolt circle is large compared with the diameter of the bolts, the shear stress on each may be taken as uniform, say, f_s pounds per square inch, and—

$$\text{Total tangential force on each bolt} = f_s \times \frac{\pi d^2}{4}$$

$$\text{Torque due to each bolt} = R \times f_s \frac{\pi d^2}{4}$$

$$\text{Total torque on coupling } T = N \times R f_s \frac{\pi d^2}{4} \text{ pound-inches}$$

where N = number of bolts.

Example.—In a shaft coupling the radius of the bolt circle is 9 inches, and there are 8 bolts each $1\frac{1}{8}$ inches diameter. Allowing a shear stress for the bolts of 4000 lbs. per square inch, what horse-power can be transmitted by the coupling at 120 revolutions per minute?

We must first find the torque transmitted—

$$\text{Area of each bolt} = \frac{\pi d^2}{4} = \frac{\pi}{4} \times \frac{9}{8} \times \frac{9}{8} = 1 \text{ square inch}$$

$$\text{Force on each bolt} = 4000 \times 1 = 4000 \text{ lbs.}$$

$$\text{Torque due to each bolt} = 4000 \times 9 = 36{,}000 \text{ pound-inches}$$

$$\text{Total torque on coupling} = 36{,}000 \times 8 \text{ pound-inches}$$

$$= \frac{36{,}000 \times 8}{12} = 24{,}000 \text{ pound-feet}$$

$$\text{Horse-power transmitted} = \frac{24{,}000 \times 2\pi \times 120}{33{,}000}$$

$$= 548{\cdot}5 \text{ H.P.}$$

Torsional Stiffness and Angle of Twist.—Torsional stiffness means resistance to twisting under the action of a torque or twisting moment. The torsional stiffness of a round rod is proportional to the fourth power of its diameter, and inversely proportional to the length which is subjected to torque. In other words, for a given torque the angle of twist produced is proportional to the length and inversely proportional to the fourth power of the diameter. This may be proved experimentally for wires or thin rods by the apparatus shown in Fig. 161. The diameter of the pulley to which the torque was applied was 10 inches. The following results were obtained from iron wires of different lengths and diameters—

Load in each scale pan including weight of scale pan (pounds).	Diameter of wire (inches).	Length of wire (inches).	Angle of twist in degrees.
1	$\frac{1}{4}$	36	2
1	$\frac{1}{4}$	72	4
1	$\frac{1}{4}$	108	6
5	$\frac{1}{4}$	36	10
5	$\frac{1}{4}$	72	20
5	$\frac{1}{4}$	108	30
1	$\frac{1}{8}$	36	32
5	$\frac{1}{8}$	36	160

It will be seen from the above results that for the $\frac{1}{4}$ inch wire, increasing the length from 36 to 72 inches, doubled the angle of twist, also that the angle of twist for the 108 inches length was, in each case, three times that for the 36 inches length; hence, the angle of twist is proportional to the length of the wire. Next consider the $\frac{1}{8}$ inch wire. For a load of 1 lb. and length 36 inches the angle of twist is 32°. For the same load and length of the $\frac{1}{4}$ inch wire the angle of twist is 2°, that is, $\frac{1}{16}$ of the angle in the case of the $\frac{1}{8}$ inch wire.

The ratio of the diameters is $\frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}$ and $(\frac{1}{2})^4 = \frac{1}{16}$, hence, we see that the angle of twist is inversely proportional to the fourth power of the diameter.

For short pieces of stout wire or rods the simple apparatus shown in Fig. 193 is very convenient. The rod to be tested is fixed in the clip A at one end, and is held rigid at the other end by means of a cap and two set screws. The clip A is part of a spindle mounted in ball bearings, on the outer end of which is keyed the pulley B. An adjustable pulley C is clipped to the fixed rod D. On rotating the pulley B, the clip A and, therefore, that end of the rod is twisted, the other end being held fast.

The graduated scale G can be moved along the rods E and F, on which it rests by two V notches and a straight edge, in order to read the angle between the two pointers P. In conducting an experiment the two pointers are clipped on to the rod at a convenient distance apart, usually one of them at the fixed end of the rod under test and their readings taken under the graduated scale. A pure torque is applied to the free end by the arrangement shown; a weight W is applied at the circumference of wheel B, and an equal and opposite force is applied at a point on the circumference diametrically opposite thereby giving a pure

couple or torque without any bending. The angle of twist due to a certain torque can therefore be read off for any convenient length of rod tested.

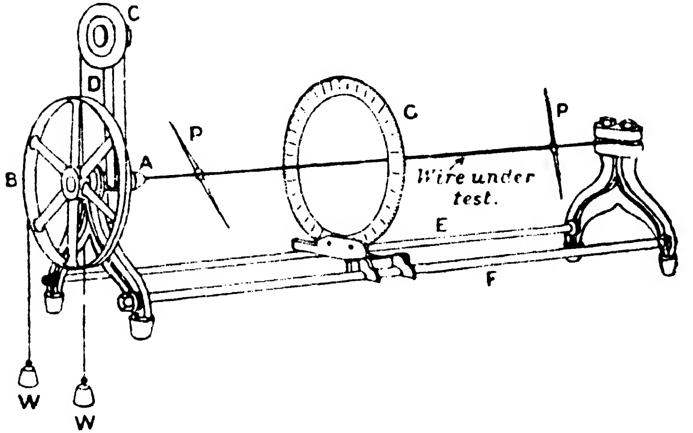


Fig. [illegible] — Torsion test of a wire or rod

Examples XVIII.

1. Find the twisting moment which will produce a stress of 9000 lbs. per square inch in a steel shaft 3 inches diameter.

2. If a shaft 3 inches diameter transmits 100 H.P. at 150 revolutions per minute, find the greatest intensity of shear stress.

3. What H.P. may be transmitted by a shaft 4 inches diameter running at 120 revolutions per minute if the maximum shear stress is 9000 lbs. per square inch?

4. Find the diameter of shaft required to transmit 50 H.P. at 200 revolutions per minute allowing a safe working shear stress of 9000 lbs. per square inch.

5. In a shaft coupling the radius of the bolt circle is 6 inches and there are 8 bolts each 1 inch diameter. Allowing a shear stress for the bolts of 4000 lbs. per square inch, what H.P. can be transmitted by the coupling at 180 revolutions per minute?

6. Two lengths of shaft each 2 inches diameter are connected by a flanged coupling whose 4 bolts have their centres on a circle concentric with the shaft centre and 8 inches diameter. Allowing a shear stress of 9000 lbs. per square inch in the shaft, what twisting moment can be transmitted? What must be the diameter of the bolts if the shear stress in them is limited to 5000 lbs. per square inch?

7. If a steel wire 6 feet long and $\frac{1}{4}$-inch diameter twists 18° when a twisting moment of 50 pound-inches is applied, what twisting moment must be applied to a steel rod $\frac{3}{8}$-inch diameter and 2 feet long in order to twist it 5°?

CHAPTER XIX

MATERIALS AND THEIR PROPERTIES

Working Stresses.—By actually pulling pieces of metal until they break, we find the ultimate strength or tenacity of the material or the number of tons per square inch of section, which a bar of the metal would stand when the load is steadily applied. But when metal is used in machines and structures the conditions differ greatly from a steadily applied pull. A force may come upon a part of a machine suddenly, and by so doing produce much greater effects. This may be illustrated by hanging, say, a 5 lb. weight on a spring balance very steadily: the balance records 5 lbs. pull; now let the 5 lbs. weight come on quite suddenly, and the balance will record about 10 lbs. pull instantaneously, while if the weight is dropped on to the pan or hook of the balance the instantaneous force recorded may be much greater. A load more or less suddenly applied to a part of a machine or structure, and then removed from time to time and applied again is called a *live* load. A force which remains always the same and is never removed is called a *dead* load. It is found from experience in making and using machines, and also from special laboratory experiments, that under repeated fluctuating or live loads materials break with only a fraction (such as $\frac{1}{3}$ or $\frac{1}{4}$) of the stress which it takes to break them by a steady pull; the cause of such failure is sometimes called the *fatigue* of a material. In designing a part of a machine or structure we try to make it of such a size that the maximum stress shall not exceed a certain amount which is called the *working stress.* This working stress differs for the same material according to various conditions, and is always far below the ultimate strength of the material because of the effect of fatigue, because we do not wish to exceed the elastic limit and get permanent strains, because we may be unable to estimate reliably all the forces involved, and because we want to have a margin for safety. If we allow a working stress of, say, 6 tons per square inch on a steel tie-bar, and the ultimate strength is 30 tons per square

inch, or 5 times as much as the working stress, we say we are using a *factor of safety* of *five*. Thus—

$$\text{working stress} = \frac{\text{ultimate strength}}{\text{factor of safety}}$$

$$\text{factor of safety} = \frac{\text{ultimate strength}}{\text{working stress}}$$

The choice of a suitable factor of safety is a matter of experience, and varies greatly with different materials and conditions.

Testing of Materials.—When quantities of material are supplied under contracts it is usual to specify, among other things, that samples must, when tested to fracture, have a certain ultimate strength, and a certain degree of ductility as shown by the ultimate extension when a piece is broken by tension. A complete tension

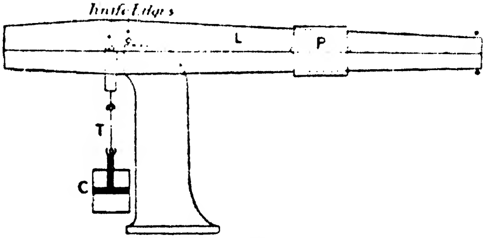


FIG. 194.—Diagram of testing machine (for tension).

test of a wire was given in Chap. XVI., and commercial tension tests of material for machines and structures differ from this mainly in being made upon larger pieces for which hand power provides insufficient straining force. Testing machines with hydraulic or screw power for straining, and one or more levers for measuring the pull are used, and differ considerably in kind: a diagrammatic representation of a simple form is shown in Fig. 194. The pull on the test piece T, is applied by a hydraulic cylinder C, and is weighed by the long lever L, on which a travelling weight P moves over a scale graduated in tons or other units. The test piece T, cut from a forging or casting, or it may be from a rolled bar is marked out with centre punch dots pitched one inch apart over a length usually of 8 inches, and held at its ends in suitable grips or sockets according to its form. The straining force is then gradually applied at its lower end, and the counterpoise P is moved along the lever to keep it balanced between the two stops. If

measurements of the elastic stretch are to be made on so short a length as 8 inches, some form of special multiplying instrument called an extensometer will be required. For the later measurements of stretch beyond the elastic limit a pair of dividers may be sufficient. The greatest load is carefully noted, and the extension of the marked length after fracture is noted. The following particulars refer to a sample of mild steel tested in such a machine as that shown in Fig. 194.

Section 2 inches by ½-inch, ultimate load 30 tons.

Original length between gauge points, 8 inches.

Final length after fracture, 10·125 inches.

$$\text{Total stretch} = 10{\cdot}125 - 8 = 2{\cdot}125 \text{ inches.}$$

$$\text{Per cent. elongation} = \frac{2{\cdot}125}{8} \times 100 = 26{\cdot}5 \text{ per cent.}$$

$$\text{Ultimate strength or tenacity} = \frac{30}{2 \times \frac{1}{2}} = 30 \text{ tons per square inch.}$$

Compression tests of metals in testing machines are also made, but are much less common than tensile tests, because the supporting material at the ends affects the results greatly in short lengths, while longer lengths buckle up. Stone, bricks, cement and concrete are frequently tested in compression.

Production and Properties of Iron and Steel.

Pig Iron.—This is produced by reducing iron ore at a high temperature with carbon contained in coal or coke in large furnaces called *blast furnaces*, limestone being used to form a fusible slag with the earthy matters mixed with the ore. Pig iron which is the crudest form used in the production of other irons is classed as grey or white according to its appearance when fractured.

The grey variety contains upwards of 2 per cent. of carbon separated from the iron as graphite, and is used for making castings. The white variety contains the carbon combined with the iron, and is used mainly for producing wrought iron and mild steel.

Cast Iron.—For making castings iron is melted in furnaces called cupolas, various grades of pig iron being used in proportions found suitable from experience. Cast iron is brittle, breaking in tension with no measurable elongation, and has an average tenacity of about 8 tons per square inch; it is liable to be porous and therefore not reliable without a high factor of safety. In compression it is very strong, breaking at about 40 to 50 tons per square inch. If a casting is rapidly cooled by being poured into a water-cooled metal mould a chilled casting of hard white iron is produced.

Malleable Castings are made by heating iron castings for long periods with red hematite (iron ore) in air-tight pots or boxes. The

hematite removes some of the carbon from the cast iron, and renders the casting stronger and much less brittle.

Wrought Iron is a purified form of iron made from cast iron by the removal of carbon and other impurities which would cause brittleness. Pig iron is first melted, and the foreign elements oxidised by an air blast or by contact with iron oxides which pass into a slag. The iron forms a pasty and not a liquid mass, which has to be hammered to weld the particles of metal together and to expel the slag. Subsequent rolling, cutting up and rolling together again improves the quality and expels more slag, but the slag finally left in the metal can always be traced, and causes a fibrous appearance of the metal when fractured.

Wrought iron has a tenacity of from 16 to 24 tons per square inch, is ductile, and can be easily welded, a property in which it is superior to steel and accounts for its continued use in various links, shackles, and connections.

Steel is a name applied to a variety of combinations of iron with carbon and other elements. *Mild Steels* containing much less than ½ per cent. of carbon have to a large extent replaced wrought iron and differ from it in being produced in a more liquid form. In the *Siemen's* Open Hearth process of making mild steel, pig iron is melted, and pure oxidised iron ores are then added to get rid of carbon and other elements. A sufficient amount of carbon is subsequently introduced by adding a special alloy of manganese and iron containing carbon. The resulting product may have as little or less carbon than wrought iron, while it has greater tenacity and ductility.

In the *Bessemer Process* the carbon and iron are burnt out of the molten pig iron in a special furnace called a Bessemer Converter, by a blast of air blowing through from underneath. The necessary amount of carbon is afterwards added by an alloy of iron and manganese as in the Siemen's process. The Bessemer process is much quicker than the open hearth process, but in consequence is not under such good control in regard to producing steel of a given composition.

Hard Steels.—Steel for cutlery, springs and tools is mostly made by the *Cementation* process. This consists in strongly heating very pure wrought iron bars with charcoal from which the iron takes up carbon. The charcoal and iron are packed in troughs in the furnace, and covered with the refuse from under grindstones consisting of iron rust and sand, a mixture which partially melts and excludes the air. The process takes several days, depending upon the quality of steel required, and the result is judged from the fracture of trial bars withdrawn as the heating proceeds. The resulting

product is called *blister steel*, and the bars are broken up and hammered till thoroughly welded. or they may be melted in crucibles to form a more uniform *cast steel*.

The tenacity and ductility of steels vary greatly with the amount of carbon they contain. Mild steels have a tenacity of from 26 to 36 tons per square inch with 20 to 30 per cent. elongation ; hard steels run up to over 70 tons per square inch, and break with very little elongation.

Hardening and Tempering.—Steels containing more than about $\frac{1}{2}$ per cent. of carbon are hardened by being heated to a cherry red colour, and then cooled rapidly by plunging into cold water or oil. The brittleness produced by this treatment is then reduced by *tempering*, which consists of reheating the steel to a certain temperature depending upon the degree of hardness or temper required.

Special Steels.—There are several other types of steel which differ from the above in being a combination of iron with elements other than carbon, such as tungsten, vanadium, nickel, etc. These steels are hardened by processes quite different from that used for ordinary steel and are used for special purposes.

Case Hardening.—Mild steel and wrought iron cannot be hardened by the above method. In cases where wrought iron or mild steel articles require to be hard on the surface only, the process called case hardening is employed. The operation is similar to the Cementation process, the articles being heated in contact with charcoal, leather or other articles containing carbon. The articles take up carbon forming a layer of steel at the surface whose depth varies with the time the process is allowed to continue (say, about $\frac{1}{8}$ inch deep). In cases where the hardness is only required to be *skin* deep the iron article is heated and then rubbed over with ferrocyanide of potassium, which forms a very thin layer of hard steel on the surface.

Timber.—The cross-section of a tree trunk consists of two parts, an inner and darker core of *heartwood* surrounded by *sapwood*. Trees which are felled in winter give more durable timber than those felled in other seasons, because the quantity of sap or juice present in the timber when felled is a minimum. The process of *seasoning* consists in drying the timber by natural or artificial means, and so removing the sap and moisture. The sapwood is not durable, is weak and unreliable, and should never be used for structural purposes ; well-seasoned heartwood should be used for all purposes.

Timbers may be classified broadly into *hard* and *soft* woods. Hard woods include oak, ash, beech, elm, mahogany, greenheart, teak, ebony ; soft woods include all pines and firs. The properties

of a timber decide whether it is suitable for the purpose intended; a detailed account of the different properties is beyond the scope of this book.

Very wet (fresh cut) timber has about half its maximum strength, and in the process of drying its strength begins to rise when the moisture present gets below 60 per cent. of the weight of *dry* timber, and rises steadily with decrease of moisture to the maximum strength when only about 4 per cent. of its own weight of moisture remains. For comparison of different woods it is therefore necessary to adopt a standard percentage of moisture; from 12 to 15 per cent. is usually chosen, this being the amount retained after good air drying. The following figures give some idea of the tenacity, and Young's modulus of different woods along the grain.

	Tenacity. Tons per square inch.	Young's modulus E. Tons per square inch.
Oak (British) . . .	4 to 8	650
Ash	2 to 7	700
Elm	2 to 6	500
Teak	2 to 7	1000
Yellow pine . . .	1 to 2	700
Red pine	2 to 6	700
Spruce	2 to 3	700

Stone.—The strength of stone subjected to crushing stress, as it usually is in buildings, varies greatly with the character of the stone, granite often having a strength of 1500 tons per square foot, while sandstone and the weaker varieties of limestone may only have about a quarter or a fifth of this crushing strength. Except in very tall structures a building stone is generally chosen rather from considerations of durability and appearance than for its strength.

Brick.—The strength of bricks varies greatly with the composition of the clay from which they are made, the method of manufacture and other causes. The average strength of a common brick is about 150 tons per square foot, and of blue Staffordshire bricks about 400 tons per square foot.

Lime.—Lime is usually made by roasting, or calcining, limestone or chalk in a lime-kiln. The limestone consists of calcium carbonate, with some alumina (clay) and silica, and the process of calcining drives off the moisture and carbonic acid, leaving *quicklime*. The quicklime, when sprinkled with water, breaks up into a powder called slaked lime, and heat is given out.

Mortar.—When lime is mixed to a paste with water and then left in the air it soon hardens or *sets*. The paste consists chiefly of calcium hydrate which, when left exposed to the air, absorbs the carbonic acid present, and forms calcium carbonate again. The lime used for mortar should be that formed from limestone containing some alumina which enables it to set without contact with air. Limes of this kind are called hydraulic limes, and their action is similar to that of cement. A mortar composed only of such lime is called a quick-setting mortar. In practice the lime is usually mixed with a certain proportion of sand and water, the result being a mortar which sets more slowly: the greater the proportion of sand the slower the mortar is in setting. Hence, by varying the proportion of sand, a quick or a slowly setting mortar is produced, but increasing the amount of sand used weakens the mortar considerably.

Cements.—*Roman cement* is made by calcining nodules found in clay. It is known as a natural cement, and when mixed with water sets quickly, but is not very strong; in engineering work it has been largely superseded by Portland cement.

Portland cement is largely used by the engineer and builder. It is manufactured by mixing about 3 parts of chalk and 1 of clay, the product being afterwards roasted or calcined and then ground to a fine powder. The fineness of grinding is a very important item in the manufacture of this cement. When mixed with water it combines chemically with a certain quantity and sets in a solid mass impervious to water. Sand is often mixed with the cement, and, as in the case of mortar, the strength diminishes as the proportion of sand is increased. The cement which is intimately mixed with the sand when both dry and wet forms a binding material which, when it sets, unites the grains of sand together; hence the finer the cement is ground the stronger the resulting product, because the fine particles of cement can fill up the very small spaces between the grains of sand. The strength of Portland cement gradually increases for several months after setting has taken place.

Concrete is largely used for foundations, walls, etc. It is made by mixing together in suitable proportions, sand, broken bricks, gravel, cement and water. The cement binds the other ingredients together, forming a solid block resembling stone.

CHAPTER XX

MOTION

THE *speed* of a body is the rate at which it moves through space. If it always moves over equal distances in the same length of time it is said to move at a uniform or constant speed. For example, if a train travels 30 miles in an hour its *average speed* is 30 miles per hour,

or $$\frac{30 \times 5280}{60} = 2640 \text{ feet per minute}$$

or $$\frac{2640}{60} = 44 \text{ feet per second.}$$

But unless it travels just 44 feet in every second, or takes exactly 1 second for each 44 feet its speed is not constant but variable. Thus, it might move 20 miles in the first 35 minutes and 10 miles in the remaining 25 minutes, in which case its speed would not be constant, but still its average speed over the 30 miles would be 44 feet per second. The average speed of a body is simply the distance moved divided by the time taken, and is expressed in miles per hour, feet per minute, feet per second, or any unit of distance per unit of time.

If at any instant the speed of a train moving at a varying rate is said to be 30 miles an hour or 44 feet per second, this means that if its speed ceased to change at that instant it would move 44 feet in the next second or 30 miles in the next hour, and so on. Actually it might move, say, 44·6 feet in the next second or 22·15 feet in the next half second—an average speed of $22 \cdot 15 \div \frac{1}{2}$ or 44·3 feet per second. But if the distance moved in a small fraction of a second could be accurately measured the speed would be at the average rate of 44 feet per second.

Velocity.—When the term velocity is used it generally signifies speed in a particular specified direction: speed may be used as meaning the rate of motion independent of direction. When motion is along a single straight line, speed and velocity will be the same.

If a motor car moves at a uniform speed of 20 miles per hour.

how many feet will it travel in 7 seconds, and how long will it take to travel 32 miles?

Distance travelled per hour $= 20 \times 5280$ feet

Distance travelled per second $= \dfrac{20 \times 5280}{60 \times 60} = \frac{88}{3}$ or $29\frac{1}{3}$ feet

Distance travelled in 7 seconds $= 29\frac{1}{3} \times 7 = 205\frac{1}{3}$ feet

Time to travel 32 miles $= \frac{32}{20} = 1{\cdot}6$ hour or 1 hour 36 minutes.

In general terms we may say if s = distance travelled in feet at constant velocity or speed v feet per second in a time t seconds—

$$v = \frac{s}{t};\ s = vt \text{ and } t = \frac{s}{v}$$

Speed-Time Diagram.—Since for constant speed the distance s moved is equal to the product of speed and time $v \times t$, it may be represented by a rectangle, the sides of which are v and t. Thus, if we represent the constant speed in Fig. 195 by OM vertically and the time by ON horizontally the rectangle OMQN represents the product of speed and time or the distance moved in that time. If the speed is plotted on a scale of x feet per second to 1 inch, and the time on a scale of y second to 1 inch, then 1 square inch represents $x \times y$ feet.

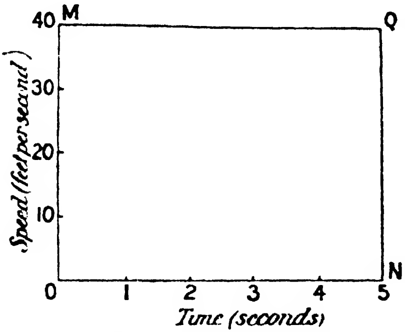


FIG. 195.—Graphical representation of speed, time, and distance.

In the case of a variable speed the area under a curve showing the velocity on a time base still represents the distance travelled. Thus, if Fig. 196 represents the varying speed of a moving body the area under the curve and above the base represents the distance moved in starting from rest (time O) and coming to rest again at time F.

A very simple case of a varying speed is that in which the speed varies uniformly, that is, increases or decreases by a constant amount in each second. Fig. 197 represents the varying speed of a body moving in this manner starting at time O with a velocity or speed u feet per second represented by OP, the speed increasing uniformly until after t seconds the speed is v feet per second

represented by NM. The area under the line PM represents the distance travelled, namely—

$$\frac{OP + NM}{2} \times ON$$

or

$$s = \frac{u + v}{2} \times t$$

Note that $\frac{u + v}{2}$ is the mean or average velocity.

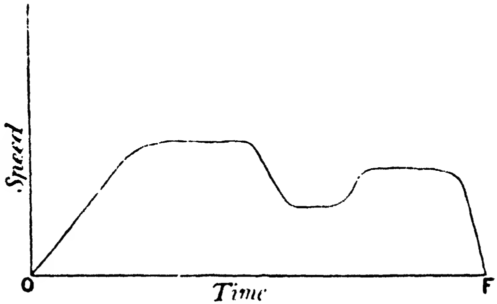


FIG. 196.

Example 1.—A body has its speed increased uniformly from 5 to 12 feet per second in one minute. How far will it travel in this time?

$$\text{Average speed} = \frac{5 + 12}{2} \text{ feet per second}$$

$$\therefore \text{ distance moved } s = \frac{5 + 12}{2} \times 60 = 510 \text{ feet.}$$

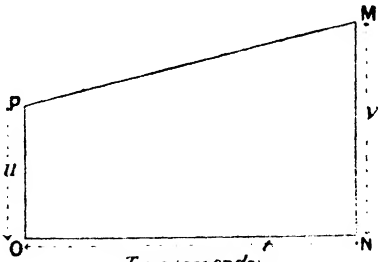


FIG. 197.—Uniformly varying speed.

Example 2.—A body starting from rest has its speed increased

uniformly until at the end of 2 minutes it is moving at the rate of 30 miles an hour. Find the distance travelled in this time.

$$30 \text{ miles an hour} = \frac{30 \times 5280}{60 \times 60} = 44 \text{ feet per second}$$

$$\text{Average speed} = \frac{0 + 44}{2} = 22 \text{ feet per second.}$$

NOTE.—When a body starts from rest (velocity $= o$) and its speed increases uniformly, the average speed is half the maximum.

Distance travelled $= 22 \times 2 \times 60 = 2640$ feet or $\frac{1}{2}$ mile.

Example 3.—The table shows the distance a body is from its starting position at certain times. Calculate the mean velocity in each 10 seconds, and from a speed-time curve find the speed after 50 seconds.

Time t (seconds) .	0	10	20	30	40	50	60
Distance s (feet) .	0	135	463	917	1392	1729	1980

Mean velocity in 1st 10 seconds $= \frac{135}{10}$
$= 13{\cdot}5$ feet per second (take this as the velocity after 5 seconds).

Mean velocity in 2nd 10 seconds $= \dfrac{463 - 135}{10}$
$= 32{\cdot}8$ feet per second at say 15 seconds

Mean velocity in 3rd 10 seconds $= \dfrac{917 - 463}{10}$
$= 45{\cdot}4$ feet per second at say 25 seconds

Mean velocity in 4th 10 seconds $= \dfrac{1392 - 917}{10}$
$= 47{\cdot}5$ feet per second at say 35 seconds

Mean velocity in 5th 10 seconds $= \dfrac{1729 - 1392}{10}$
$= 33{\cdot}7$ feet per second at say 45 seconds

Mean velocity in 6th 10 seconds $= \dfrac{1980 - 1729}{10}$
$= 25{\cdot}1$ feet per second at say 55 seconds.

Plotting these mean velocities and times we obtain the curve shown in Fig. 198, from which we see that at 50 seconds the velocity is about 29 feet per second.

Angular Velocity.—If a body moves in a circle its angular velocity which may be constant or variable is the rate of its angular

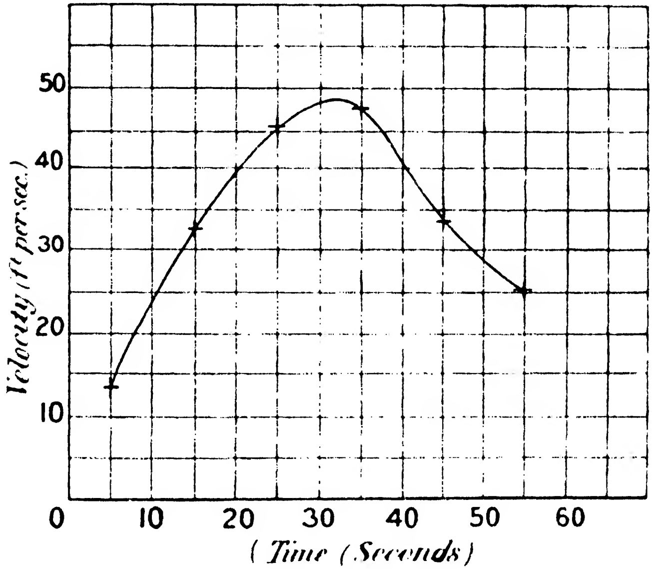


Fig. 198.

movement about the centre of the circle. Angular velocity is usually measured in radians per second, or in revolutions per minute or per second. If a point is moving in a circle of radius, say 5 feet, at a uniform speed of 15 feet per second, it will travel over an arc of the circle 5 feet long in $\frac{5}{15} = \frac{1}{3}$ second, and this arc (see Introduction, p. 4) is the arc defining 1 radian angle at the centre, so that the point moves through 1 radian angle in $\frac{1}{3}$ second and in 1 second it moves through $1 \div \frac{1}{3} = 3$ radians, or its angular velocity is 3 radians per second.

If the point moves in a circle of radius r feet, with a uniform speed of v feet per second along the circumference, and its angular velocity is ω radians per second, in one second the arc moved over is v feet, and the corresponding angle (see Introduction, p. 4) at the centre is $\frac{v}{r}$ radians or—

$$\omega = \frac{v}{r} \text{ or the speed } v = \omega r$$

And—

$$\omega = \frac{\text{angle traversed}}{\text{time taken}}\text{ ; angle traversed} = \omega \times \text{time taken, and}$$

$$\text{time} = \frac{\text{angle traversed}}{\omega}$$

relations which are exactly similar to those given above for linear speed.

If a shaft makes N revolutions per minute, what is its angular velocity?

The number of radians moved by any point distant r from the centre of the shaft is—

$$\frac{2\pi r}{r} = 2\pi \text{ per revolution of the shaft}$$

or

$$2\pi N \text{ per minute}$$

$$\frac{2\pi N}{60} = \frac{\pi N}{30} \text{ radians per second.}$$

Example.—A pulley 3 feet diameter is keyed to a shaft which makes 240 revolutions per minute. Find the linear and angular speed of the pulley rim.

240 revolutions per minute $= \frac{240}{60}$ or 4 revolutions per second

Circumference of pulley $= \pi \times 3 = 9{\cdot}42$ feet.

Linear speed of rim $= 9{\cdot}42 \times 4 = 37{\cdot}68$ feet per second

And—

$\left.\begin{array}{r}\text{Radians turned through}\\ \text{per revolution}\end{array}\right\} = 2\pi$

Angular speed of pulley $= 2\pi \times 4 = 8\pi = 25{\cdot}13$ radians per second

Acceleration.—When the speed of a body increases it is said to have an acceleration, and when the speed decreases it is said to have a retardation or negative acceleration. An increase or decrease of speed may take place at a uniform rate, in which case the acceleration is said to be constant or uniform. For example, if a body is moving at a speed of 8 feet per second, and after 2 seconds it is moving at 14 feet per second in the same direction, the total increase of speed during the two seconds is $14 - 8 = 6$ feet per second. This is at the average rate of 3 feet per second increase of speed per second, or 3 feet *per second per second.* If the acceleration is *uniform* its velocity will be—

after 1 second $8 + 3 = 11$ feet per second
after 2 seconds $11 + 3$ or $8 + (3 \times 2) = 14$ feet per second
after 3 seconds $14 + 3$ or $8 + (3 \times 3) = 17$ feet per second
after 4 seconds $17 + 3$ or $8 + (3 \times 4) = 20$ feet per second

and so on. And if the acceleration is f feet per second per second in the direction of motion, the increase of speed in t seconds will be $f \times t$ feet per second, so that a velocity u feet per second after t second becomes—

$$v = u + ft \text{ feet per second}$$

We have seen that when speed varies uniformly, the average speed during any period is half the sum of the initial and final velocity, or—

$$\text{average velocity} = \frac{u + v}{2} = \frac{u + (u + ft)}{2} = u + \tfrac{1}{2} ft$$

And distance travelled =

$$s = \text{average velocity} \times \text{time} = (u + \tfrac{1}{2} ft) \times t \text{ or } ut + \tfrac{1}{2} ft^2$$

Thus, in the above numerical case with initial velocity 8 feet per second, during 4 seconds the average speed is—

$$\frac{8 + 20}{2} = 14 \text{ feet per second}$$

hence the distance travelled is—

$$14 \times 4 = 56 \text{ feet.}$$

When a body starts from rest and attains a speed v feet per second under a constant rate of increase, the average speed is $\frac{v}{2}$ or $\frac{1}{2} ft$, and the distance travelled is—

$$\tfrac{1}{2} vt \quad \text{or } \tfrac{1}{2} ft^2$$

The commonest case is that of a freely falling body which increases its downward speed at a rate of about 32·2 feet per second every second; hence, after falling from rest for say 7 seconds, it attains a speed of—

$$7 \times 32{\cdot}2 = 225{\cdot}4 \text{ feet per second.}$$

Its average speed during the 7 seconds is—

$$\frac{225{\cdot}4}{2} = 112{\cdot}7 \text{ feet per second.}$$

And hence it falls—

$$112{\cdot}7 \times 7 = 788{\cdot}9 \text{ feet.}$$

In any time t seconds it gets a final speed of $32{\cdot}2\ t$, and hence an average speed of $\frac{32{\cdot}2}{2} t = 16{\cdot}1\ t$ feet per second and falls

$$16{\cdot}1\ t \times t \text{ or } 16{\cdot}1\ t^2 \text{ feet.}$$

The acceleration of a body falling freely under the action of gravitation is generally written g which is about 32·2 feet per

second per second, so that the final velocity after starting from rest is gt feet per second, and the depth of fall is—

$$\frac{1}{2}gt^2 \text{ feet.}$$

It is useful to know the relation between the distance travelled s and the maximum velocity v when a body starts from rest and has a uniform acceleration f in the direction of motion. We have seen that—

$$v = ft \quad \text{and time } t = \frac{s}{\frac{1}{2}v}$$

$$\text{hence } v = f \times \frac{s}{\frac{1}{2}v} = \frac{2fs}{v} \quad \text{or } v^2 = 2fs.$$

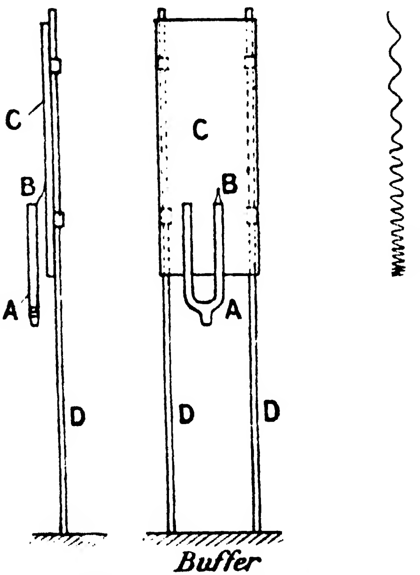


FIG. 199.—Experiment of uniform acceleration.

Experiment.—Experiments on bodies moving with uniform acceleration may be easily carried out on the simple apparatus shown diagrammatically in Fig. 199 in which C is a heavy smoked plate which slides freely down the two rods D. A tuning fork A, which vibrates at a constant known speed, has a tracing point attached to one leg. The plate is allowed to fall freely, the tracing point on the fork tracing out a wavy line as shown on the right of Fig. 199. It will be noticed that as the speed of the falling plate increases, the length of the waves described on the plate increases also. In a particular experiment the tuning fork drew 5 waves in each $\frac{1}{40}$ second, the length of successive 5 waves was measured, the results being tabulated on page 257.

It will be seen from the table on p. 257 that the increase in the length of successive 5 waves, and therefore the increase in the distance moved during the successive intervals of time taken to draw 5 waves (namely $\frac{1}{40}$ second), is practically constant and equal to 0·02 foot.

	Length of 5 waves. inches.	Length of 5 waves. feet.	Increase in length feet.	Average velocity feet per second
First 5 waves .	0·75	0·0625	—	2·50
Second 5 waves	1·00	0·0833	0·0208	3·332
Third 5 waves .	1·23	0·1025	0·0192	4·10
Fourth 5 waves.	1·46	0·1216	0·0191	4·864
Fifth 5 waves .	1·71	0·1425	0·0209	5·70
Sixth 5 waves .	1·95	0·1625	0·0200	6·50
Seventh 5 waves	2·19	0·1825	0·020	7·30
Eighth 5 waves.	2·44	0·2033	0·0208	8·132
Ninth 5 waves .	2·07	0·2225	0·0192	8·9
Tenth 5 waves .	2·91	0·2425	0·0200	9·7

The mean velocity for each 5 waves (column 4) is increasing at the practically uniform rate of 0·8 foot per second, *i.e.* $0{\cdot}02 \div \frac{1}{40}$; hence we see that the rate of increase of velocity is constant, or, in other words, the acceleration is constant.

To find this acceleration.—The time taken for each 5 waves is $\frac{1}{40}$ second, hence—

increase in velocity

$= 0{\cdot}8$ foot per second in $\frac{1}{40}$ second

$= \dfrac{0{\cdot}8}{\frac{1}{40}} = 32$ feet per second in 1 second

Or, the acceleration is 32 feet per second.

Similar experiments on gravitational acceleration are made by partially counterbalancing the falling load, to reduce the acceleration sufficiently so that the time of falling is longer, and can be measured by other means. Such an apparatus is called an Atwood's machine.

Another simple apparatus for finding the velocity of a falling body, and the acceleration due to gravity is shown diagrammatically in Fig. 200. A wooden lath swings like a pendulum on a knife edge at A. The time of swing may be varied by altering the position of the sliding weight B. An

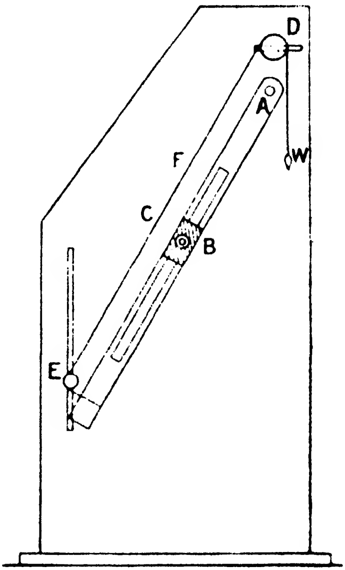


FIG. 200.

angular weight W is suspended by the thread C, passing over pulleys D and E, the other end of the thread being tied to the lath. The pulley D has a screw adjustment, and its position is adjusted so that when the lath hangs vertically, the corner of the weight W just touches it. To use the apparatus the weight W has its corner chalked, and a mark is made on the lath with it. A lighted match is then applied to the thread at F, the weight falls and is struck by the swinging lath exactly one quarter of its periodic time from starting, and makes a second mark upon it. The distance between the two chalk marks is the vertical distance the weight has fallen, and one quarter the complete swing period is the time taken for the weight to fall from which g is calculated. The period (T) is found by setting the lath swinging and counting the number (N) of complete (to and fro) oscillations it makes in, say, 2 minutes, then—

$$T = \frac{120}{N} \text{ seconds.}$$

Let s be the distance in feet between the two chalk marks on the lath, then—

$$s = \tfrac{1}{2}gt^2 \text{ and } t = \frac{T}{4}$$

$$\text{hence } s = \tfrac{1}{2}g\frac{T^2}{16} \text{ or } g = \frac{32s}{T^2}$$

the velocity v of the weight after falling s feet, *i.e.* when the lath strikes it, is found as follows—

$$s = \text{average velocity} \times t$$

$$= \frac{0 + v}{2} \times \frac{T}{4} = \frac{vT}{8}$$

$$\text{hence } v = \frac{8s}{T} \text{ feet per second.}$$

The experiment should be repeated by clamping the sliding weight B in different positions on the lath, and the average value of g found.

Example 1.—An electric train gets up a speed of 30 miles an hour in 20 seconds: find the average acceleration in feet per second per second. How far does it go before it gets up this speed if the acceleration is constant? What retardation is necessary to bring it to rest in 300 feet?

$$30 \text{ miles an hour} = \frac{30 \times 5280}{60 \times 60} = 44 \text{ feet per second.}$$

In 20 seconds the gain of velocity is 44 feet per second; in 1 second the gain of velocity is $\frac{44}{20} = 2{\cdot}2$ feet per second; hence, in each second

the velocity increases 2·2 feet per second, or the acceleration is 2·2 feet per second per second.

$$\text{Average speed} = \frac{0 + 44}{2} = 22 \text{ feet per second}$$

$$\begin{aligned}\text{hence, distance moved} &= \text{average speed} \times \text{time}\\ &= 22 \times 20 = 440 \text{ feet}\end{aligned}$$

When being brought to rest the average speed = 22 feet per second as before, and time taken to come to rest $= \dfrac{\text{distance moved}}{\text{average speed}}$

$$= \tfrac{300}{22} = 13{\cdot}63 \text{ seconds.}$$

That is, in 13·63 seconds a velocity of 44 feet per second is destroyed, hence, in 1 second a velocity of $\dfrac{44}{13{\cdot}63} = 3{\cdot}22$ feet per second is destroyed, or the retardation is 3·22 feet per second per second.

Example 2.—With what velocity must a jet of water be projected vertically upwards in order to reach a height of 80 feet?

Let v be the velocity—

$$\begin{aligned}\text{Then} \qquad v^2 &= 2gs\\ &= 2 \times 32{\cdot}2 \times 80\\ \therefore\ v &= \sqrt{64{\cdot}4 \times 80} = 72 \text{ feet per second.}\end{aligned}$$

Example 3.—A rifle bullet is fired vertically upwards with a velocity of 2000 feet per second. Neglecting air resistance, find the height to which it will rise, and the time taken to reach the ground again.

$$\begin{aligned}v^2 &= 2gs\\ s &= \frac{v^2}{2g} = \frac{2000 \times 2000}{2 \times 32{\cdot}2} = 62{,}110 \text{ feet.}\end{aligned}$$

The time taken for the bullet to rise will be equal to the time taken in falling $s = \frac{1}{2}gt^2$.

$$\begin{aligned}\therefore\ t^2 &= \frac{2s}{g} = \frac{2 \times 62{,}110}{32{\cdot}2}\\ t &= \sqrt{\frac{62{,}110}{16{\cdot}1}} = \sqrt{3857} = 62{\cdot}1 \text{ seconds}\end{aligned}$$

Hence, total time from firing to reaching the ground again—

$$= 62{\cdot}1 \times 2 = 124{\cdot}2 \text{ seconds.}$$

Changes of Velocity.—Changes of velocity may be changes in speed or changes in direction or both. So far we have only considered changes of speed or accelerations in the direction of motion. But a body moving in a straight line may have an acceleration in some other direction, with a result that the body changes its direction of motion, generally by following a curved path, its speed either remaining the same or changing according to the acceleration. Changes in direction of velocity are most conveniently dealt with by vectors. Velocities, like forces, have the

two characteristics of magnitude and direction, and as in Chap. I. we represented forces by straight lines of definite length and direction, so can we represent velocities.

Suppose a point in a body is at P (Fig. 201) moving with the velocity V_1 feet per second, and after a time t seconds it is at Q, and moving with velocity V_2 feet per second. If the vectors ac and bc represent completely the velocities V_2 and V_1 respectively, then since in vectors—

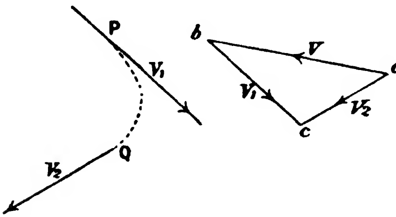


FIG. 201.—Changes of velocity.

$$ab + bc = ac$$

ab represents the velocity V feet per second added to V_1 (or bc) to give the resultant velocity V_2 (or ac), that is, ab represents in magnitude and direction the *change V in velocity* during motion from P to Q. Note that the change is not in the direction of motion and may be much larger numerically than the actual velocity V_1 at P or V_2 at Q, and that there need not be any change in speed for V_1 and V_2 may be equal. The average acceleration during the interval occupied in moving from P to Q is $\frac{V}{t}$ feet per second per second in the direction a to b.

Example 1.—A body is moving due north at 80 feet per second; after 0·3 second it is moving north-east at 60 feet per second: find the average acceleration during this time.

The vectors ac and bc (Fig. 202) represent completely the velocities 80 and 60 feet per second respectively to a scale of 20 feet per second to 1 inch, then ba represents the velocity added to ac in the 0·3 second. The length of ba is 2·84 inches which represents $2·84 \times 20 = 56·8$ feet per second. Hence the average acceleration is $\frac{56·8}{0·3} = 189·3$ feet per second per second in the direction b to a.

Example 2.—A body moves from a position A to a position B in $\frac{1}{10}$ second. These positions measured in feet from two perpendicular axes were found to be as follows—

	x	y
Position A	0·205	0·090
Position B	0·420	0·185

Find the x and the y components of the mean velocity of the body, and also the resultant mean velocity.

Fig. 203 shows the two positions of the body to scale. In moving from A to B the x component of the displacement is—

$$0{\cdot}420 - 0{\cdot}205 = 0{\cdot}215 \text{ foot,}$$

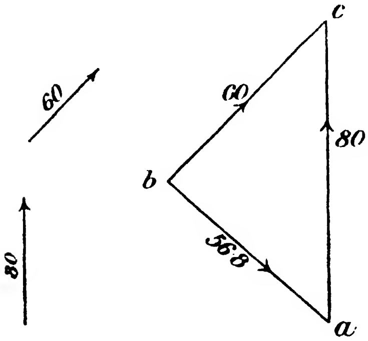


FIG. 202.

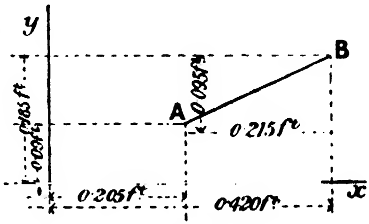


FIG. 203.

and since this takes place in $\frac{1}{10}$ second, the x component of the velocity is—

$$0{\cdot}215 \div \tfrac{1}{10} = 2{\cdot}15 \text{ feet per second from left to right on Fig. 203.}$$

The y component of the displacement is—

$$0{\cdot}185 - 0{\cdot}090 = 0{\cdot}095 \text{ foot,}$$

and the y component of the velocity is—

$$0{\cdot}095 \div \tfrac{1}{10} = 0{\cdot}95 \text{ foot per second upwards.}$$

The resultant mean velocity will be the resultant of the x and y component velocities at right angles, that is—

$$\sqrt{2{\cdot}15^2 + 0{\cdot}95^2} = \sqrt{4{\cdot}6225 + 0{\cdot}9025} = \sqrt{5{\cdot}525} = 2{\cdot}35 \text{ feet per second.}$$

The resultant or actual mean velocity of the body in moving from A to B might also be found without finding the x and y component velocities. The body moves a distance AB (Fig. 203) which scales 0·235 foot in $\frac{1}{10}$ second, hence the mean velocity is—

$$0{\cdot}235 \div \tfrac{1}{10} = 2{\cdot}35 \text{ feet per second.}$$

Example 3.—A body is moving in such a manner that at a particular instant its component velocity due east is 10 feet per second, and its component velocity due north is 12 feet per second. In $\frac{1}{5}$ second afterwards its component velocities are 15 feet per second due east, and 40 feet per second due north. Find the mean component accelerations during this time, and also the resultant acceleration.

Increase in velocity in direction due east $= 15 - 10 = 5$ feet per second. Hence, component acceleration due east $= 5 \div \frac{1}{5} = 25$ feet per second per second. Increase in velocity in direction due north $= 40 - 12 = 28$ feet per second. Hence, component acceleration due

north $= 28 \div \frac{1}{5} = 140$ feet per second per second. Now, these two component accelerations are at right angles, hence—

Resultant acceleration

$$= \sqrt{140^2 + 25^2}$$
$$= \sqrt{19600 + 625} = \sqrt{20225} = 142 \cdot 2 \text{ feet per second per second.}$$

Check.—The resultant acceleration may also be found graphically as follows: The actual initial velocity of the body is represented in magnitude and direction by the vector *ac* (Fig. 204) which scales 15·6 feet per

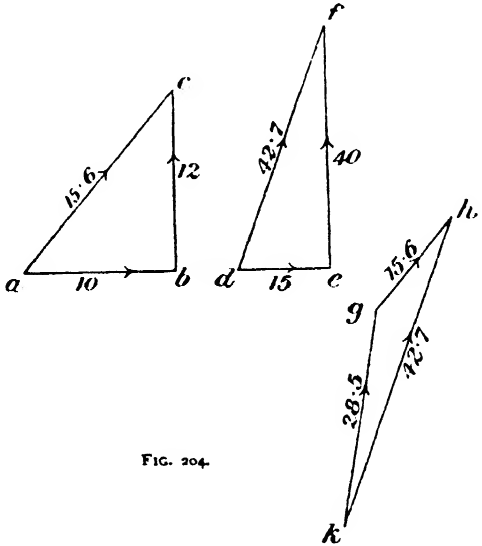


FIG. 204.

second. The actual final velocity is represented completely by *df* (Fig. 204) which scales 42·7 feet per second.

If now we draw the vectors *gh* and *kh* to represent completely the velocities of 15·6 feet per second, or *ac*, and 42·7 feet per second, or *df*, we have in vectors—

$$kg + gh = kh$$

kg represents the velocity in feet per second added to *gh* or *ac* to give the resultant velocity *kh* or 42·7 feet per second; *kg* scales 28·5 feet per second, hence, since this velocity is added in $\frac{1}{5}$ second, the average acceleration during this period is $28 \cdot 5 \div \frac{1}{5} = 142 \cdot 5$ feet per second per second, which agrees very closely with the result found above.

Relative Velocity.—The velocity of a point A relative to a point B is the rate of change of position (or displacement per unit of time) of A with respect to B.

Let v be the velocity of A and u that of B.

If A remained stationary, its velocity *relative to* B would be $-u$, since, if an observer were stationed on B and therefore moving with B, A would appear to be moving in the opposite direction with velocity u, hence the minus sign.

But as A has itself a velocity v, its total velocity relative to B is $v + (-u)$ or $v - u$, the subtraction to be performed by vectors.

Example.—To a person seated in a railway train the telegraph poles on the side of the track appear to be moving backwards with the same velocity as the train is going forwards.

If the direction of motion of A and B be in the same straight line then the subtraction can more readily be made arithmetically.

Example 1.—Suppose A is moving in the same direction as B and in the same straight line, the velocity of A being 10 feet per second and that of B 30 feet per second, then the velocity of A relative to B will be $10 - 30 = -20$ feet per second, that is, A appears to an observer on B to be moving away from him at 20 feet per second.

If now the bodies A and B are moving in opposite directions, *i.e.* towards or away from each other, the velocity of A relative to B will be $10 + 30 = 40$ feet per second.

Example 2.—Two railway lines intersect: on the first a train A is approaching the crossing and moving due east; on the second another train B is approaching the crossing coming from a direction 15° E. of N. Find the velocity of A relative to B.

First set out the two lines at the proper angles as in the left side of Fig. 205. Draw the vector *ab* parallel to the velocity of A, and *cb*

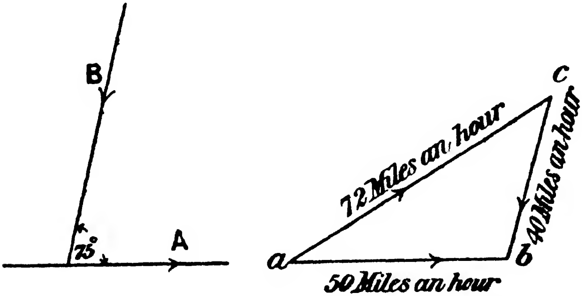


FIG. 205.

parallel to the velocity of B to completely represent the velocity of A, and the velocity of B *reversed* in direction respectively, then the length of *ac* represents the relative velocity of the two trains: it will be found to scale 72 miles an hour, and the velocity of A relative to B will be 72

miles an hour in the direction a to c. which is found to be in a direction going $57\frac{1}{2}^\circ$ E. of N. The velocity of B relative to A will be 72 miles an hour in the direction from c to a.

NOTE.—In Chap. I. we added vectors. To find the relative velocity between two moving bodies we have to *subtract* vectors. To avoid confusion in subtracting vectors, *reverse the direction of the* one to be subtracted *and then add the two together.*

EXAMPLES XX.

1. If a train moves at a uniform speed of 50 miles an hour, how many yards will it travel in half a minute, and how long will it take to travel 19 miles?

2. What is the average speed of a train which travels from Nottingham to King's Cross a distance of 123 miles in $2\frac{1}{2}$ hours?

3. A body starting from rest has its speed increased uniformly until at the end of $\frac{1}{2}$ minute it is moving at the rate of 15 miles an hour: find the distance travelled in this time.

4. A pulley is rotating at a speed of 250 revolutions per minute. What is its angular velocity in radians per second?

5. A locomotive is travelling at a speed of 60 miles an hour. The driving wheels are 7 feet diameter, and the stroke of the engine is 18 inches. Assuming that no slipping of the wheels on the rails takes place, find the angular velocity of the driving wheels, and the mean speed of the engine piston.

6. A gas engine of 19 inches piston stroke runs at 225 revolutions per minute. What is the angular velocity of the flywheel in radians per second, and what is the mean piston speed of the engines?

7. If the diameter of the flywheel in Question 6 is 6 feet 6 inches, find the linear speed of the wheel rim.

8. The velocity of a body changes from 10 feet per second to 50 feet per second in the same direction in half a minute. Find the acceleration.

9. Express an acceleration of 30 miles per hour per minute in feet per second per second

10. A train starts from rest with a uniform acceleration, and in $1\frac{1}{2}$ minutes attains a speed of 60 miles an hour. Find (a) the acceleration, (b) the distance travelled in the first minute, (c) the distance travelled in the $1\frac{1}{2}$ minutes, (d) the time taken to reach a speed of 50 miles an hour.

11. A body is projected vertically upwards with a velocity of 250 feet per second; to what height will it travel?

12. A stone is dropped from the top of a tower 200 feet high: how long will it take to reach the ground?

13. A body is moving due west at 10 feet per second and 6 seconds later is moving due east at 8 feet per second. What has been its acceleration?

14. A wheel making 180 revolutions per minute has its speed increased to 210 revolutions per minute in 10 seconds. Find, (a) the acceleration in revolutions per minute per second, (b) in radians per second per second.

15. A body is moving due east at 40 feet per second; after 0·6 second it is moving north-west at 80 feet per second. Find the average acceleration during this time.

16. The centre of gravity G of the crank balance weight of a gas engine

moves from position G_1 to G_2 in $\frac{1}{10}$ second. These positions measured in feet from two perpendicular axes were found to be as follows :—

	x	y
G_1	0·185	0·208
G_2	0·516	0·405

Find the x and the y components of the mean velocity of the centre of gravity during the movement and also the resultant mean velocity.

17. A body A is moving north at 15 feet per second, and a body B crosses its path south of A moving east at 20 feet per second. What is the velocity of A relative to B?

18 Two ships leave a port at the same time, the first steams north-west at 15 knots, and the second 30° south of west at 17 knots. What is the speed of the second relative to the first?

CHAPTER XXI

MOMENTUM, INERTIA, AND FORCE

Mass and Inertia.—The mass of a body is the quantity of matter in it. We might measure the mass of a body by its weight in pounds, for its weight is simply the gravitational force with which the earth attracts it; masses are actually measured and compared by weighing. At the beginning of Chap. I. we took unit force as the weight of 1 lb. of matter in London, and in engineers' units we take the unit mass as—

$$32\cdot2 \text{ lbs. or } g \text{ lbs.}$$

32·2 being the uniform acceleration of a body falling freely in London. The weight of this quantity of matter varies a trifle at other parts of the earth's surface, and likewise the acceleration due to gravity.

Inertia is a property which all bodies possess, of inertness or sluggishness in getting into motion from rest when acted upon by a force, and of ceasing or decreasing motion when opposed by a force. The measure of the inertia of a body is its mass. Thus, if two bodies at rest weighing 10 and 5 lbs. are each acted upon by a force of, say, 3 lbs., the heavier body will take twice as long as the lighter one to get up a speed of, say, 6 feet per second in the direction of the applied force; its acceleration will be only half as great or, its inertia is twice as great.

Momentum.—Momentum is sometimes called the quantity of motion of a body. It is proportional to the mass of the body and to its velocity. The engineers' unit of momentum is that of unit mass (32·2 lbs.) moving at unit velocity (1 foot per second) so that for any moving body—

$$\text{Momentum} = \frac{\text{weight in pounds}}{32\cdot2} \times \text{velocity in feet per second.}$$

Or, for a body of weight W lbs. moving at v feet per second—

$$\text{momentum} = \frac{W}{32\cdot2} \times v \text{ or } \frac{Wv}{g}$$

As velocity has definite direction as well as magnitude so has momentum, and it can be represented by a vector of length proportional to the magnitude of the momentum in the direction of motion.

Example 1.—A truck weighing 4 cwts. is moving at 20 miles per hour. What is its momentum in engineers' units?

$$\text{Mass} = \frac{4 \times 112}{32\cdot2}, \text{ velocity} = \frac{20 \times 5280}{60 \times 60} = \frac{88}{3} \text{ feet per second}$$

$$\text{Momentum} = \frac{4 \times 112 \times 88}{32\cdot2 \times 3} = 408 \text{ units.}$$

Example 2.—Water issues from a round nozzle 1 inch diameter at a speed of 30 feet per second. What weight of water leaves per second and what is its momentum? Water weighs 62·4 lbs. per cubic foot.

$$\text{Area of nozzle orifice} = \frac{\pi}{4} \text{ square inch}$$

$$\text{volume of water issuing per second} = \frac{\pi}{4} \times 30 \times 12 \text{ cubic inches}$$

$$= \frac{\pi \times 30 \times 12}{4 \times 1728} \text{ cubic feet}$$

$$= \frac{90\pi}{1728} \text{ cubic feet}$$

$$\text{weight of water issuing per second} = \frac{90\pi}{1728} \times 62\cdot4 = 10\cdot2 \text{ lbs.}$$

$$\text{momentum of this water} = \frac{10\cdot2}{32\cdot2} \times 30 = 9\,5 \text{ units.}$$

Change of Momentum.—If a body changes its velocity due to the action of a force upon it, it changes its momentum by a corresponding amount. Thus, if a body is moving at 20 feet per second, and it weighs 10 lbs. its momentum is $\frac{10}{32\cdot2} \times 20$ units. If its speed increases to, say, 35 feet per second in the same direction its momentum is then $\frac{10}{32\cdot2} \times 35$ units, and the increase in momentum is—

$$\frac{10}{32\cdot2} \times 35 - \frac{10}{32\cdot2} \times 20 \text{ or } \frac{10}{32\cdot2} \times (35 - 20) = \frac{10}{32\cdot2} \times 15 \text{ units}$$

or, its increase of momentum is—

mass $\times$ increase of velocity.

If this increase of velocity takes place in 3 seconds the average rate of increase of momentum is—

$$\frac{10}{32\cdot2} \times 15 \div 3 = \frac{50}{32\cdot2} \text{ or } 1\cdot55 \text{ units per second}$$

Momentum having direction as well as magnitude, changes in momentum when the motion is not all in the same straight line must be found by vectors as were changes of velocity.

Example.—A body weighing 96·6 lbs. is moving at 20 feet per second due east. After 2 seconds it is moving south-east at 24 feet per second and after 2 more seconds it is moving due south at 21 feet per second. Find the changes of momentum during each 2 seconds and the total change during the 4 seconds. Also find the average change of momentum during each interval.

First find the change of velocity during each interval by the method of Chap. XX. Thus, *ab* (Fig. 206) represents 20 feet per second due

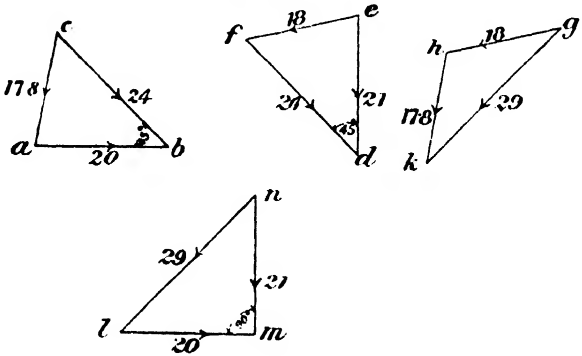


FIG. 206.

east, *cb* 24 feet per second south-east, then *ca* which scales 17·8 feet per second represents completely the change of velocity during the first two seconds ; hence, for the first 2 seconds

$$\text{change of momentum} = \frac{96·6}{32·2} \times 17·8 = 53·4 \text{ units, and}$$

$$\text{average change per second} = \frac{53·4}{2} = 26·7 \text{ units.}$$

Similarly, by drawing *fd* to represent 24 feet per second south-east, and *ed* 21 feet per second due south we have during the second two seconds the change of velocity *ef* which scales 18 feet per second ; hence, for the second two seconds

$$\text{change of momentum} = \frac{96·6}{32·2} \times 18 = 54 \text{ units, and}$$

$$\text{average change per second} = \frac{54}{2} = 27 \text{ units.}$$

To find the total change of velocity during the 4 seconds we must now add the two changes vectorally thus, *gh* represents the change of 18 feet per second (parallel to *ef*), and *hk* the change of 17·8 feet per

second (parallel to *ca*), hence ***gk*** which scales 29 feet per second represents completely the total change in velocity, and

$$\text{total change of momentum} = \frac{96{\cdot}6}{32{\cdot}2} \times 29 = 87 \text{ units.}$$

The total change of momentum can also be found independently of intermediate conditions as follows: draw *lm* (Fig. 206) to represent the initial velocity of 20 feet per second due east, and ***nm*** to represent the final velocity of 21 feet per second due south; then the difference ***nl*** which scales 29 feet per second represents the total change of velocity and

$$\text{total change of momentum} = \frac{96{\cdot}6}{32{\cdot}2} \times 29 = 87 \text{ units as before.}$$

The Law of Force and Motion.—The rate of change of momentum of a body in motion is proportional to the force applied, and is in the direction in which the force acts. This is known as Newton's second law of motion. The engineers' unit of mass is so chosen that the rate of change of momentum (per second) is numerically *equal* to the applied force. If we do not know the instantaneous rate of change of momentum, but only the average rate over a given time, we can only calculate the *average* force during that time; we call such an average force the *time average* of force.

$$\text{Average force} = \text{average rate of change of momentum}$$
$$= \frac{\text{total change of momentum}}{\text{time taken to change}}$$

Hence, also—

$$\text{Average force} \times \text{time of action} = \text{total change of momentum.}$$

Example 1.—A truck weighing 4 cwts. is moving at 6 miles per hour, and after 10 seconds is moving at 21 miles per hour. What is the average force in the direction of motion, acting upon it during that time?

$$\text{change of velocity} = 21 - 6 = 15 \text{ miles per hour} = 22 \text{ feet per second,}$$

$$\text{change of momentum} = \frac{4 \times 112}{32{\cdot}2} \times 22 = 306 \text{ units,}$$

$$\text{average force} = \text{average change of momentum per second}$$
$$= \frac{306}{10} = 30{\cdot}6 \text{ lbs.}$$

Example 2.—A jet of water issuing horizontally from a nozzle 1½ inches diameter strikes a vertical wall and is thereby diverted at right

angles. none splashing back. If the speed of the jet is 25 feet per second, find the force exerted on the wall by the jet.

$$\text{volume of water striking wall per second} = \frac{\pi}{4} \times \frac{(1 \cdot 5)^2}{144} \times 25 = 0 \cdot 306 \text{ cubic foot,}$$

$$\text{weight of water striking wall per second} = 0 \cdot 306 \times 62 \cdot 4 \text{ lbs.}$$

$$\text{force on wall} = \text{change of momentum per second}$$

$$= \frac{0 \cdot 306 \times 62 \cdot 4}{32 \cdot 2} \times 25 = 14 \cdot 8 \text{ lbs.}$$

Example 3.—A car starting from rest is drawn by a varying force F pounds, which, after t seconds, is as shown in the following table—

t (seconds) .	0	2	5	8	11	13	16	19	20
F (pounds)	1280	1270	1220	1110	905	800	720	670	660

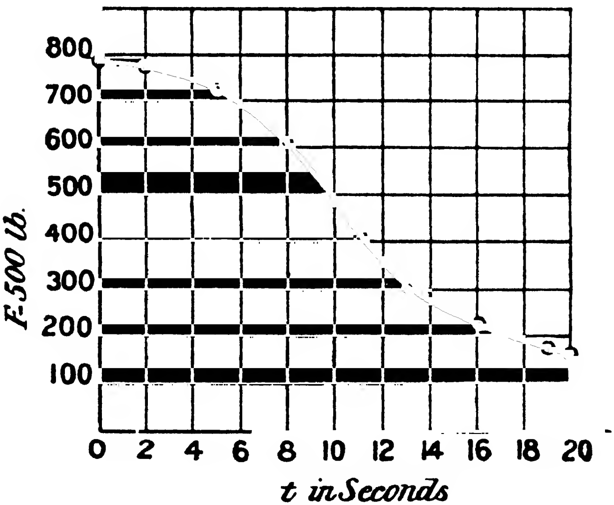


FIG. 207.

If the frictional resistance is constant and equal to 500 lbs., what is the time average of the force acting during the 20 seconds? What is the total gain of momentum? If the car weighs 12 tons find its velocity at the end of the 20 seconds.

The force to produce motion is F − 500 : plot the curve F − 500 and t as shown in Fig. 207. In that figure. shown here half size, the

force scale was 200 lbs. to the inch, and time scale 4 seconds to the inch. Now find the area of this curve by counting the number of square inches, it will be found to be 11·93 square inches. Dividing the area by the base we get the average height—

$$\frac{11·93}{5} = 2·382 \text{ inches,}$$

and since 1 inch represents 200 lbs., the average value of F − 500 is 2·382 × 200 = 476·4 lbs. Hence the time average of the force (F) acting during the 20 seconds is—

$$500 + 476·4 = 976·4 \text{ lbs.}$$

The area of the curve also represents to scale the momentum produced. Since 1 inch vertically represents 200 lbs., and 1 inch horizontally represents 4 seconds, 1 square inch represents 200 × 4 = 800 units of momentum.

$$\begin{aligned} \text{Hence momentum of car at end of 20 seconds} &= 800 \times 11·93 \\ &= 9544 \text{ units} \end{aligned}$$

$$\text{and mass of car} = \frac{12 \times 2240}{32·2} \text{units}$$

$$\begin{aligned} \text{hence velocity} &= \frac{\text{momentum}}{\text{mass}} = \frac{9544}{12 \times 2240} \times 32·2 \\ &= 11·4 \text{ feet per second.} \end{aligned}$$

The reader should plot Fig. 207 full size on squared paper.

Force and Acceleration.—We have seen that force is numerically equal to the rate of change of momentum, but this may be put another way which is useful for calculations on moving bodies.

Rate of change of momentum = rate of change of (mass × velocity).

But if the mass of a body remains constant this becomes—

Rate of change of momentum = mass × rate of change of velocity
or = mass × acceleration

So that—

$$\text{Force} = \frac{W}{g} \times f, \text{ and } f = \frac{\text{force}}{W} \times g$$

where f is the acceleration.

For instance, taking the first of the examples on p. 269. In the 10 seconds the change of velocity was 22 feet per second. Hence—

$$\text{Average acceleration} = \frac{22}{10} = 2·2 \text{ feet per second per second.}$$

Hence—

$$\text{Average force acting on truck} = \frac{4 \times 112}{32·2} \times 2·2 = 30·6 \text{ lbs. as before.}$$

Acceleration, which is change of velocity per second, may, like change of velocity, be found by vectors when the direction of motion is changing.

Example 1.—A truck weighs $\frac{1}{2}$ ton, and is pulled on a level line by a force of 40 lbs in excess of the frictional resistance. Find the acceleration of the truck. How far will it move in 20 seconds? What horse-power is then being exerted?

$$\text{Mass of truck} = \frac{112 \times 10}{32\cdot2} = 34\cdot8 \text{ units.}$$

$$\text{Acceleration} = \frac{40}{34\cdot8} = 1\cdot15 \text{ feet per second per second.}$$

Velocity after 20 seconds $= 1\cdot15 \times 20 = 23$ feet per second.
Average velocity during the 20 seconds $= \frac{23}{2}$ feet per second.
Distance moved $= \frac{23}{2} \times 20 = 230$ feet.
Work being done per second $= 23$ feet per second $\times$ 40 lbs.
$= 920$ foot-pounds per second.
Horse-power $= \frac{920}{550} = 1\cdot67$ H.P.

Example 2.—A train weighing 150 tons has a frictional resistance of 16 lbs. per ton. What average pull will be required to give it a speed of 30 miles per hour from rest in $1\frac{1}{2}$ minutes on the level, and what horse-power would be required at the end of this time?

30 miles an hour = 44 feet per second.

$$\text{Average acceleration} = \frac{44}{60 \times 1\cdot5} = 0\cdot489 \text{ foot per second per second.}$$

$$\left.\begin{array}{l}\text{Average accelerating}\\ \quad\text{force}\end{array}\right\} = \frac{150 \times 2240}{32\cdot2} \times 0\cdot489 = 5101 \text{ lbs.}$$

Force to overcome frictional resistance $= 150 \times 16 = 2400$ lbs.
Total pull $= 5101 + 2400 = 7501$ lbs.

$$\text{Horse-power} = \frac{7501 \times 44}{550} = 600 \text{ H.P.}$$

Example 3.—Part of a machine weighs 644 lbs. and is moving at a speed of 40 feet per second. After 0·25 second it is moving at 30 feet per second in the same plane, but in a direction inclined 60° to its former path. Find the mean acceleration during this time, and the average force acting upon it to produce this change in velocity.

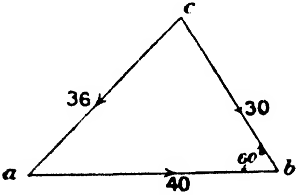


FIG. 208.

Let *ab* (Fig. 208) represent completely the velocity of 40 feet per second, and *cb* the velocity of 30 feet per second, then the change of velocity is completely represented by *ca* which scales 36 feet per second. This change takes place in 0·25 second, hence—

$$\text{mean acceleration} = \frac{36}{0\cdot25} = 144 \text{ feet per second per second.}$$

$$\begin{aligned}\text{Average force} &= \text{change of momentum per second}\\ &= \frac{W}{g} \times \text{acceleration}\\ &= \frac{644}{32\cdot2} \times 144 = 2880 \text{ lbs.}\end{aligned}$$

Acceleration of Gravity on Inclines.—We have seen that velocity, change of velocity, and change of velocity per second which is acceleration, are vector quantities, and therefore, like forces, can be resolved into components as in Chap. XIII. Thus for bodies acted upon by the force of gravity on a smooth slope we may resolve the vertical acceleration of about 32·2 feet per second per second into two components, one down the slope and the other perpendicular to it and resisted by the material of the slope. We have seen in Chap. XI. that the force up the plane required to just balance the pull of gravity on a body of weight W down a smooth plane inclined a to the horizontal is—

$$W \sin a \text{ or } \frac{1}{n} W$$

where the slope is 1 vertically to n along the slope. Hence the unresisted pull of gravity down the slope would give an acceleration of—

$$\frac{\text{Force}}{\text{Mass}} = \frac{W \sin a}{\frac{W}{g}} = g \sin a$$

or— $$\frac{W}{n} \div \frac{W}{g} = \frac{g}{n} \quad \text{or} \quad \frac{32 \cdot 2}{n} \text{ feet per second per second}$$

which is the acceleration required.

Example 1.—If the frictional resistance amounts to 15 lbs. per ton, how far will a train, starting from rest, travel down an incline of 1 in 70 in 20 seconds? If started up this incline at 30 miles an hour how far would it go before coming to rest?

Effective force per ton down plane $\Big\} = \frac{1}{70} \times 2240 - 15 = 32 - 15 = 17$ lbs.

Acceleration down plane $= \dfrac{\text{force}}{\text{mass}} = 17 \div \dfrac{2240}{32 \cdot 2}$

$= 0 \cdot 2446$ feet per second per second.

Average speed in 20 seconds $= \frac{1}{2}$ maximum speed.

$= \frac{1}{2} \times 20 \times 0 \cdot 2446 = 2 \cdot 446$ feet per second.

distance moved down incline in 20 seconds $= 20 \times 2 \cdot 446 = 48 \cdot 9$ feet.

Total retarding force per ton uphill $= \frac{1}{70} \times 2240 + 15 = 32 + 15 =$ 47 lbs.

Retardation $= 47 \div \dfrac{2240}{32 \cdot 2} = 0 \cdot 676$ feet per second per second.

Time to overcome speed of 30 miles an hour or 44 feet per second

$$= \frac{44}{0 \cdot 676} = 65 \cdot 2 \text{ seconds.}$$

Distance travelled up incline at average speed of 22 feet per second

$= 65 \cdot 2 \times 22 = 1434$ feet.

Example 2.—Find the H.P. required for a car weighing 10 tons if it has to attain a speed of 30 miles an hour in 30 seconds from rest up an incline of 1 in 100, the frictional resistance being 20 lbs. per ton.

Force to overcome friction $= 20 \times 10 = 200$ lbs.

Force to overcome gravity up to the plane $= \frac{1}{100} \times 10 \times 2240$
$= 224$ lbs.

The acceleration is 30 miles an hour or 44 feet per second in 30 seconds
$= \frac{44}{30}$ feet per second per second, hence

$$\text{accelerating force} = \text{mass} \times \text{acceleration} = \frac{10 \times 2240}{32 \cdot 2} \times \frac{44}{30} = 1020 \text{ lbs.}$$

Hence total force required $= 200 + 224 + 1020 = 1444$ lbs.

$$\text{and horse-power} = \frac{1444 \times 44}{550} = 115 \cdot 5 \text{ H P.}$$

Action and Reaction in Motion.—We have seen in the early Chapters that for two bodies at rest the pull or push of each on the other is equal. Although perhaps not quite so easy to realize, the same is true of bodies in motion. This is called Newton's third law of motion and is often stated in the words "to every action there is an equal and opposite reaction." Thus, the forward pull of a horse on a cart is not greater than but just equal to the backward pull of the cart upon the horse. Take, for example, the pull of a locomotive upon a moving train upon a level line. If this pull is just equal to the various frictional resistances, the train moves on without alteration of speed. If the pull exceeds the frictional resistances acceleration of the train takes place; in this case the equal backward pull of the train consists of two parts, the frictional resistance and the inertia forces or resistance to taking up speed. If the pull is less than the frictional resistances of the train, retardation takes place and the frictional resistances of the train are being overcome partly by the pull of the locomotive and partly by the (forward) inertia forces of the train which resist a change of speed.

Example.—If a passenger lift has an upward acceleration of 2 feet per second per second, what force will a man weighing 140 lbs. exert on the floor of the lift? Find the force if the lift is descending with an acceleration of 2 feet per second per second.

Here the downward pressure of the man on the floor is exactly equal to the upward pressure of the floor on the man, so we can calculate the latter quantity. The upward pressure exceeds the man's weight by an amount sufficient to give him an acceleration of 2 feet per second per second. Hence the total upward pressure is—

$$140 + \text{mass} \times \text{acceleration} = 140 + \frac{140}{32 \cdot 2} \times 2 = 140 + 8 \cdot 7 = 148 \cdot 7 \text{ lbs.}$$

In descending, the upward pressure will be less than the man's

weight by an amount taken to give the downward acceleration of 2 feet per second per second, namely—

$$140 - \frac{140}{32 \cdot 2} \times 2 = 140 - 8 \cdot 7 = 131 \cdot 3 \text{ lbs.}$$

When the lift is at rest, or moving with constant speed, the pressure will of course be 140 lbs.

Impulsive Forces: **Blows**.—The forces acting in blows and collisions although they act for short periods of time cause considerable impulses or changes of momentum, and are called impulsive forces. The time average of a force has already been explained as the total change of momentum divided by the time in which the change takes place. The (time) average force of a blow is exactly the same except that in proportion to the mass of the body acted upon the change of momentum is considerable and the time short; hence the force is large.

Example 1.—The head of a hand hammer weighs 4 lbs., and when moving at 25 feet per second is brought to rest in $\frac{1}{400}$ of a second. Find the average force of the blow.

$$\text{Change of momentum} = \frac{4}{32 \cdot 2} \times 25 = \frac{100}{32 \cdot 2} = 3 \cdot 10 \text{ units.}$$

$$\begin{aligned} \text{Average force} &= \text{change of momentum per second.} \\ &= 3 \cdot 10 \div \tfrac{1}{400} = 3 \cdot 1 \times 400 = 1240 \text{ lbs.} \end{aligned}$$

Example 2.—The diameter of the piston of a steam hammer is 36 inches, total weight of hammer and piston 20 tons, effective steam pressure 40 lbs. per square inch. Find the acceleration with which the hammer descends, and its velocity after falling 4 feet. If the hammer then comes in contact with the iron and is brought to rest in $\frac{1}{200}$ second, find the average force of the blow.

The force producing acceleration of the hammer is equal to the sum of its weight, and the total steam pressure on the piston, namely—

$$\begin{aligned} &20 + \frac{0 \cdot 7854 \times 36 \times 36 \times 40}{2240} \\ &\qquad = 20 + 18 \cdot 2 = 38 \cdot 2 \text{ tons.} \end{aligned}$$

$$\text{Acceleration} = \frac{\text{force}}{\text{mass}} = \frac{38 \cdot 2 \times 2240}{20 \times 2240} \times 32 \cdot 2 = 61 \cdot 5 \text{ feet per second per second.}$$

$$\begin{aligned} \text{Now } v^2 &= 2fs \text{ (Chap. XX.)} \\ &= 2 \times 61 \cdot 5 \times 4 = 492 \\ v &= \sqrt{492} = 22 \cdot 2 \text{ feet per second.} \end{aligned}$$

$$\text{Change of momentum} = \frac{20 \times 2240}{32 \cdot 2} \times 22 \cdot 2 \text{ units}$$

$$\begin{aligned} \text{Average force} &= \text{change of momentum per second.} \\ &= \frac{20 \times 2240 \times 22 \cdot 2}{32 \cdot 2} \div \tfrac{1}{200} \text{ lbs.} \\ &= \frac{20 \times 22 \cdot 2}{32 \cdot 2} \times 200 = 2760 \text{ tons.} \end{aligned}$$

Momentum after Collision.—It follows from the law of equal action and reaction, that if two bodies collide, the rates of change of momentum being equal and opposite, the total gain of momentum by one body is just equal to the loss of momentum by the other, and therefore looking upon the two bodies as one system there is no gain or loss of momentum in the collision, that is, the total momentum of the two is the same after as it was before the impact. This is called the conservation of momentum. Thus, if a heavy hammer moving quickly strikes a nail driving it into a large immovable body the hammer and nail move forward together with such a velocity that the total momentum of the two after the blow is equal to that of the hammer alone before the blow. If the hammer weighs say 3·22 lbs. and is moving at 25 feet per second, and the nail weighs 0·0322 lb.—

$$\text{Momentum before impact} = \frac{3 \cdot 22}{32 \cdot 2} \times 25 = 2 \cdot 5 \text{ units}$$

$$\text{Momentum after impact} = 2 \cdot 5 \text{ units}$$

$$\text{Velocity of hammer and nail} = \frac{2 \cdot 5}{\text{total mass}} = \frac{2 \cdot 5}{\frac{3 \cdot 32}{32 \cdot 2} + \frac{0 \cdot 0322}{32 \cdot 2}}$$

$$= \frac{2 \cdot 5}{0 \cdot 1 + 0 \cdot 001} = \frac{2 \cdot 5}{0 \cdot 101} = 24 \cdot 75 \text{ feet per second,}$$

which is the velocity with which the nail starts to enter.

Experiment on Momentum.—An experiment to find the velocity of a rifle bullet and involving the law of momentum may be performed by firing a rifle bullet into a heavy block of wood which is free to move and in which the bullet remains imbedded. It will be necessary to know the initial velocity of the block of wood after impact, which could easily be found by having the block suspended and finding how far it rises. Suppose the block of wood weighs 25 lbs. and the bullet 1 ounce, and the block after being struck starts moving at 6 feet per second, what was the velocity of the bullet?

$$\text{Momentum after impact} = \frac{25\frac{1}{16}}{32 \cdot 2} \times 6 = \frac{401 \times 6}{16 \times 32 \cdot 2} = 4 \cdot 67 \text{ units}$$

This is also the momentum of the bullet before impact, hence—

$$\text{velocity of bullet} = 4 \cdot 67 \div \left(\frac{1}{16 \times 32 \cdot 2}\right) = 2406 \text{ feet per second.}$$

It may occur to the reader that the use of units of 1 ounce mass and feet per second would be simpler, thus—

$$\text{momentum} = \text{initial velocity} \times 1 = 401 \times 6 = 2406$$

hence, velocity of bullet = 2406 feet per second.

Units of Force.—The reader having worked so far will now be able to see clearly why the engineers' unit of force being fixed at the weight of 1 lb. mass in London where the acceleration of gravity is approximately 32·2 feet per second per second, the unit of mass is chosen as 32·2 lbs. For if this mass is acted upon by a force of 1 lb. its acceleration is—

$$\text{force} \div \text{mass} = 1 \div \frac{32 \cdot 2}{32 \cdot 2} = 1 \text{ foot per second per second}$$

and thus 1 lb. weight is the force which acting on unit mass (32·2 lbs.) gives it unit acceleration (1 foot per second per second), or force is numerically equal to—

mass × acceleration or change of momentum per second

In the C.G.S. (centimetre, gramme, second) system of units the unit of force is the dyne. This is the force which acting on a mass of 1 gramme would give it an acceleration of 1 centimetre per second per second. The acceleration of gravity in London is about 981 centimetres per second per second, hence the weight of 1 gramme exerts a force of about 981 dynes.

Rotation.—If the rim of a wheel of radius r is turning at a speed of v feet per second, we have seen that its angular velocity ω is $\frac{v}{r}$ radians per second. The rate of increase of angular velocity is called the angular acceleration. If a is the angular acceleration then a = rate of increase of ω = rate of increase of $\frac{v}{r} = \frac{1}{r} \times$ rate of increase of v, or $a = \frac{1}{r} \times$ rate of increase of speed of rim in the direction of motion.

Example.—A wheel is turning at 40 revolutions per minute and after 20 seconds is turning at 70 revolutions per minute. Find the average acceleration.

$$\begin{aligned}\text{Average acceleration} &= \frac{70 - 40}{20} = 1 \cdot 5 \text{ revolutions per minute per second} \\ &= 1 \cdot 5 \times \frac{2\pi}{60} \text{ radians per second per second} \\ &= \frac{\pi}{20} \text{ or } 0 \cdot 157 \text{ radians per second per second}\end{aligned}$$

We may regard a flywheel rim as so thin that the linear speed of all the parts of it are the same and equal to v feet per second, and the force F necessary to get up speed as applied at the rim. Then the relation between the force, mass, and change of speed is

the same as if the rim moved in a straight line instead of a circular path, and if W is the weight of the rim and r its radius—

$$F = \frac{W}{g} \times \text{rate of change of } v = \frac{W}{g}\alpha r$$

But the turning effect of the force F is proportional to its *moment* about the centre of the axle wherever it is applied, or multiplying by r—

$$Fr = \frac{W}{g} \times r \times \text{rate of change of } v \text{ or } \frac{W}{g} r^2 \alpha$$

that is—

Torque or moment of turning force for an angular acceleration α is—

$$\frac{W}{g} r^2 \alpha$$

which is proportional to the mass, the square of the radius, and the angular acceleration.

For other wheels when the rim is not thin and consequently all the material is not at the same radius r we write—

$$\text{Torque or turning moment} = \frac{W}{g} k^2 \alpha \text{ or } \frac{W}{g} . k \times \text{rate of change of } v$$

where k is called the radius of gyrations, *i.e.* a radius at which if all the mass were concentrated, the action of the wheel under torque would be unaltered, and v is the speed at that radius.

Example 1.—A flywheel having a radius of gyration of 3 feet weighs 4000 lbs; if there is a driving torque of 600 pound-feet and a frictional or other resisting torque of 200 pound-feet, how long will it take to increase in speed from 20 to 70 revolutions per minute?

Effective accelerating torque = 600 − 200 = 400 pound-feet.

$$\frac{W}{g} k^2 \alpha = \text{torque}$$

$$\alpha \times \frac{4000}{32 \cdot 2} \times 3^2 = 400$$

$$\text{Angular acceleration } \alpha = \frac{400 \times 32 \cdot 2}{4000 \times 9}$$

$$= 0 \cdot 358 \text{ radians per second per second.}$$

$$\text{Total increase of speed} = 70 - 20 = 50 \text{ revolutions per minute}$$

$$= 50 \times \frac{2\pi}{60} = 5 \cdot 23 \text{ radians per second}$$

$$\text{Time to increase} = \frac{5 \cdot 23}{0 \cdot 358} = 14 \cdot 6 \text{ seconds}$$

Example 2.—A flywheel rim 10 feet mean diameter, *i.e.* 5 feet radius of gyration, weighs 10 tons and is making 80 revolutions per minute.

If it is disconnected from any other machinery and its axle subjected to a constant frictional torque of 500 pound-feet, how many revolutions will it make before coming to rest, and how much work will it do against friction?

$$\text{Resisting torque} = 500 \text{ pound-feet}$$
$$= \frac{2240 \times 10}{32 \cdot 2} \times 5^2 \times \text{angular retardation}$$
$$\text{angular retardation} = \frac{500 \times 32 \cdot 2}{2240 \times 10 \times 25}$$
$$= 0 \cdot 0287 \text{ radians per second per second}$$
$$\left.\begin{array}{r}\text{80 revolutions}\\ \text{per minute}\end{array}\right\} = \frac{80 \times 2\pi}{60} = 8 \cdot 38 \text{ radians per second}$$

me to come to rest at 0·0287 radians per second reduction per second

$$= \frac{8 \cdot 38}{0 \cdot 0287} = 292 \text{ seconds or 4 minutes 52 seconds}$$
$$\text{Average angular speed} = \tfrac{1}{2} \text{ maximum} = 40 \text{ revolutions per minute}$$
$$\text{Revolutions in coming to rest} = 40 \times \frac{292}{60} = 194 \cdot 7$$
$$\text{Work done per revolution} = \text{torque} \times \text{angle in radians}$$
$$= 500 \times 2\pi$$
$$= 3142 \text{ foot-pounds}$$
$$\text{Total work done} = 3142 \times 194 \cdot 7$$
$$= 611{,}750 \text{ foot-pounds}$$

Experiment.—That a constant torque produces uniform angular

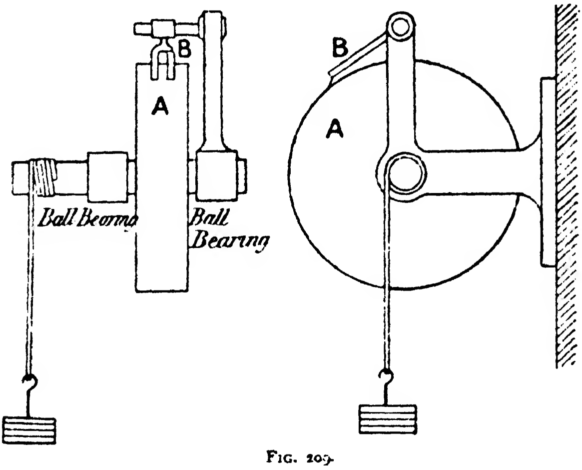


FIG. 209.

acceleration may be shown experimentally by the apparatus shown in Fig. 209. A is a wheel mounted in ball bearings, to reduce the friction

to a minimum, round the axle of which a cord is coiled, to the free end of which may be hung a convenient weight. A strip of paper is placed tightly round the rim of the wheel and the overlapping ends pasted together. The paper is then smoked until evenly coated with soot. A tuning fork B is provided with a tracing point and draws a curve in the same way as described for Fig. 199. A convenient weight having been attached to the cord the wheel A is released, and the tuning work set in vibration. The weight descends causing rotation of the wheel, and the tracing point draws on the smoked paper a series of waves similar to that shown in Fig. 199. The increase in distance between successive waves will be found to be constant, and therefore the linear acceleration of the rim of the wheel is constant. The linear acceleration of the rim being constant, it follows that the angular acceleration of the wheel is constant, since we have already shown (p. 277) that—

$$\text{angular acceleration} = \frac{1}{r} \times \text{rate of increase of speed of rim.}$$

EXAMPLES XXI.

1. A train weighing 100 tons is moving at 60 miles an hour. What is its momentum in engineers' units?

2. Water issues from a round nozzle $1\frac{1}{2}$ inches diameter with a speed of 40 feet per second. What weight of water leaves per second and what is its momentum? (Water weighs 62·4 lbs. per cubic foot.)

3. A body weighing 100 lbs. is moving at 30 feet per second due north. After 3 seconds it is moving south-west at 40 feet per second. Find the change of momentum, and the average change in momentum per second.

4. A car weighing 1 ton is moving at 8 miles an hour, and after 12 seconds is moving at 20 miles an hour. What is the average force acting on it in the direction of motion during that time?

5. A train weighing 250 tons is moving at 60 miles an hour. What retarding force will be required to bring it to rest in $\frac{1}{4}$ minute?

6. A jet of water issuing horizontally from a nozzle 1 inch diameter strikes a vertical wall and is thereby diverted at right angles, none splashing back. It exerts a force of 12 lbs. on the wall. Find the speed of the jet.

7. A jet of water delivering 50 gallons per second with a velocity of 20 feet per second impinges perpendicularly on a fixed plate. Find the pressure on the plate.

8. A train starting from rest is drawn by a varying force F pounds which, after t seconds, is as shown in the following table:—

t (seconds)	0	11	20	34	45	55	60
F (pounds)	4010	3915	3760	3530	3370	3210	3000

If the frictional resistance is constant and equal to 1200 lbs., what is the time average of the force acting during the 60 seconds? What is the total gain of momentum? If the weight of the train is 300 tons, find its velocity at the end of the 60 seconds.

9. A truck weighing 15 cwts. is pulled on a level line by a force of 60 lbs.

in excess of the frictional resistance. Find the acceleration of the truck. How far will it move in $\frac{1}{2}$ minute? What horse-power is then being exerted if the frictional resistance is 14 lbs.?

10. A train weighing 200 tons has a frictional resistance of 12 lbs. per ton. What average pull will be required to give it a speed of 40 miles an hour in 2 minutes from rest on the level, and what horse-power would be required at the end of this time?

11. A body weighs 483 lbs. and is moving at a speed of 20 feet per second. After $\frac{1}{2}$ second it is moving at 30 feet per second in the same plane, but in a direction perpendicular to its former path. Find the mean acceleration during this time, and the average force acting upon it to produce this change in velocity.

12. A tramcar weighs 5 tons. Find the horse-power required if it has to attain a speed of 12 miles an hour in 15 seconds from rest up an incline of 1 in 30, the frictional resistance being 15 lbs. per ton.

13. How far will the car in Question 12 travel from rest down an incline of 1 in 80 in 1 minute? If started up this incline at 12 miles an hour how far would it go before coming to rest?

14. A man weighing 180 lbs. is in a lift which descends with an acceleration of 2 feet per second per second. What force is exerted by the man on the floor of the lift? What would be the force if the lift were ascending at (1) a uniform speed, and (2) with an acceleration of 2 feet per second per second.

15. The head of a hammer weighs 10 lbs., and when moving at 30 feet per second is brought to rest in $\frac{1}{100}$ of a second. Find the average force of the blow.

16. A chipping hammer weighs 2 lbs. The chisel edge is $\frac{3}{8}$ inch long and $\frac{1}{30}$ inch wide. The hammer strikes the chisel with a velocity of 25 feet per second. What will be the pressure in pounds per square inch on the chisel edge if it is brought to rest in $\frac{1}{200}$ second?

17. A steam hammer weighs 10 tons, the piston is 21 inches diameter and the effective steam pressure is 50 lbs. per square inch. Find (*a*) the acceleration with which the hammer comes down, (*b*) velocity of hammer after descending 3 feet, (*c*) the time then taken for the hammer to come to rest if the material is compressed $\frac{1}{2}$ inch.

18. A bullet weighing 1 ounce and moving at 1000 feet per second strikes and remains imbedded in a block of wood weighing 32 lbs. which is free to move. With what velocity will the block start to move?

19. A pulley is running at 200 revolutions per minute and after half a minute is running at 250 revolutions per minute. Find the average angular acceleration.

20. A wheel is making 200 revolutions per minute and after 10 seconds its speed has fallen to 150 revolutions per minute. If the angular retardation be constant how many more revolutions will it make before coming to rest?

21. A wheel having a radius of gyration of 4 feet weighs 10,000 lbs. If there is a driving torque of 1200 pound-feet, and a frictional or other resisting torque of 300 pound-feet, how long will it take to increase in speed from 10 to 240 revolutions per minute?

22. A flywheel having a mean diameter of 6 feet, *i.e.* radius of gyration 3 feet, weighs 2 tons, and is making 120 revolutions per minute. What resisting torque must be applied in order to bring it to rest in 1 minute, and how much work will it do against the resistance?

CHAPTER XXII

ENERGY

A BODY is said to possess energy when on account of its condition it is capable of doing work. Thus, in virtue of its position, velocity, high temperature, electrical pressure or chemical composition a body may be capable of doing work and is said to possess energy. In addition to the various forms of mechanical energy the engineer has to deal particularly with heat energy and electrical energy.

Relation between Heat and Mechanical Work.—When work is spent in overcoming friction, heat is produced, and for every foot-pound of work so spent a perfectly definite amount of heat is produced. Heat is measured by engineers in British thermal units written briefly B.Th.U. A British thermal unit is the amount of heat required to warm 1 lb. of water 1° F. in temperature; to warm, say, 20 lbs. of water 8° F. would take $20 \times 8 = 160$ B.Th.U. To produce 1 British thermal unit of heat by mechanical work would require about 778 foot-pounds.

Example.—How many British Thermal Units are produced per hour in a bearing in which half a horse-power is being absorbed in friction?

$$\text{Foot-pounds of work converted into heat per hour} = \tfrac{1}{2} \times 33{,}000 \times 60 = 990{,}000$$

$$\text{Equivalent British Thermal Units} = \frac{990{,}000}{778} = 1272 \text{ B.Th.U.}$$

Although 778 foot-pounds of mechanical work can produce 1 B.Th.U. of heat it is not possible to produce 778 foot-pounds from 1 B.Th.U. of heat. The mechanical work obtainable from 1 unit of heat depends upon the temperature of the available heat and the method of converting heat into work. The conversion of heat into work is studied in the subject of Heat Engines.

Heat Energy in Fuels.—Heat energy for conversion into mechanical work is usually obtained by burning substances in furnaces or engine cylinders. It is interesting to see how much

heat energy is obtainable from various substances. The following short table gives approximate values :—

Substance.	Energy in 1 lb.	
	B.Th.U.	Foot-pounds.
Good coal	14,500	11,280,000
Wood (ordinary) . . .	6,000	4,668,000
Petroleum oil . . .	20,000	15,560,000
Crude coal tar . .	16,000	13,538,000
Petrol	20,000	15,560,000

Example 1.—An oil engine uses 0·7 lbs. of petroleum per horse-power per hour. What proportion of the available energy in the oil is converted into mechanical work?

$$\text{One horse-power for 1 hour} = 33{,}000 \times 60 \text{ foot-pounds}$$
$$= \frac{33{,}000 \times 60}{778} = 2545 \text{ B.Th.U.}$$

Now, 1 lb. of petroleum contains 20,000 B.Th.U., hence—

$$\text{Proportion converted into mechanical work} = \frac{2545}{20{,}000 \times 0{\cdot}7} = 0{\cdot}1817$$
$$= 18{\cdot}17 \text{ per cent.}$$

Example 2.—If a steam engine plant converts 8 per cent. of the available energy in the coal into mechanical work, how many pounds of coal per hour will be required for a plant with an output of 700 H.P.?

$$700 \text{ H.P. for 1 hour} = 700 \times 33{,}000 \times 60 \text{ foot-pounds}$$
$$= \frac{700 \times 33{,}000 \times 60}{778} \text{ B.Th.U.}$$

and this is 8 per cent of the total energy in the coal, hence—

$$\text{Total energy in the coal} \times \tfrac{8}{100} = \frac{700 \times 33{,}000 \times 60}{778}$$

$$\text{Total energy in the coal} = \frac{700 \times 33{,}000 \times 60}{778} \times \tfrac{100}{8} \text{ B.Th.U.}$$

Assuming 1 lb. of coal to contain 14,000 B.Th.U., we have—

$$\text{coal required per hour} = \frac{\text{total energy}}{14{,}000}$$
$$= \frac{700 \times 33{,}000 \times 60 \times 100}{778 \times 8 \times 14{,}000} = 1590 \text{ lbs.}$$

Relation of Electrical and Mechanical Energy.—The electrical engineers' unit of work is the *joule*, see Chap. IX., p. 118, and one joule per second is called the *watt*. For commercial purposes the unit of electrical energy is the *Board of*

Trade unit, this being one kilowatt (or 1000 watts) for one hour, which is a quantity of work. Since 1 E.H.P. is equal to 746 watts, we have, as on p. 119,

$$1 \text{ kilowatt} = \tfrac{1000}{746} \text{ E.H.P.}$$

and 1 kilowatt hour or 1 Board of Trade unit $= \frac{1000}{746} \times 33{,}000 \times 60$
$= 2{,}654{,}150$ foot-pounds.

Example 1.—An electric locomotive draws a train of gross weight 500 tons up an incline of 1 in a 100 at a steady speed of 15 miles an hour. If the frictional resistance is constant and equal to 15 lbs. per ton, what is the total pull and horse-power? If the voltage supply is 500 volts, and 60 per cent. of the energy supplied is usefully employed, what current in ampères is taken by the motors?

$$\text{Frictional resistance} = 500 \times 15 = 7500 \text{ lbs.}$$
$$\text{Force to draw train up incline} = \tfrac{1}{100} \times 500 \times 2240 = 11{,}200 \text{ lbs.}$$
$$\text{Total pull required} = 7500 + 11{,}200 = 18{,}700 \text{ lbs.}$$

Horse-power at 15 miles an hour or 22 feet per second

$$= \frac{\text{pull} \times \text{distance moved per second}}{550}$$
$$= \frac{18{,}700 \times 22}{550} = 748 \text{ H.P.}$$

$$\text{Electrical horse-power supplied to motors} = 748 \times \tfrac{100}{60}$$
$$\text{Watts supplied to motors} = 748 \times \tfrac{100}{60} \times 746.$$

And since watts = ampères × volts, we have—

$$\text{current supplied} = \frac{748 \times 100 \times 746}{60 \times 500}$$
$$= 1860 \text{ ampères.}$$

Example 2.—A countershaft driven by an electric motor drives a lathe by means of a belt. If the shaft and belt absorb 0·3 H.P., and the frictional resistance of the lathe absorbs 0·4 H.P., and 0·9 H.P. are expended in turning the work in the lathe, how many watts must be supplied to the motor, if its efficiency is 85 per cent.? If the pressure of supply is 200 volts what current is taken?

$$\text{Output of motor} = 0{\cdot}3 + 0{\cdot}4 + 0{\cdot}9 = 1{\cdot}6 \text{ H.P.} = 1{\cdot}6 \times 746 \text{ watts.}$$
$$\text{Watts supplied to motor} = 1{\cdot}6 \times 746 \times \tfrac{100}{85} = 1404 \text{ watts}$$
$$\text{Current taken} = \frac{1404}{200} = 7{\cdot}02 \text{ ampères.}$$

Mechanical Energy.

Potential Energy.—Mechanical energy takes various forms. If work is spent in lifting a body, the body is then said to possess *potential energy* or energy of position. Suppose a body weighing 80 lbs. is lifted 20 feet above the ground, an amount of work—

$$20 \times 80 = 1600 \text{ foot-pounds}$$

is spent in lifting it, and the body possesses 1600 foot-pounds of potential energy, and is capable of doing 1600 foot-pounds of work.

Thus it might be employed by means of pulley blocks to lift a heavier weight of, say, 100 lbs. through a smaller height than 20 feet. If there were no friction, the work spent in lifting the second weight would also be 1600 foot-pounds, so that the height lifted would be—

$$\frac{1600}{100} = 16 \text{ feet.}$$

If the mechanical efficiency of the lifting machine were, say, 70 per cent., the work available for lifting the 100 lbs. weight would be—

$$1600 \times \frac{70}{100} = 1120 \text{ foot-pounds}$$

and the height of lift would be—

$$\frac{1120}{100} = 11 \cdot 2 \text{ feet.}$$

In this case if the remaining work were wasted in heat by friction, the heat produced would be—

$$\frac{30}{100} \times 1600 \text{ or } 1600 - 1120 = 480 \text{ foot-pounds}$$
$$= \frac{480}{778} = 0 \cdot 62 \text{ B.Th.U.}$$

Another form of potential energy is that of a strained spring. We have seen in Chap. V. that to stretch or compress a spring requires a certain amount of work, and the same holds good for spiral springs such as are used in clocks and watches. The work so spent is stored as strain energy in the spring, and can be employed to do useful work; clocks, watches, clockwork instruments and toys are well-known examples in which work is stored as strain energy.

Kinetic Energy.—Another very important form of mechanical energy is that possessed by a body in motion. We have seen that to put a body in motion requires force to overcome its inertia, and this force acting through the displacement of the body does work. This work is accumulated in the body and is called the *kinetic energy* (*K.E.*) or energy of motion of the body.

Suppose a constant unresisted force of 10 lbs. acts for 5 seconds on a body originally at rest and weighing 40 lbs., how much kinetic energy will the body then have stored up in it?

$$\text{Acceleration of the body} = \frac{\text{accelerating force}}{\text{mass}} = \frac{10}{\frac{40}{32 \cdot 2}}$$
$$= 8 \cdot 05 \text{ feet per second per second}$$
$$\text{Final speed after 5 seconds} = 5 \times 8 \cdot 05 = 40 \cdot 25 \text{ feet per second}$$
$$\text{Average speed} = \frac{40 \cdot 25}{2} = 20 \cdot 125 \text{ feet per second}$$
$$\text{Distance travelled in 5 seconds} = 20 \cdot 125 \times 5 = 100 \cdot 625 \text{ feet}$$
$$\text{Work spent} = 10 \times 100 \cdot 625$$
$$= 1006 \cdot 25 \text{ foot-pounds}$$

which is the kinetic energy of the body.

Suppose the body weighed W pounds and was acted on by a constant force F pounds until it attained a speed of v feet per second in the direction of the force, what would be its K.E.?

$$\text{Acceleration} = \frac{F}{\frac{W}{32 \cdot 2}} = \frac{32 \cdot 2F}{W} \text{ feet per second per second}$$

$$\text{Time to attain a velocity of } v \text{ feet per second} = \frac{v}{\text{acceleration}} = \frac{Wv}{32 \cdot 2\,F}$$

Distance moved in this time at average velocity $\frac{v}{2}$ feet per second

$$= \frac{v}{2} \times \text{time}$$

$$= \frac{v}{2} \times \frac{Wv}{32 \cdot 2\,F} = \frac{Wv^2}{64 \cdot 4\,F}$$

$$\text{Work done} = \text{distance} \times \text{force} = \frac{Wv^2}{64 \cdot 4\,F} \times F = \frac{Wv^2}{64 \cdot 4} \text{ or } \frac{Wv^2}{2g} \text{ foot-pounds}$$

Thus we see that the kinetic energy of the body depends only on its weight and on the *square* of its speed, and not upon the force F which sets it in motion. It will be the same whether set in motion by a large force acting for a short time only, or by a smaller force acting for a longer time. When a body of weight W pounds has a velocity v feet per second its

$$\text{kinetic energy} = \frac{Wv^2}{2g} \text{ or } \frac{Wv^2}{64 \cdot 4} \text{ foot-pounds}$$

Example 1.—A truck weighs 10 cwt. and is running at 30 miles per hour. What is its K.E. in foot-pounds? If the resistance to motion on the level is 30 lbs. per ton, how far will it run before coming to rest?

30 miles per hour = 44 feet per second.

$$\text{Kinetic energy} = \frac{Wv^2}{2g} = \frac{10 \times 112 \times 44 \times 44}{2 \times 32 \cdot 2} = 33{,}640 \text{ foot-pounds}$$

Total resistance to running = $\frac{1}{2} \times 30 = 15$ lbs.

Work spent against resistance = 15 lbs. × distance run = the K.E. of 33,640 foot-pounds,

$$\text{distance run} = \frac{33{,}640}{15} = 2242 \text{ feet.}$$

Example 2.—A projectile has 1,500,000 foot-pounds of kinetic energy at a velocity of 2500 feet per second. How much kinetic energy will it have lost when its velocity has fallen to 1500 feet per second?

$$\text{K.E.} = \frac{Wv^2}{2g} = \frac{W}{2g} \times (2500)^2 = \frac{W}{2g} \times 6{,}250{,}000 \text{ foot-pounds,}$$

and this is equal to 1,500,000 foot-pounds. Hence—

$$\frac{W}{2g} \times 6{,}250{,}000 = 1{,}500{,}000$$

$$\frac{W}{2g} = \frac{1{,}500{,}000}{6{,}250{,}000} = \frac{30}{125}$$

at 1500 feet per second.

$$\text{K.E.} = \frac{W}{2g} \times (1500)^2 = \frac{30}{125} \times 2{,}250{,}000 = 540{,}000 \text{ foot-pounds.}$$

$$\text{Loss of K.E.} = 1{,}500{,}000 - 540{,}000 = 960{,}000 \text{ foot-pounds.}$$

Example 3.—A bullet weighing 1 ounce has a velocity of 1200 feet per second, what is its kinetic energy in foot-pounds? If it is fired into a freely suspended block of wood weighing 1 lb. in which it remains imbedded, how much kinetic energy is lost in the impact, and how many units of heat are generated?

$$\text{K.E. of bullet} = \frac{1}{16} \times \frac{1}{64{\cdot}4} \times 1200 \times 1200 = 1397 \text{ foot-pounds.}$$

Since the momentum remains the same after impact, and the weight of the block and bullet is 17 ounces or 17 times as great as that of the bullet, the velocity is $\frac{1}{17}$ of 1200 feet per second,

$$\text{or } \frac{1200}{17} = 70{\cdot}6 \text{ feet per second.}$$

$$\text{Kinetic energy of block and bullet} = \frac{17}{16} \times \frac{1}{64{\cdot}4} \times (70{\cdot}6)^2 = 82 \text{ foot-pounds.}$$

$$\text{Loss of kinetic energy} = 1397 - 82 = 1315 \text{ foot-pounds.}$$

$$\text{Equivalent heat generated} = \frac{1315}{778} = 1{\cdot}69 \text{ B.Th.U.}$$

Example 4.—A train weighing 150 tons has a frictional resistance of 16 lbs. per ton. What average pull will be required to give it a speed of 30 miles an hour in $1\frac{1}{2}$ minutes on the level?

The reader may note that this problem has already been solved on p. 272, and should note the different method adopted here.

Kinetic energy at 30 miles per hour or 44 feet per second

$$= \frac{150 \times 2240}{2 \times 32{\cdot}2} \times 44 \times 4 = 10{,}100{,}869 \text{ foot-pounds.}$$

Distance travelled in $1\frac{1}{2}$ minutes at average speed of $\frac{44}{2}$ or 22 feet per second $= 90 \times 22 = 1980$ feet.

$$\text{Average force to do the above work in 1980 feet} = \frac{10{,}100{,}869}{1980} = 5101 \text{ lbs.}$$

Force to overcome frictional resistance $= 150 \times 16 = 2400$ lbs.

Total pull $= 5101 + 2400 = 7501$ lbs. as on p. 272.

Important. Example 5.—The motion of a body of weight 5200 lbs. is opposed by a constant frictional resistance of 2800 lbs. It starts from rest under the action of a force F pounds whose value is here given at the instants at which the body has passed x feet from rest:—

F . .	5500	5200	4900	5100	5000	4500
x . .	0	5	10	15	20	25

What is the speed of the body when it has moved 25 feet from rest?

Plot the curve connecting F and x as shown half size in Fig. 210 where the scale of F is 1000 lbs. to 1 inch, and that of x is 5 feet to 1 inch, then the total area under the curve represents to scale the total work done during the 25 feet. Now draw the horizontal line AB across, representing the constant frictional resistance of 2800 lbs., then the area of the rectangle ABCD below AB represents the work expended against the frictional

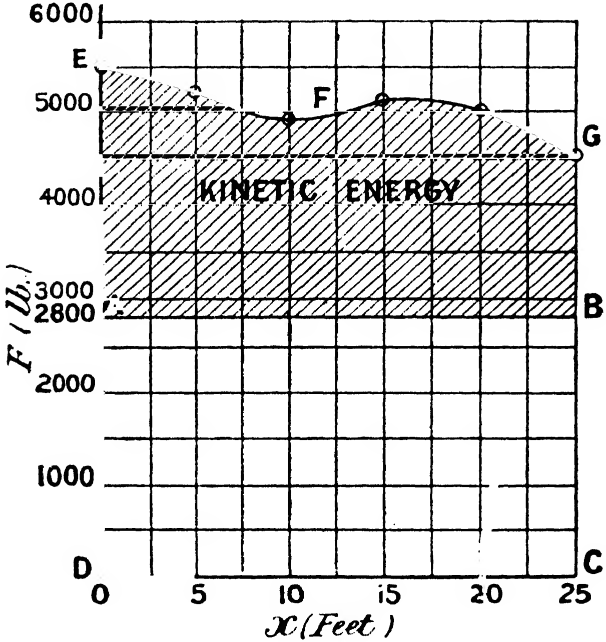


FIG. 210

resistance of 2800 lbs., and the remaining area shown shaded ABGFE represents the remainder of the work done, and this must be stored up in the body as kinetic energy. Counting up the number of squares in the area ABGFE we find its area to be 11·2 square inches, and since the scale is 1 square inch, represents $1000 \times 5 = 5000$ foot-pounds, the kinetic energy of the body after it has moved 25 feet is—

$$5000 \times 11{\cdot}2 = 56{,}000 \text{ foot-pounds.}$$

Let v be the velocity of the body in feet per second, then—

$$\text{K.E.} = \frac{Wv^2}{2g} = 56{,}000$$

$$\frac{5200v^2}{2 \times 32 \cdot 2} = 56{,}000$$

$$v^2 = \frac{56{,}000 \times 64 \cdot 4}{5200}$$

$$v = \sqrt{\frac{56{,}000 \times 64 \cdot 4}{5200}} = 26 \cdot 33 \text{ feet per second.}$$

Fly Press.—There are several machines for utilizing the kinetic energy of a moving body to overcome a resistance and to do useful work. A common example is the fly press shown in Fig. 211 used for stamping medals, etc., or for punching holes in

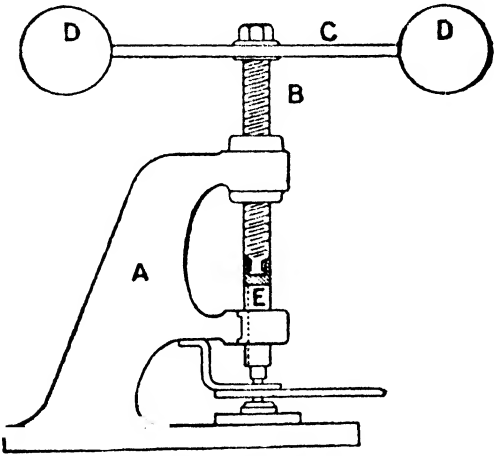


FIG. 211.—Fly press.

metal plates. The frame of the machine consists of a casting A, provided with a nut in which the screw B works. To the top of the screw is fixed a lever, C, which has a heavy cast-iron ball D at each end, and to the lower end is attached a bar, E, which slides in a bearing in A. E is attached to the screw in such a manner that the screw can rotate independently of it. The punch is attached to E, and the die rests on the base of the press. The plate in which the hole is to be punched is placed on the top of the die, the screw is revolved by swinging the lever C round, and so forcing the punch down through the plate. The kinetic energy of the balls D is absorbed in overcoming the resistance offered by the plate to punching.

Example.—In a fly press the weight of each ball is 50 lbs., and their velocity is 15 feet per second; the die on the end of the screw moves through $\frac{1}{16}$ inch in coming to rest. What average pressure is exerted on the metal subjected to stamping?

$$\text{Kinetic energy of the balls} = \frac{100 \times 15 \times 15}{64{\cdot}4} = 349 \text{ foot-pounds.}$$

$$\text{Pressure in pounds} \times \text{distance in feet} = 349$$

$$\text{Pressure} \times \tfrac{1}{16} \times \tfrac{1}{12} = 349$$

$$\text{Pressure} = 349 \times 16 \times 12$$

$$\text{,,} \quad = 67{,}008 \text{ lbs.}$$

Kinetic Energy of a Falling Weight.—If a body weighing W pounds is lifted h feet above the ground, it has stored in it W $\times$ h foot-pounds of energy. If it falls freely to the ground again through h feet this potential energy will all be converted into kinetic energy. If its velocity on reaching the ground is v feet per second—

$$\text{Potential energy } Wh = \text{kinetic energy } \frac{Wv^2}{2g}.$$

This follows from Chap. XX, p. 256, for after falling a height h feet under the acceleration of gravity the velocity v is given by—

$$v^2 = 2gh \text{ or } v = \sqrt{2gh} \text{ or } h = \frac{v^2}{2g}$$

$$\text{and } Wh = \frac{Wv^2}{2g}$$

When the weight has fallen, say, $\frac{1}{4}$ of the height h the K.E. will be equal to $\frac{1}{4}Wh$ and the potential energy $\frac{3}{4}Wh$, and in all positions the sum of the potential and the kinetic energy will be equal to Wh. Note, then, when a body has fallen vertically through a height h its velocity $v = \sqrt{2gh}$ whether it is moving vertically or in any other direction, and its K.E. is $\frac{Wv^2}{2g}$ or Wh.

Fly-wheels: Kinetic Energy of Rotation.—One of the most convenient and usual methods of storing energy is in the rim of a fly-wheel. Suppose the thin rim of a wheel weighs W pounds, and its linear speed (which is the same throughout) is v feet per second, the kinetic energy of the wheel is—

$$\frac{Wv^2}{2g} \text{ or } \frac{Wv^2}{64{\cdot}4} \text{ foot-pounds.}$$

Example.—The diameter of the rim of a fly-wheel is 10 feet and it weighs 8 tons: find its kinetic energy when rotating at 120 revolutions per minute.

120 revolutions per minute = 2 revolutions per second

velocity of rim = $10\pi \times 2 = 62{\cdot}83$ feet per second

$$\text{kinetic energy} = \frac{8 \times 2240}{2 \times 32\cdot2} \times 62\cdot83 \times 62\cdot83 = 1{,}098{,}000 \text{ foot-pounds.}$$

or in foot-tons—

$$\text{kinetic energy} = \frac{8}{2 \times 32\cdot2} \times 62\cdot83 \times 62\cdot63 = 490 \text{ foot-tons.}$$

If the radius of the rim of the fly-wheel is r feet and it makes n revolutions per second—

$$v = 2\pi r \times n \text{ feet per second}$$

$$\text{kinetic energy} = \frac{Wv^2}{2g} = \frac{W}{2g} \times (2\pi rn)^2 = \frac{2W\pi^2r^2n^2}{32\cdot2} = 0\cdot61Wr^2n^2.$$

Or, again, if the wheel makes N revolutions per minute $N = 60n$,

$$\text{or } v = \frac{2\pi rN}{60} = \frac{\pi rN}{30}$$

$$\text{kinetic energy} = \frac{Wv^2}{2g} = \frac{W}{64\cdot4} \times \frac{\pi^2r^2N^2}{900} = 0\cdot00017Wr^2N^2$$

For a given wheel, W and r being fixed, its K.E. at any given speed of rotation (N revolutions per minute) is *proportional to the square of the angular speed.* Thus its K.E. at 100 revolutions per minute is four times its K.E. at 50 revolutions per minute.

In the case of rotating wheels, all the material of which is not concentrated in a thin rim, different parts of the wheel are moving at different linear speeds, and in order to calculate the K.E. we must take the speed at the radius of gyration. This, as explained on page 278, is a radius at which, if the whole mass of the wheel were concentrated, its effect would be unaltered.

Thus, the kinetic energy of any wheel is—

$$\frac{Wv^2}{2g}$$

where v is the linear speed in feet per second at the radius of gyration k, and at N revolutions per minute $v = \dfrac{2\pi kn}{60} = \dfrac{\pi kn}{30}$

$$\frac{Wv^2}{2g} = \frac{W}{2g} \times \frac{\pi^2k^2N^2}{900} \text{ or } 0\cdot00017Wk^2N^2$$

Example 1.—A fly-wheel at rest in frictionless ball bearings has a torque of 60 pounds-inches applied to it while it makes 5 revolutions. If the wheel weighs 200 lbs., what is then its linear speed at the radius of gyration? If the radius of gyration is 0·75 foot, what is the speed of rotation of the wheel in revolutions per minute?

$$\text{Work converted into K.E.} = \left(\frac{60}{12}\right) \text{pound-feet} \times (5 \times 2\pi) \text{ radians}$$

$$= 50\pi = 157\cdot1 \text{ foot-pounds}$$

If v is the speed of the radius of gyration—

$$\frac{Wv^2}{2g} = \frac{200v^2}{64{\cdot}4} = 157{\cdot}1$$

$$v^2 = \frac{157{\cdot}1 \times 64{\cdot}4}{200} = 50{\cdot}6$$

$$v = \sqrt{50{\cdot}6} = 7{\cdot}11 \text{ feet per second}$$

Circumference at 0·75 foot radius $= 2 \times \pi \times 0{\cdot}75 = 4{\cdot}71$ feet

$$\text{Revolutions per second} = \frac{7{\cdot}11}{4{\cdot}71}$$

$$\text{Revolutions per minute} = \frac{7{\cdot}11}{4{\cdot}71} \times 60 = 90{\cdot}5$$

Example 2.—A fly-wheel weighs 1 ton and has a radius of gyration of 2·5 feet. If it is running at 120 revolutions per minute and 75 foot-pounds are absorbed by friction per revolution, find the revolutions made in coming to rest.

$$\text{K.E. of wheel} = \frac{Wv^2}{2g} = \frac{2240}{64{\cdot}4} \times \left(\frac{2\pi \times 2{\cdot}5 \times 120}{60}\right)^2$$

$$= \frac{2240}{64{\cdot}4} \times 10\pi \times 10\pi = 34{,}320 \text{ foot-pounds}$$

$$\text{Revolutions made in coming to rest} = \frac{34320}{75} = 457{\cdot}6$$

Example 3.—A fly-wheel weighs 6 tons, and has a radius of gyration of 4 feet 6 inches. How many foot-pounds of energy would be stored in it when it is making 120 revolutions per minute? If it is supported in bearings 8 inches diameter, and the coefficient of friction is 0·01 at all speeds, how much work would be spent in friction in 1 revolution, and how many revolutions would the wheel make before coming to rest?

$$\text{Speed at radius of gyration} = \frac{120 \times 2\pi \times 4{\cdot}5}{60} = 56{\cdot}55 \text{ feet per second.}$$

$$\text{Kinetic energy stored} = \frac{6 \times 2240}{64{\cdot}4} \times 56{\cdot}55 \times 56{\cdot}55 = 667{,}300 \text{ foot-pounds.}$$

$$\text{Frictional force at circumference of shaft} = 0{\cdot}01 \times 6 \times 2240 = 134{\cdot}4 \text{ lbs.}$$

$$\text{Circumference of shaft} = 8\pi \text{ inches} = \frac{8\pi}{12} \text{ feet} = 2{\cdot}094 \text{ feet.}$$

$$\text{Work spent per revolution} = 134{\cdot}4 \times 2{\cdot}094 = 281{\cdot}4 \text{ foot-pounds.}$$

Revolutions to absorb all the K.E. and bring the wheel to rest—

$$= \frac{667300}{281{\cdot}4} = 2371 \text{ revolutions.}$$

Changes in Speed and in Kinetic Energy.—Suppose a fly-wheel is rotating at N_1 revolutions per minute, and its linear speed at the radius of gyration is v_1 feet per second; then, if it gives out energy to a machine, and is reduced in speed to N_2 revolutions per minute and velocity v_2 feet per second at its radius of gyration—

$$\text{Work given out} = \text{change in kinetic energy}$$
$$= \frac{Wv_1^2}{2g} - \frac{Wv_2^2}{2g} = \frac{W}{2g}(v_1^2 - v_2^2)$$

or—

$$\text{Work given out} = \frac{W}{2g} \times \frac{\pi^2k^2N_1^2}{900} - \frac{W}{2g} \times \frac{\pi^2k^2N_2^2}{900} = \frac{W\pi^2k^2}{64{\cdot}4 \times 900}(N_1^2 - N_2^2)$$
$$\text{or} = 0{\cdot}00017Wk^2(N_1^2 - N_2^2)$$

The quantity $0{\cdot}00017Wk^2$ is the kinetic energy of the wheel when rotating at 1 revolution per minute. This quantity, often called the M of the wheel, when multiplied by the square of the number of revolutions per minute gives the K.E. of the wheel.

Example 1.—A fly-wheel making 120 revolutions per minute has stored in it 80,000 foot-pounds. What is its speed after it has given up 30,000 foot-pounds of energy?

Let N be its speed in revolutions per minute, its K.E. is then 80,000 − 30,000 = 50,000 foot-pounds. Then since the K.E. is proportional to the square of the speed—

$$\frac{N^2}{120^2} = \frac{50{,}000}{80{,}000} = \frac{5}{8}$$
$$N^2 = \frac{5}{8} \times 120 \times 120 = 9000$$
$$N = \sqrt{9000} = 94{\cdot}87 \text{ revolutions per minute.}$$

Example 2.—The fly-wheel of a shearing machine weighs 4000 lbs., and has a radius of gyration of 3 feet. At the beginning of the cutting stroke its speed is 100 revolutions per minute, and at the end its speed is 90 revolutions per minute. How much energy has been given out, and if 80 per cent. of this is usefully employed on a 3-inch stroke, what average cutting force is exerted?

Kinetic energy at 100 revolutions per minute $= 0{\cdot}00017 \times 4000 \times 3^2 \times 100^2$

" 90 " $= 0{\cdot}00017 \times 4000 \times 3^2 \times 90^2$

$$\text{Energy given out} = 0{\cdot}00017 \times 4000 \times 9(100^2 - 90^2)$$
$$= 17 \times 4 \times 9\,(10^2 - 9^2)$$
$$= 17 \times 4 \times 9 \times 19 = 11{,}628 \text{ foot-pounds.}$$
$$\text{Average force in pounds} \times \tfrac{3}{12} \text{ feet} = \tfrac{80}{100} \times 11\,628 \text{ foot-pounds.}$$
$$\text{Average force} = 3{\cdot}2 \times 11{,}628 = 37{,}210 \text{ lbs.}$$

Example 3.—The fly-wheel of a punching machine has stored in it 80,000 foot-pounds of energy when making 120 revolutions per minute. What reduction of kinetic energy takes place if after punching a hole the speed is reduced to 90 revolutions per minute? If the working stroke is $\frac{3}{4}$ inch, and 75 per cent. of the energy is usefully employed in punching, what is the average force exerted?

Let M be the kinetic energy of the wheel at 1 revolution per minute, then, $M \times 120^2 = 80{,}000$ foot-pounds

$$M = \frac{80{,}000}{14{,}400} = \frac{50}{9}$$
$$= 5\tfrac{5}{9} \text{ foot-pounds; and at 90 revolutions per minute.}$$

$$\text{Kinetic energy} = \text{M} \times 90^2 = \tfrac{50}{9} \times 8100 = 45{,}000 \text{ foot-pounds}$$
$$\text{Reduction in kinetic energy} = 80{,}000 - 45{,}000 = 35{,}000 \text{ foot-pounds}$$
$$\text{Average force in pounds} \times \tfrac{1}{12} \times \tfrac{3}{4} = 35{,}000 \times \tfrac{75}{100}$$
$$\text{Average force} = 12 \times 35{,}000 = 420{,}000 \text{ pounds.}$$

We might have stated the reduction of kinetic energy thus—

$$\frac{\text{reduction of kinetic energy}}{80{,}000} = \frac{120^2 - 90^2}{120^2} = \frac{144 - 81}{144} = \frac{63}{144} = \frac{7}{16}$$

$$\text{hence, reduction of kinetic energy} = \tfrac{7}{16} \times 80{,}000$$
$$= 35{,}000 \text{ foot-pounds.}$$

Experiments on the Kinetic Energy of a Fly-wheel.—(a) *To find the Kinetic Energy when rotating at* 1 *revolution per minute.*—Fig. 209 shows an experimental fly-wheel mounted in ball bearings to reduce the frictional resistance to a minimum. Round the axle of the wheel is coiled a cord to the free end of which a convenient weight is attached. The length of the cord is so arranged that when the weight is released it rotates the wheel during its descent, and on reaching the ground the cord is detached from the axle. The distance the weight falls is carefully measured, and the total number of revolutions the wheel makes from the instant the weight is released until the wheel comes to rest are counted. By means of a stop watch the time taken for the weight to reach the ground is measured, and also the total time from the instant the weight is released until the wheel comes to rest is measured.

Let d be the effective diameter of the axle, *i.e.* diameter of axle + diameter of cord in feet; t the time in seconds for the weight to reach the ground; T the total time in seconds until the wheel comes to rest; N the total number of revolutions made by the wheel before it comes to rest; and h the distance the weight falls in feet, n = maximum speed of wheel in revolutions per minute, then—

$$\text{Effective circumference of axle} = \pi d \text{ feet}$$

$$\text{Revolutions made by the wheel during the descent of the weight} = \frac{h}{\pi d}$$

The average speed of the wheel during the time t seconds

$$= \frac{h}{\pi d} \div t = \frac{h}{\pi d t} \text{ revolutions per second.}$$

Since the speed will be uniformly accelerated,

Maximum speed of wheel at the instant the weight reaches the ground = twice the average speed

$$= \frac{2h}{\pi d t} \text{ revolutions per second}$$

$$n = \frac{2h}{\pi d t} \times 60 \text{ revolutions per minute.}$$

This maximum speed of the wheel should be checked from the total revolution made (N) and the total time T as follows :—

Average speed of wheel during T seconds $= \frac{N}{T}$ revolutions per sec

$$\therefore \quad \text{Maximum speed} = \frac{2N}{T} \text{ revolutions per second.}$$

or $$n = \frac{2N}{T} \times 60 \text{ revolutions per minute.}$$

These two results will not generally agree if the wheel is mounted in plain journal bearings, but will for all practical purposes be the same for a wheel mounted in ball bearings, the friction of which is more uniform at various speeds.

Let v be the speed of the weight in feet per second when it reaches the ground, then—

$$\text{average speed of weight} = \frac{h}{t} \text{ feet per second}$$

and $$v = \frac{h}{t} \times 2 \text{ feet per second}$$

Now, the total potential energy of the weight W pounds before it is released will be—

$$W \times h \text{ foot-pounds.}$$

When the weight reaches the ground it is moving with velocity v feet per second and has K.E. equal to $\frac{Wv^2}{2g}$ foot-pounds, the wheel will also at this instant have kinetic energy equal to $0{\cdot}00017wk^2n^2$ foot-pounds; hence, since the total energy remains constant, we have, neglecting friction—

Potential energy of weight

= kinetic energy of wheel + kinetic energy of weight

$$Wh = 0{\cdot}00017wk^2n^2 + \frac{Wv^2}{2g}$$

and

$$0{\cdot}00017wk^2n^2 = Wh - \frac{Wv^2}{2g} = \text{K.E. of wheel at } n \text{ revolutions per min.}$$

and

$$0{\cdot}00017wk^2 = \frac{Wh - \frac{Wv^2}{2g}}{n^2} \text{ foot-pounds}$$

where w = weight of wheel in pounds

which is the kinetic energy of the wheel when making 1 revolution per minute.

Another method of measuring the maximum speed of the wheel is to use a tuning fork to trace the vibration on the rim of the wheel as in Fig. 209. This enables the maximum linear speed of the rim to be found from which the maximum revolutions per minute is deduced.

(b) *To find the radius of gyration in feet.*—Remove the wheel from its bearings and weigh it. Let w be its weight in pounds, then—

$$0{\cdot}00017wk^2 = \text{K.E. at 1 revolution per minute}$$

$$k^2 = \frac{\text{K.E. at 1 revolution per minute}}{0{\cdot}00017w}$$

$$= \frac{Wh - \frac{Wv^2}{2g}}{0{\cdot}00017wn^2}$$

hence

$$k = \sqrt{\frac{Wh - \frac{Wv^2}{2g}}{0{\cdot}00017wn^2}}$$

(c) *To correct for friction.*—The kinetic energy of the wheel will be less than $Wh - \frac{Wv^2}{2g}$ by the amount of work lost in friction during the time the weight is descending, and all the kinetic energy of the wheel is wasted in friction during the time $T - t$ seconds, *i.e.* from the instant the weight reaches the ground until the wheel comes to rest. If the wheel makes, say, 5 revolutions while the weight is descending, and 100 more while it is coming to rest, work lost in friction per revolution if constant $= \frac{1}{100}$ of kinetic energy of the wheel, and loss due to friction during the fall of the weight $= \frac{5}{100}$ of kinetic energy of the wheel, hence—

$$Wh = 0{\cdot}00017wk^2n^2 + \tfrac{5}{100} \times 0{\cdot}00017wk^2n^2 + \frac{Wv^2}{2g}$$

$$Wh = \tfrac{105}{100} \times 0{\cdot}00017wk^2n^2 + \frac{Wv^2}{2g}$$

from which we find $0{\cdot}00017wk^2$ and k as before.

If N is the *total* revolutions made by the wheel and N_1 the number of revolutions made after the weight has reached the ground—

$$Wh = \frac{N}{N_1} \times 0{\cdot}00017wk^2n^2 + \frac{Wv^2}{2g}$$

which gives $0{\cdot}00017wk^2$ the kinetic energy at 1 revolution per minute.

Example 1.—A fly-wheel is carried on a spindle 3 inches diameter. A string is wrapped round the spindle to which one end is loosely attached. The other end of the string carries a weight which starting from rest pulls the fly-wheel round and falls 2·5 feet to the ground in 5 seconds. Find the speed of the weight and the speed of the wheel when the weight touches the ground.

$$\text{Average speed of falling weight} = \frac{2{\cdot}5}{5} = 0{\cdot}5 \text{ feet per second}$$

$$\text{Speed of weight on reaching ground} = 2 \times 0{\cdot}5 = 1 \text{ foot per second.}$$

Revolutions made by the wheel while the weight descends

$$= \frac{2{\cdot}5}{\pi \frac{3}{12}} = \frac{10}{\pi}$$

$$\text{Average speed of wheel} = \frac{10}{\pi \times 5} = \frac{2}{\pi} \text{ revolutions per second}$$

$$\text{Speed when weight reaches the ground} = \frac{2}{\pi} \times 2 \text{ revolutions per second}$$

$$= \frac{4}{\pi} \times 60 = 76{\cdot}5 \text{ revs. per min.}$$

Example 2.—A fly-wheel weighing 200 lbs. is carried on a spindle 2·5 inches diameter. A string is wrapped round the spindle to which one end is loosely attached. The other end of the string carries a weight of 40 lbs., 4 lbs. of which is necessary to overcome the friction (assumed constant) between the spindle and its bearings. Starting from rest, the weight, pulling the fly-wheel round, falls vertically through 3 feet in 7 seconds. Find the energy stored in the wheel when it makes 100 revolutions per minute, and the radius of gyration of the wheel.

Average speed of the falling weight $= \frac{3}{7}$ feet per second, the maximum velocity is $2 \times \frac{3}{7}$ or $\frac{6}{7}$ foot per second.

The net work done by the falling weight, *i.e.* the whole work done minus that spent in overcoming friction, is—

$$(40 - 4) \times 3 = 108 \text{ foot-pounds.}$$

The kinetic energy of the weight after falling 3 feet is—

$$\tfrac{1}{2} \times \frac{40}{32{\cdot}2} \times \left(\tfrac{6}{7}\right)^2 = 0{\cdot}456 \text{ foot-pound.}$$

Revolutions made by wheel during descent of weight

$$= \frac{3}{\frac{2{\cdot}5\pi}{12}} = \frac{36}{2{\cdot}5\pi}$$

$$\text{Average speed of wheel} = \frac{36}{2{\cdot}5\pi \times 7}$$

$$\text{Maximum speed} = \frac{36}{2{\cdot}5\pi \times 7} \times 2 = \frac{72}{17{\cdot}5\pi} \text{ revolutions per second}$$

$$= \frac{72 \times 60}{17{\cdot}5\pi} = 78{\cdot}5 \text{ revolutions per minute}$$

Hence—

$$\text{K.E. of wheel} + 0{\cdot}456 = 108$$

$$0{\cdot}00017wk^2n^2 = 108 - 0{\cdot}456 = 107{\cdot}544 \text{ ft.-lbs.}$$

$$\text{K.E. at 1 revolution per minute } 0{\cdot}00017wk^2 = \frac{107{\cdot}544}{n^2}$$

$$= \frac{107{\cdot}544}{78{\cdot}5 \times 78{\cdot}5} = 0{\cdot}0174 \text{ ft.-lb.}$$

$$\text{K.E. at 100 revolutions per minute} = 0{\cdot}0174 \times 100^2$$

$$= 174 \text{ foot-pounds}$$

$$\text{also since } 0{\cdot}00017wk^2 = 0{\cdot}0174$$

$$k^2 = \frac{0{\cdot}0174}{0{\cdot}00017w} = \frac{0{\cdot}0174}{0{\cdot}00017 \times 200}$$

$$\therefore k = \sqrt{\frac{0{\cdot}0174}{0{\cdot}00017 \times 200}}$$

$$= 0{\cdot}716 \text{ foot or } 8{\cdot}6 \text{ inches.}$$

Example 3.—An engine in starting exerts on the crank-shaft for one minute a constant turning moment of 1000 pound-feet, and there is a uniform resisting moment of 800 pound-feet. The fly-wheel has a radius of gyration of 5 feet and weighs 2000 lbs. Neglecting the inertia of all parts except the fly-wheel, what speed will the engine attain after one minute from starting?

Let ω = angular velocity required in radians per sec., then

$\frac{\omega}{2}$ = average angular velocity.

$$\text{Total angle turned through in one minute} = 60 \times \frac{\omega}{2} = 30\omega \text{ radians}$$

$$\text{Net work done in one minute} = (1000 - 800)30\omega$$

$$= 200 \times 30\omega = 6000\omega \text{ ft.-lbs.}$$

$$\text{Velocity at radius of gyration} = \text{angular velocity} \times \text{radius}$$

$$= \omega \times 5 = 5\omega \text{ feet per second}$$

$$\text{K.E. of wheel} = \frac{Wv^2}{2g} = \frac{2000 \times (5\omega)^2}{2 \times 32{\cdot}2} = 6000\omega$$

$$\frac{2000 \times 25\omega}{64{\cdot}4} = 6000$$

$$\omega = \frac{6000 \times 64{\cdot}4}{2000 \times 25} = 7{\cdot}74 \text{ radians per second}$$

$$= \frac{7{\cdot}74 \times 60}{2\pi} = 74 \text{ revolutions per minute}$$

EXAMPLES XXII.

1. A steam plant uses 2·5 lbs. of coal per horse-power per hour. What proportion of the available energy in the coal is converted into mechanical work? Take the number of heat units in 1 lb. of the coal as 13,000 B.Th.U.

2. An oil engine converts 18 per cent. of the available energy in the oil into mechanical work, how many pounds of oil will be required per hour for an engine of 50 H.P. (1 lb. of oil contains 20,000 B.Th.U.)?

3. An electric tramcar weighs 5 tons, and is driven up an incline of 1 in 50 at a steady speed of 12 miles an hour. If the frictional resistance is constant and equal to 20 lbs. per ton, and the efficiency of the motors is 85 per cent., what E. H.P. is supplied to the motors? How many Board of Trade units will be consumed per hour, and if the voltage of supply is 500 volts, what current in ampères is taken by the motors?

4. If 1 lb. of coal contains 14,500 B.Th.U., how many pounds of coal contain the same energy as 100 Board of Trade units of electric supply?

5. Calculate the kinetic energy of a rifle bullet weighing 1 ounce when moving at a speed of 1000 feet per second.

6. A tramcar weighs 3 tons, and is running at 15 miles an hour. What is its K.E. in foot-pounds? If the frictional resistance is constant and equal to 20 lbs. per ton, how far will it run up an incline of 1 in 100 before coming to rest?

7. A body when moving at 88 feet per second has 8000 foot-pounds of kinetic energy. How much kinetic energy will it have lost when its velocity has fallen to 15 feet per second?

8. A projectile has 2,300,000 foot-pounds of kinetic energy at a velocity of 2000 feet per second. What will be its velocity when its kinetic energy has been reduced to 340,000 foot-pounds?

9. A hammer of weight 2 lbs. strikes a nail of weight 1 ounce with a velocity of 34 feet per second, and drives it 1 inch into a fixed block of wood; find the average resistance offered by the wood. If the block be perfectly free to move and weighs 67 lbs., how far would the nail be driven?

10. The hammer of a pile driver weighs 5 cwts.; if it falls 5 feet on to the head of a pile weighing 1 ton, and drives it 3 inches into the ground, what is the average resistance of the ground, and how much energy is lost in the impact?

11. A train weighing 200 tons has a frictional resistance of 12 lbs. per ton. An average pull of 7000 lbs. is exerted on it. Starting from rest, what will be its velocity in miles per hour at the end of 1 minute?

12. The balls of a fly-press weigh 40 lbs., and are moving at 30 feet per second. What force will be exerted if the punch travels a distance of ½ inch?

13. The motion of a body of weight 6400 lbs. is opposed by a constant frictional resistance of 3400 lbs. It starts from rest under the action of a varying force F pounds, whose value is here given at the instants at which the body has passed x feet from rest—

F . .	7100	7000	6900	6600	6500	6400	6500	6300	6200
x . .	0	5	10	15	20	25	30	35	40

What is the speed of the body after it has moved 40 feet, and also 20 feet from rest?

14. The diameter of the thin rim of a fly-wheel is 6 feet, and it weighs 4 tons; find its kinetic energy when rotating at 240 revolutions per minute.

15. A fly-wheel at rest in frictionless ball bearings has a torque of 500 pound-feet applied to it while it makes 8 revolutions. If the wheel weighs 1 ton, what is its linear speed at the radius of gyration? If the radius of gyration is 3 feet, what is the speed of the wheel in revolutions per minute?

16. A fly-wheel weighs 10,000 lbs., and has a radius of gyration of 3·5 feet. It is running at 120 revolutions per minute, and makes 500 revolutions in coming to rest. How much work in foot-pounds is absorbed by friction per revolution?

17. A fly-wheel weighs 5 tons, and has a radius of gyration of 4 feet. How many foot-pounds of energy would be stored in it when it is making 60 revolutions per minute? If it is supported in bearings 7 inches diameter, and the coefficient of friction is 0·01 at all speeds, how much work would be spent in friction in 1 revolution, and how many revolutions would the wheel make before coming to rest?

18. A fly-wheel making 90 revolutions per minute gives out 27,000 foot-pounds of energy while the speed falls to 85 revolutions per minute. If the radius of gyration of the wheel is 5 feet, what should be its weight?

19. The fly-wheel of a shearing machine has 180,000 foot-pounds of kinetic energy stored in it when its speed is 240 revolutions per minute. What energy does it part with during a reduction of speed to 200 revolutions per minute? If 80 per cent. of this energy given out is imparted to the shears during a stroke of $2\frac{1}{2}$ inches, what is the average force due to this on the blade of the shears?

20. A machine stores 10,050 foot-pounds of kinetic energy when the speed of its driving pulley rises from 100 to 101 revolutions per minute. How much kinetic energy would it have stored in it when its driving pulley is making 100 revolutions per minute?

21. A fly-wheel weighing 212 lbs. is carried on a spindle 2 inches diameter. A string is wrapped round the spindle to which one end is loosely attached. The other end of the string carries a weight of 60 lbs. of which 8 lbs. are required to overcome the friction (assumed constant) of the bearings. Starting from rest, the weight, pulling the fly-wheel round, falls vertically through 3 feet in 5 seconds. Find the energy stored in the wheel when it makes 100 revolutions per minute, and its radius of gyration.

22. A fly-wheel of weight 200 lbs. is carried on a spindle 2 inches diameter. A string is wrapped round the spindle as in Question 21 and carries a weight of 50 lbs. on the free end. Starting from rest the weight falls vertically through 3 feet in 6 seconds, and from the time of starting until the wheel stops again it makes 210 revolutions. Find the K.E. when making 100 revolutions per minute, and also the radius of gyration of the wheel.

23. An engine in starting exerts on the crankshaft for one minute a constant turning moment of 1500 pound-feet, and there is a uniform resisting moment of 900 pound-feet. The fly-wheel has a radius of gyration of 4 feet, and weighs 4000 lbs. Neglecting the inertia of all parts except the fly-wheel, what speed will the engine attain after one minute from starting?

CHAPTER XXIII

CIRCULAR MOTIONS AND SIMPLE VIBRATIONS

IN Chap. XX. we saw that when a body moved in a curved path its acceleration was not in the direction of its motion but was inclined to it; also that the mean acceleration between two points on its path was to be found by dividing the change in velocity by the time taken, the change in velocity not being a mere arithmetic change in speed but the vector change found by drawing vectors to scale.

Motion in a Circle.—A very important kind of motion is that of a body which moves in a circular path at constant speed v feet per second, and we wish to find its acceleration. Let r be the radius in feet of the circle in which its c.g. moves. The time taken for a complete circuit will be—

$$\frac{\text{distance}}{\text{speed}} = \frac{2\pi r}{v} \text{ seconds.}$$

The time taken to travel through one radian of angle or r feet of arc will be $\frac{r}{v}$ seconds and the angular velocity ω will be—

$$\omega = \frac{v}{r} \text{ radians per second.}$$

Let us find the change of velocity between two points P and Q (Fig. 212), say, 60° apart round the circle. Draw the vector qr to represent the velocity v at Q on a scale of v feet per second = 3 inches; add to this the vector rp equal to the speed v at P but in the opposite direction (this is subtracting the velocity at P from that at Q); the vector resultant is qp (from q towards p) and qp represents the change of velocity between P and Q, or qp represents—

$$\text{velocity at Q} - \text{velocity at P.}$$

Now, measuring qp, it will be found to be also 3 inches long (pqr is an equilateral triangle) and represents a velocity of v feet per second. Hence the change of velocity between P and Q is v feet per second in a direction qp or SO. where S is midway between

P and Q. The time t taken to move from P to Q 60° apart is $\frac{1}{6}$ of the time taken for a complete circuit or—

$$t = \frac{1}{6} \times \frac{2\pi r}{v} = \frac{\pi r}{3v} \text{seconds}$$

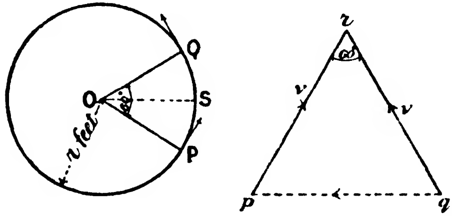


FIG. 212.

Hence the magnitude of the average acceleration or change of velocity per second is—

$$\frac{\text{change of velocity}}{\text{time}} = \frac{v}{t} = \frac{v}{\frac{\pi r}{3v}} = \frac{3}{\pi} \cdot \frac{v^2}{r} = 0{\cdot}956 \frac{v^2}{r} \text{ feet per sec. per sec.}$$

This is approximately the acceleration at S midway between P and Q, and it is in the direction *from S towards O*, that is, *towards* the centre of the circle.

Now find the average acceleration in the same way between points P′ and Q′ (Fig. 213) 30° apart. Taking v = say, 10 inches,

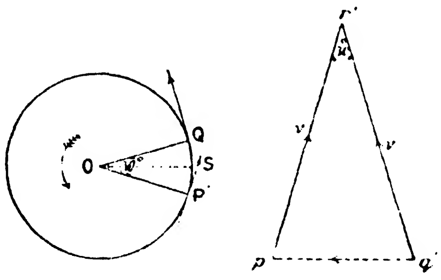


FIG. 213.

an actual drawing (which the reader should make for himself) shows $q'p'$ to be 5·17 inches representing 0·517v feet per second. The time taken is half as much as before or $\frac{1}{12} \times \frac{2\pi r}{v} = \frac{\pi}{6} \cdot \frac{r}{v}$ seconds.

Hence the mean acceleration or the approximate acceleration at S′ is—

$$\frac{0{\cdot}517v}{\frac{\pi}{6} \times \frac{r}{v}} = 0{\cdot}988\,\frac{v^2}{r} \text{ feet per second per second}$$

or rather more than in the previous case. If P and Q are taken closer together, the mean acceleration will be found slightly higher and approaching—

$$\frac{v^2}{r} \text{ feet per second per second}$$

the nearer together P and Q are taken.

The acceleraton may also be calculated, for in Fig. 213—

$$\frac{\frac{1}{2}p'q'}{q'r'} = \sin\frac{30^\circ}{2} = \sin 15^\circ$$

$$p'q' = 2q'r' \sin 15^\circ \text{ representing } 2v \sin 15^\circ$$

and from the tables (p. 367), $\sin 15^\circ = 0{\cdot}2588$. Hence—

$$p'q' \text{ represents } 2 \times v \times 0{\cdot}2588 = 0{\cdot}5176v$$

which when divided by the time $\frac{\pi}{6}\cdot\frac{r}{v}$ gives $0{\cdot}99\frac{v^2}{r}$.

The acceleration of a body moving in a circle with uniform speed is then at any point—

$$\frac{v^2}{r} \text{ feet per second per second}$$

or since $v = \omega r$ the acceleration may also be written—

$$\frac{(\omega r)^2}{r} = \omega^2 r \text{ feet per second per second.}$$

Example.—Find the acceleration of a point on the rim of a fly-wheel 8 feet diameter rotating at 100 revolutions per minute.

$$\omega = 100 \times \frac{2\pi}{60} = \frac{10\pi}{3} \text{ radians per second} = 10{\cdot}472 \text{ radians per second}$$

$$\text{Acceleration} = \omega^2 r$$

$$= \left(\frac{10\pi}{3}\right)^2 \times 4 = 10{\cdot}470 \times 10{\cdot}472 \times 4$$

$$= 437 \text{ feet per second per second.}$$

Centrifugal Force.—We have just seen that a body moving in a circle of radius r feet with constant speed v feet per second must have an acceleration $\frac{v^2}{r}$ feet per second per second towards the

centre of the circle. To give it this acceleration we have seen in Chap. XXI. that it must be acted upon by a force—

$$\text{mass} \times \text{acceleration} = \frac{W}{g} \times \frac{v^2}{r} \text{ or } \frac{Wv^2}{gr} \text{ or } \frac{W}{g}\omega^2 r \text{ pounds}$$

towards the centre of the circle when W is its weight in pounds and ω is its angular velocity in radians per second. This *inward* force acting on the body is called the centripetal force. The reader should notice that there is *no outward force on the body*. If a body on the end of a string is turned in a circle by the *inward* pull of the *string*, the body exerts an *outward* pull *on the string*, and the string pulls at the centre of the circle; this *outward* pull is called the *centrifugal force*. It is not a force on the body although *equal and opposite* to the inward centripetal force on the body.

The reader will readily appreciate the centrifugal force which a body exerts on whatever guides it in a circular path by whirling round by hand a stone or weight on the end of a string. If the stone becomes detached from the string it will not have acting on it the centripetal force to keep it moving in a circular path, and will travel along in the direction and at the speed which it had on becoming detached; this is the principle of a sling.

In calculating the centrifugal force—

$$\frac{W}{g} \cdot \frac{v^2}{r}$$

for a body of any shape it is important to notice that this speed v and radius r should be taken as the values at the centre of gravity of the body.

Example 1.—A block of cast iron 4 inches by 3 inches by 4 inches is fastened to the arm of a wheel with its centre of gravity 3 feet from the axis. The wheel makes 600 revolutions per minute. What is the force tending to fracture the fastening? (One cubic inch of cast iron weighs 0·26 lb.)

$$\begin{aligned} \text{The weight W of the block} &= \text{volume in cubic inches} \times 0{\cdot}26 \text{ lb.} \\ &= 4 \times 3 \times 4 \times 0{\cdot}26 \\ &= 12{\cdot}48 \text{ lbs.} \end{aligned}$$

$$\text{Force on fastening} = \text{centrifugal force} = \frac{Wv^2}{gr}$$

$$\text{and } v = 2\pi \times 3 \times \tfrac{600}{60} = 60\pi = 188{\cdot}5 \text{ feet per second}$$

$$\text{hence } \frac{Wv^2}{gr} = \frac{12{\cdot}48 \times 188{\cdot}5 \times 188{\cdot}5}{32{\cdot}2 \times 3}$$
$$= 4592 \text{ lbs.}$$

Example 2.—A pulley is built up in two halves. Each half weighs 50 lbs., and its centre of gravity is 1·5 feet from the centre of the wheel. The two halves are held together by two bolts whose centres are 1·5

feet from the axis of the wheel. Find the pull in the bolts when the pulley makes 360 revolutions per minute.

Angular velocity $\omega = 2\pi \times \frac{360}{60} = 12\pi = 37{\cdot}7$ radians per second

$$\text{Centrifugal force} = \frac{W}{g}\omega^2 r$$

$$= \frac{50}{32{\cdot}2} \times 37{\cdot}7 \times 37{\cdot}7 \times 1{\cdot}5 = 3310 \text{ lbs.}$$

and this is the pull in the bolts.

Example 3.—A pulley weighs 5 cwt. and its centre of gravity is $\frac{1}{4}$ inch from the centre of rotation. Find the pull on the shaft at 500 revolutions per minute.

$$\text{The radius} = \tfrac{1}{4} \times \tfrac{1}{12} = \tfrac{1}{48} \text{ foot,}$$

and $$\omega = \frac{2\pi \times 500}{60} = \frac{100\pi}{6} \text{ radians per second}$$

$$\text{Pull on shaft} = \frac{5 \times 112}{32{\cdot}2} \times \left(\frac{100\pi}{6}\right)^2 \times \tfrac{1}{48}$$
$$= 993 \text{ lbs.}$$

Balancing of Machines.—If any pulley or other body weighing W pounds attached to a rotating shaft has not its centre of gravity in the axis of rotation, it will exert on the shaft a centrifugal force—

$$\frac{W}{g} \cdot \frac{v^2}{r} \text{ or } \frac{W}{g}\omega^2 r \text{ pounds}$$

where r is the distance of the centre of gravity from the axis *in feet*, and v is the speed *in feet per second* of a point at this distance from the axis of rotation, or ω is the angular velocity of the shaft in radians per second. Such a rotating body is said to be *out of balance*, and may cause troublesome vibrations. It may be *balanced* by adding other weights called balance weights, which will exert a centrifugal force equal and opposite to that of the unbalanced body.

Example 1.—A body weighing 1 cwt. is bolted to the face plate of a lathe with its centre of gravity 18 inches from the axis of rotation. What weight placed diametrically opposite at a radius of 15 inches will give perfect balance at a speed of 60 revolutions per minute?

$$\text{Centrifugal force of weight out of balance} = \frac{W}{g}\omega^2 r$$

Here $\omega = \frac{60}{60} \times 2\pi = 2\pi$ radians per second, and $r = \frac{18}{12} = 1{\cdot}5$ feet; hence—

$$\frac{W}{g}\omega^2 r = \frac{112}{32{\cdot}2} \times (2\pi)^2 \times 1{\cdot}5 = 205{\cdot}9 \text{ lbs.}$$

Now, the centrifugal force of the balance weight must be equal to 205·9 lbs.; hence if W_1 is the weight required—

$$\frac{W_1}{g}\omega^2 r = 205{\cdot}9$$

$$\frac{W_1}{32{\cdot}2} \times (2\pi)^2 \times \frac{15}{12} = 205{\cdot}9$$

$$W_1 = \frac{205{\cdot}9 \times 32{\cdot}2 \times 12}{(2\pi)^2 \times 15} = 134{\cdot}4 \text{ lbs.}$$

NOTE.—Although in the above solution the centrifugal force has been actually calculated, it is really not necessary. Let W be the out of balance weight at a radius r feet, W_1 the balance weight required at a radius r_1 feet; ω the angular velocity, then since the centrifugal forces must be equal and opposite, we have—

$$\frac{W}{g}\,\omega^2 r_1 = \frac{W_1}{g}\,\omega^2 r_2$$

or $Wr_1 = W_1 r_2$ which might also be obtained by taking moments about the axis of the weights taken perpendicular to the axis

and $W_1 = W \times \frac{r_1}{r_2}$, being independent of the speed

Applying this to the above example, we have—

$$W_1 = 112 \times \frac{18}{15} = 134{\cdot}4 \text{ lbs. as before.}$$

Example 2.—A fly-wheel weighs 10 tons and is out of balance. It is balanced by placing a weight of 200 lbs. at 50 inches from the centre. How far was the c.g. of the fly-wheel from the axis?

Let x be the distance in inches.

Taking moments about the axis we have—

$$10 \times 2240 \times x = 200 \times 50$$

$$x = \frac{200 \times 50}{10 \times 2240} = 0{\cdot}446 \text{ inch.}$$

Conical Pendulum and Centrifugal Governor.—If a weight at the end of a string is whirling in a horizontal circle the loaded end will lie below the fixed end of the string, and the string will move over a conical surface. If r is the radius of the circle in feet, W the weight of the body in pounds, and v its constant speed in feet per second, the two forces keeping the body moving in its circular path are (Fig. 214)—

(1) the pull of the string T pounds
(2) the weight of the body W pounds.

The resultant of these must be a force $\frac{W}{g}\cdot\frac{v^2}{r}$ pounds towards the

centre of the circle O. This resultant is shown by *ac* in the triangle of forces *abc* (Fig. 214), where—

$$ab + bc = ac$$

and comparing the triangle of forces with the triangle ABO showing the string, it is evident that—

$$\frac{ab}{bc} \text{ or } \frac{T}{W} = \frac{AB}{BO} \text{ or } \frac{l}{h}$$

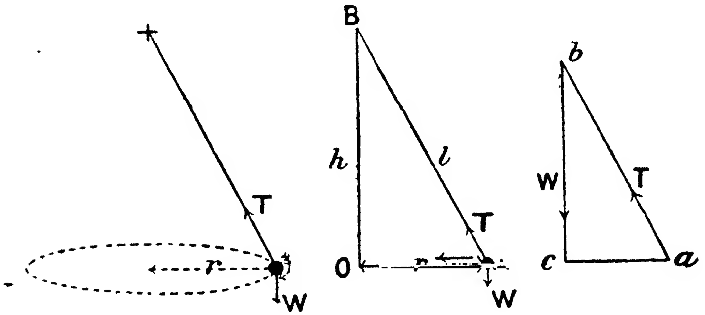


FIG. 214.—Conical pendulum.

and— $$\frac{bc}{ac} \text{ or } \frac{W}{\frac{Wv^2}{gr}} = \frac{BO}{AO} \text{ or } \frac{h}{r}$$

or dividing by W, $$\frac{gr}{v^2} = \frac{h}{r}$$

hence— $$h = \frac{gr^2}{v^2}$$

or since the angular velocity $\omega = \frac{v}{r}$

$$h = \frac{g}{\omega^2}$$

The height is inversely proportional to the square of the angular velocity or to the square of the revolutions per minute. This is the principle of the centrifugal governor, in which the rising of a rotating ball (decrease of *h*) with increase of speed is made to control the speed of an engine by operating levers to check the supply of steam, gas, or oil as the case may be.

Example 1.—Find the height of a governor of the above type when running at 120 revolutions per minute.

$$\omega = 2\pi \times \frac{120}{60} = 4\pi = 12{\cdot}56 \text{ radians per second}$$

$$h = \frac{g}{\omega^2} = \frac{32{\cdot}2}{12{\cdot}56 \times 12{\cdot}56} = 0{\cdot}2036 \text{ foot} = 2{\cdot}44 \text{ inches.}$$

Example 2.—What must be the increase in speed of the above governor in order for h to decrease $\frac{1}{4}$ inch?

Here $h = 2{\cdot}44 - 0{\cdot}25 = 2{\cdot}19$ inches $= 0{\cdot}1825$ foot.

$$h = \frac{g}{\omega^2}$$

$$\omega = \sqrt{\frac{g}{h}} = \sqrt{\frac{32{\cdot}2}{0{\cdot}1825}} = 14{\cdot}9 \text{ radians per second}$$

$$= \frac{14{\cdot}9 \times 60}{2\pi}$$

$$= 142{\cdot}2 \text{ revolutions per minute,}$$

hence the speed must increase $142{\cdot}2 - 120 = 22{\cdot}2$ revolutions per minute.

Reciprocating Motion.—There is one kind of reciprocating or to and fro motion which is very common, specially simple and very important. It may be described as follows:—

Suppose a point Q (Fig. 215) moves in a circular path of radius r feet with constant speed v feet per second, and another point P is always at the foot of the perpendicular PQ on the diameter AB. The motion of P is then called simple reciprocating motion, simple vibration, or simple harmonic motion. While Q makes one complete circuit P moves along the diameter from B to A and back again to B; this time is called the *period* of a complete vibration t seconds.

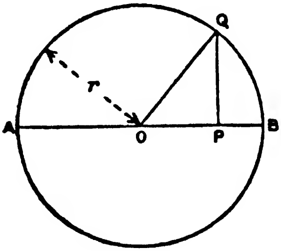


FIG. 215.

$$t = \frac{\text{circumference}}{v} = \frac{2\pi r}{v}$$

$$\text{or if } \frac{v}{r} = \omega \text{ the angular velocity of Q}$$

$$t = \frac{2\pi}{\omega}$$

The radius of the circle which is half the travel or stroke of the

motion is called the *amplitude*. We have seen that the acceleration of Q moving in the circle would be $\frac{v^2}{r}$ along QO. If we represent this acceleration by the length QO as a vector, QO is equal to the sum of two vectors QP + PO, and the vector PO represents the horizontal acceleration of Q, which is also the acceleration of **P**, hence the acceleration of P is—

$$\frac{v^2}{r} \times \frac{\text{OP}}{\text{OQ}} = \frac{v^2}{r^2} \times \text{OP}$$

$$\text{or } \omega^2 r \times \frac{\text{OP}}{\text{OQ}} = \omega^2 \times \text{OP in the direction P to O}$$

when OP = 1 (foot) the acceleration is $\frac{v^2}{r^2}$ or ω^2 feet per second per second, and the acceleration at any other displacement OP feet from O is OP times as much. Hence a body having a simple vibration or reciprocation has an acceleration proportional to its distance from the middle of its path and directed towards that point.

To give a body of weight W pounds this acceleration $\frac{v^2}{r^2} \times \text{OP}$ a force—

$$\text{mass} \times \text{acceleration} = \frac{W}{g}\frac{v^2}{r^2} \times \text{OP}$$

is necessary, and a body will have simple harmonic motion if the force acting upon it is proportional to its distance from the middle of its path.

Notice that the time of vibration—

$$t = \frac{2\pi}{\omega} \text{ or } 2\pi \div \frac{v}{r}$$

$$\text{and } \frac{v}{r} = \sqrt{\frac{v^2}{r^2}} = \sqrt{\text{acceleration per foot of displacement}}$$

so—

$$t = \frac{2\pi}{\sqrt{\text{acceleration per foot of displacement}}}$$

or since force = mass × acceleration,

$$t = 2\pi\sqrt{\frac{\text{mass}}{\text{force per foot of displacement}}}$$

$$= 2\pi\sqrt{\frac{W \div g}{\text{force per foot of displacement}}}$$

If there are n vibrations per second the time of one vibration—

$$t = \frac{1}{n} \qquad \text{or } n = \frac{1}{t}$$

Bodies attached to springs or elastic supports have a simple motion of vibration, and the reciprocating motion of a steam engine piston is nearly of the same kind unless the connecting rod is very short.

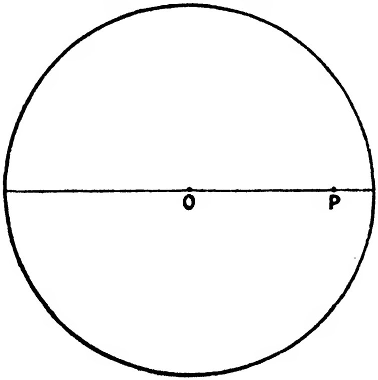


FIG. 216.

Example 1.—A crank 15 inches long is driven by a piston and makes 140 revolutions per minute. Find the acceleration of the piston when it is $3\frac{1}{2}$ inches from the end of its stroke, taking the motion as simple harmonic motion.

Fig. 216 shows the position of the piston P $3\frac{1}{2}$ inches from one end of the stroke. The distance OP is $15 - 3\frac{1}{2} = 11\frac{1}{2}$ inches and

$$\text{acceleration} = \frac{v^2}{r} \times \frac{\text{OP}}{r}.$$

Now $$v = 2\pi \times \frac{15}{12} \times \frac{140}{60} = 18{\cdot}33 \text{ feet per second}$$

and $$\frac{\text{OP}}{r} = \frac{11{\cdot}5}{15} \text{ and } r = \frac{15}{12} = 1{\cdot}25 \text{ feet}$$

$$\text{acceleration} = \frac{18{\cdot}33^2}{1{\cdot}25} \times \frac{11{\cdot}5}{15} = 206 \text{ feet per second per second.}$$

Example 2.—A weight of 6 lbs. is suspended on the end of a spring which stretches 1 inch for each 10 lbs. of load. Find the number of vibrations it will make per minute when disturbed from rest.

Force per foot of displacement = 12 × 10 = 120 lbs.

$$t = 2\pi\sqrt{\frac{\frac{W}{g}}{\text{force per foot of displacement}}}$$

$$= 2\pi\sqrt{\frac{\frac{6}{32{\cdot}2}}{120}} = 2\pi\sqrt{\frac{1}{32{\cdot}2 \times 20}} = 0{\cdot}2475 \text{ sec.}$$

$$\text{Vibrations per minute} = \frac{60}{0{\cdot}2475} = 242{\cdot}4$$

Experiment.—Hang a certain weight on the free end of a spring like the one shown in Fig. 154, and observe the time taken to make, say, 50 complete oscillations. One-fiftieth of this time will therefore be the time of one oscillation. Measure the force required to stretch the spring 1 foot as on p. 192, Chap. XV. Repeat the experiment for different weights hung on the spring, and compare the observed time of oscillation with the time calculated as above. The results obtained in a particular test are tabulated below. The force per foot of displacement was found to be 120 lbs., and it will be observed how closely the two values of the time of oscillation agree.

Load (pounds).	Observed time of one oscillation (seconds).	Calculated time (seconds). $t = 2\pi\sqrt{\dfrac{W \div g}{\text{force per foot of displacement}}}$
9	0·320	0·301
11	0·330	0·337
13	0·360	0·364
15	0·390	0·391
17	0·420	0·417
19	0·440	0·441
21	0·465	0·464
23	0·484	0·485
25	0·500	0·505
27	0·520	0·525

The Pendulum.—If a small weight is suspended from O

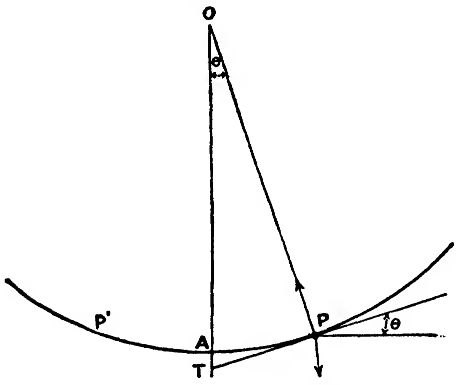


FIG. 217.—Simple pendulum.

(Fig. 217) by a long string OP, l feet long, and swings as a pendulum in a small arc PAP', its acceleration at P along PA is almost exactly

proportional to the distance AP from A. The forces acting on the small body are the pull of the string and its own weight, and we have seen that in travelling down a slope θ, the acceleration of gravity is—

$$g \times \sin\theta$$

Now if θ is small (for a long string and small movement it will be)

$$\frac{AP}{OP} \text{ or } \frac{AP}{l} = \sin\theta \text{ very nearly.}$$

Hence acceleration $= g \times \dfrac{AP}{l}$ approximately, at a distance AP which is $\dfrac{g}{l}$ per foot of displacement from A, and the time of vibration—

$$t = \frac{2\pi}{\sqrt{\text{acceleration per foot of displacement from A}}}$$

$$t = \frac{2\pi}{\sqrt{\frac{g}{l}}} \text{ or } 2\pi\sqrt{\frac{l}{g}}$$

Example.—Find the length of a simple pendulum to beat seconds. One beat is half a vibration, hence the time of vibration will be 2 seconds, and—

$$t = 2\pi\sqrt{\frac{l}{g}}, \qquad t = 4\pi^2\frac{l}{g}$$

or

$$l = \frac{t^2 g}{4\pi^2} = \frac{2^2 \times 32{\cdot}2}{4 \times \pi^2} = \frac{32{\cdot}2}{\pi^2} = 3{\cdot}26 \text{ feet.}$$

Experiment.—Determine the periods of vibration corresponding to at least three different lengths of a simple pendulum. To do this accurately the time to make 100 complete vibrations should be observed, the whole time taken being divided by 100 to obtain the time of one complete vibration (t). Then plot a curve connecting t^2 and l, and using the relation—

$$t = 2\pi\sqrt{\frac{l}{g}} \text{ or } t^2 = \frac{4\pi^2 l}{g}$$

determine the value of "g" in feet per second per second. The following results were obtained with three different lengths of pendulum :—

Length in feet (l).	Time of 100 vibrations (seconds).	t	t^2	$g = \frac{4\pi^2 l}{t^2}$
1	110	1·10	1·21	32·6
2	157	1·57	2·46	32·1
3	192	1·92	3·68	32·2

The reader should plot t^2 and l and the curve will be found to be a straight line, showing that the square of the period is directly proportional to the length of the pendulum.

EXAMPLES XXIII.

1. The stroke of a steam engine is 2 feet 6 inches, and it runs at 120 revolutions per minute. Find the acceleration of the crank pin.

2. A stone weighing 2 lbs. is whirled in a horizontal circle, making 60 revolutions per minute, at the end of a string 3 feet long. Find the pull in the string.

3. The out-of-balance weight on an engine crankshaft is 100 lbs. at a radius of 9 inches. What will be the outward pull on the bearings due to centrifugal force when the engine makes 300 revolutions per minute?

4. A steam turbine makes 20,000 revolutions per minute. Each blade weighs $\frac{1}{28}$ lb. and its c.g. is 15 inches from the axis of rotation. Calculate the centrifugal force due to each blade.

5. A fly-wheel weighs 10 tons and its c.g. is $\frac{1}{50}$ inch from the centre of the shaft. Find the pull on the shaft at 240 revolutions per minute. If the shaft is 6 inches diameter, and the coefficient of friction 0·01, how many H.P. will be wasted due to the wheel being out of balance?

6. A body weighing 56 lbs. is bolted to the face plate of a lathe with its c.g. 8 inches from the axis of rotation. What weight placed diametrically opposite at a radius of 10 inches will give perfect balance at a speed of 100 revolutions per minute?

7. What weight at a radius of 2 feet will be required to balance the fly-wheel of Question 5?

8. Calculate the height of a common unloaded governor when making 60 revolutions per minute. If the speed falls to 58 revolutions per minute, what will be the change in height?

9. Part of a machine has a simple reciprocating motion making 200 complete vibrations in one minute. Its stroke is 9 inches, find the acceleration when 3 inches from mid stroke.

10. The reciprocating parts of an engine of 2-feet stroke weigh 300 lbs. If the engine runs at 240 revolutions per minute, what is the accelerating force at the beginning of the stroke, the motion being simple harmonic?

11. A light helical spring is found to stretch 0·4 inch when an axial load of 4 lbs. is hung on it. How many vibrations per minute will this spring make when carrying a weight of 15 lbs.?

12. How many complete vibrations per minute will be made by a pendulum 3 feet long? $g = 32{\cdot}2$.

13. A pendulum makes 3000 beats per hour at the equator, and 3011 per hour near the pole. Compare the values of g at the two places.

CHAPTER XXIV

VARIOUS MOTIONS

Crank and Connecting Rod.—The crank and connecting rod is a simple link motion for converting a reciprocating movement into a rotary movement or *vice versa*. The crank OB (Fig. 218) rotates about the fixed centre O, and the end A of the connecting rod is constrained to move in guides backwards and forwards along the line AO. In the case of steam, gas or oil engines, the reciprocating motion of A is converted into the rotary motion of the crank OB;

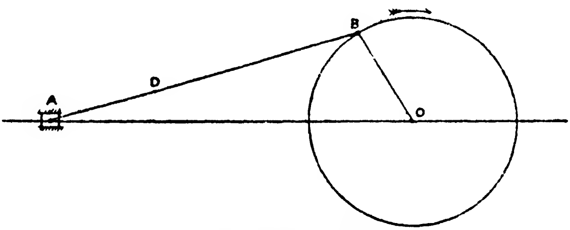


FIG. 218.—Crank and connecting rod.

in the shaping machine, and several forms of pumps, the crank OB derives its rotary motion from a source of power, and drives A through the connecting rod AB, and so produces the reciprocating motion of A.

The end A of the connecting rod reciprocates along AO, while the other end B is constrained to move in a circle of radius OB known as the crank pin circle. The path traced out in space of any other point such as D in the connecting rod may easily be drawn as follows—

Suppose the length of the connecting rod is three times the length of the crank, the crank being, say, 1 foot long and the point D is 1 foot from the end A. Draw in the crank pin circle to a scale of, say, 1 inch to 6 inches. Divide it into 12 equal parts as shown in Fig. 219, and draw in position the connecting rod for

each position of the crank, marking the point D in each case 2 inches from the end A. Join the points so obtained and we have the oval shown. This is known as the *locus* of the point D.

Curve of Piston Displacement.—The end A of the connecting rod is attached to the piston and reciprocates with it, hence

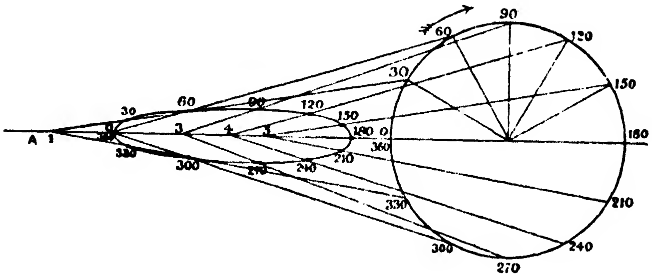


FIG. 219.

if we have the position of A for each of the above positions of the crank, we also have the position of the piston from the end of the stroke. For instance, when the crank pin B is at 0 (Fig. 219), the piston is at the end of its stroke at A, when the crank has turned through 30° the piston is at 1, when the crank angle is 90° the piston is at 3, and so on. If, then, we draw a base line CD (Fig. 220), to

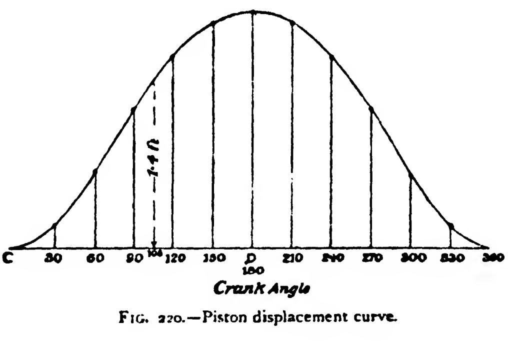


FIG. 220.—Piston displacement curve.

represent the stroke of 2 feet and divide it into 6 equal parts corresponding to the crank angles of 30°, 60°, 90°, 120°, 150°, we can draw the curve of piston displacement on a crank angle base. Erect perpendiculars from these points and mark off on them the distance the piston is from the end of the stroke, *i.e.* on the perpendicular for

$30°$ make the distance equal to length A 1 in Fig. 219, on that for $60°$ mark off the distance A2, on that for $90°$, mark off the distance A3, and so on, join these points by a smooth curve and we have the curve shown in Fig. 220, whose ordinates represent the displacement of the piston from the end of its stroke for any crank angle from $0°$ to $180°$, *i.e.* half a revolution. Repeat the construction for the other half of the revolution from $180°$ to $360°$. If, now, we wish to find the position of the piston when the crank has turned through $105°$ we measure the ordinate at $105°$; it will be found to be 1·4 feet from the end A of the stroke.

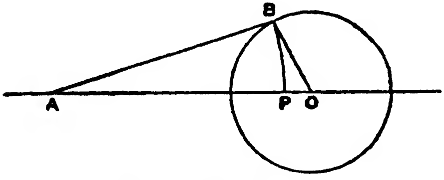


FIG. 221.—Piston position.

The most usual way of drawing the curve of piston displacement is to plot the displacements from *mid stroke* as ordinates. Dividing the crank pin circle into equal divisions of $30°$ we proceed as follows: Draw in the crank and connecting rod for, say, a crank angle of $60°$ (Fig 221). With centre A and radius AB (the length of the connecting rod) draw the arc BP to cut the line of stroke AO in point P. Then when the crank is at B the piston is OP from the middle of its stroke. Repeat this construction for each of the crank angles from $0°$ to $360°$. Then on a crank angle base plot the curve shown in Fig. 222, whose ordinates represent the

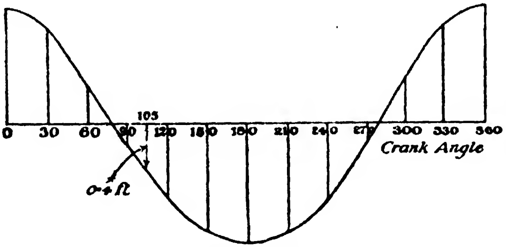


FIG. 222.—Piston displacement from mid-stroke.

distance of the piston from mid-stroke. At $105°$ this distance will be found to be 0·4 foot or 1·4 feet from the left-hand end of the stroke as above.

Curve of Piston Velocity.—Suppose the above engine runs at a speed of 100 revolutions per minute, then—

$$\text{revolutions per second} = \frac{100}{60}$$
$$\text{angle turned through per second} = \frac{100}{60} \times 360^\circ = 600^\circ$$

and to turn through an angle of 30° requires $\frac{1}{20}$ second, or 15° in $\frac{1}{40}$ second. From the curve (Fig. 222) we find that while the crank moves 15°, from 30° to 45°, the piston moves 0·206 foot, hence the average velocity during this time is—

$$\frac{0\cdot206}{\frac{1}{40}} = 8\cdot24 \text{ feet per second at } 37\tfrac{1}{2}^\circ, \text{ the middle of the interval.}$$

$$\text{The velocity of the crank pin} = \frac{\text{circumference of circle}}{\text{time}}$$
$$= \frac{2\pi \times 100}{60} = 10\cdot47 \text{ feet per second}$$

and the average velocity ratio of piston to crank pin from 30° to 45° is—

$$\frac{8\cdot24}{10\cdot47} = 0\cdot787$$

When the crank turns from 75° to 90° the piston moves 0·274 foot, and the average velocity of the piston is—

$$\frac{0\cdot274}{\frac{1}{40}} = 10\cdot96 \text{ feet per second (at } 82\tfrac{1}{2}^\circ)$$

and the average velocity ratio of piston to crank pin is—

$$\frac{10\cdot96}{10\cdot47} = 1\cdot04$$

When the crank turns from 120° to 135° the piston moves 0·166 foot, and its average velocity is—

$$\frac{0\cdot166}{\frac{1}{40}} = 6\cdot64 \text{ feet per second (at } 127\tfrac{1}{2}^\circ)$$

and the average velocity ratio of piston to crank pin is—

$$\frac{6\cdot64}{10\cdot47} = 0\cdot635$$

Proceeding in this way for equal intervals of 15° all round the crank pin circle the reader should plot for himself the piston velocity curve on a base of crank angles.

The average velocity of the piston during a whole stroke, called in practice the piston speed of the engine, will be—

$$\frac{\text{length of stroke}}{\text{time of stroke}}$$

There are two strokes every revolution, hence the time per stroke is—

$$\frac{60}{100 \times 2} = 0{\cdot}3 \text{ second}$$

$$\text{hence piston speed} = \frac{2}{0{\cdot}3} = 6{\cdot}66 \text{ feet per second}$$

$$\text{or } 6{\cdot}66 \times 60 = 400 \text{ feet per minute.}$$

Quick Return Motion.—Fig. 223 shows a quick return motion known as the *crank and slotted lever.* The crank OC rotates with uniform speed, and the block C sliding in the slotted lever gives

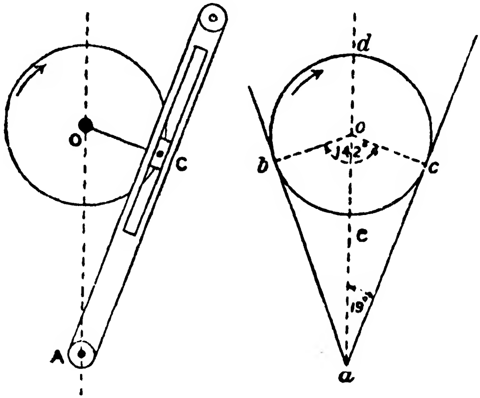


FIG. 223.—Quick return motion.

to it an oscillating motion about the fixed centre A. A connecting rod attached to the end of the slotted lever drives the ram of, say, a shaping machine. The two extreme positions of the lever are *ab* and *ac*, and the ram (carrying the cutting tool) of the machine makes a cutting stroke whilst the crank C travels round the arc *bdc*. The return stroke of the ram is made whilst the crank turns through the smaller arc *ceb*, and since the angular velocity of the crank is uniform it will take longer to move through *bdc* than *ceb*. The return stroke therefore will be made in less time than the cutting stroke.

Fig. 223 is drawn for a mechanism in which the crank OC is 2 inches long, and the distance OA between the centre of the crank shaft and fulcrum of the oscillating lever is 6 inches. The angle *boc* measures 142°, therefore during the cutting stroke along the arc

ide the crank turns through 360 − 142 = 218, and during the return stroke it turns through 142. Suppose the crank makes 60 revolutions per minute, then it makes 1 revolution in 1 second.

$$\text{Fraction of a revolution during cutting stroke} = \tfrac{218}{360}$$
$$\text{and time of cutting stroke} = \tfrac{218}{360} \times 1 = 0{\cdot}61 \text{ second.}$$
$$\text{Fraction of a revolution during return stroke} = \tfrac{142}{360}$$
$$\text{and time of return stroke} = \tfrac{142}{360} = 0{\cdot}39 \text{ second,}$$

or, in other words, the average speed of the ram during the return stroke is $\frac{218}{142}$, or 1·57 times the speed during the cutting stroke, or the time of cutting is to the time of return as 1·57 is to 1.

By drawing in the position of the crank and lever for, say, every 30° displacement of the crank from mid position, we can measure the angular displacement of the lever from its mid position as in Fig. 224. These values are tabulated below.

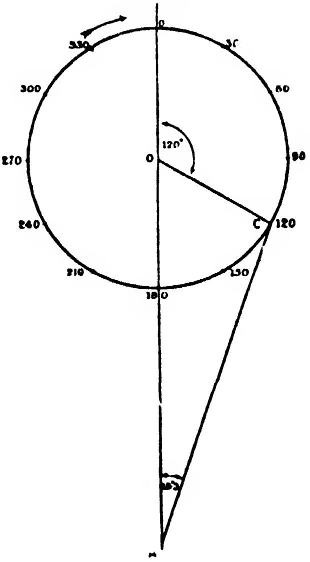


FIG. 224.

The curve of angular displacement of lever is shown plotted on a crank angle base in Fig. 225.

Since the crank makes 60 revolutions per minute, or turns 360° in 1 second, we can find approximately the angular velocity of the lever when in mid stroke for both cutting and return. During the time the crank OC, Fig. 224, turns from 10° on one side to 10° on the other side of the mid-stroke position, *i.e.* from 350° to 10°, it will be found that the lever swings about A through 2·25° on each side of its mid position, or at the mid position during the cutting stroke the crank turns 20° while the lever turns 4·5°.

Angular displacement of crank	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
Angular displacement of lever	0	7·5	13·8	18·2	19·0	12·8	0	12·8	19·0	18·2	13·8	7·5	0

$$\text{Now, time for crank to turn } 20° = \tfrac{20}{360} = \tfrac{1}{18} \text{ second;}$$

hence, angular velocity of lever in mid position $= 4 \cdot 5 \times 18^\circ$ per second

$$= 4 \cdot 5 \times 18 \times \frac{\pi}{180} = 1 \cdot 41 \text{ radians per second approximately.}$$

Also in the return stroke during the time the crank turns from 170° to 190° in Fig. 224, it will be found that the lever swings through 9·6° about A;

Hence, during the return stroke the angular velocity of the lever when in its mid position is about

$$9 \cdot 6 \times 18^\circ \text{ per second}$$
$$= 9 \cdot 6 \times 18 \times \frac{\pi}{180}$$
$$= 3 \cdot 01 \text{ radians per second}$$

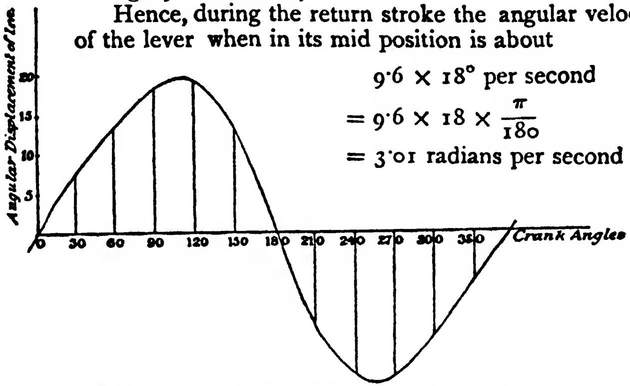


FIG. 225.—Curve of angular displacements of slotted lever.

Reversing and Quick Return Motion by Belting.—In Chap X., Fig. 98, the ordinary reversing motion by belting has been described. When a quick return is desired, the belt which gives the return motion works on a smaller pulley than that which gives the forward motion, as will be seen from Fig. 226. The machine shaft has four pulleys on it, the pulley C and D riding loosely on the shaft. When driving forward the belts are as shown in Fig. 226, the open belt being on the fixed pulley B and drives the machine shaft in a clockwise direction. The crossed belt runs on the loose pulley D. For the return stroke the open belt is pushed on to the loose pulley C by the fork K, and the crossed belt is pushed on to the pulley E by the fork H. The pulley E is keyed to the machine shaft, and therefore the machine is driven in a contra-clockwise direction at a higher speed.

Toggle Joint.—This joint has already been described in Chap. III., Fig. 48. It is reproduced diagrammatically in Fig. 227. Suppose the point A is moving in the direction of the arrow with uniform speed. Draw the mechanism in different positions so that

for each position of A we have the corresponding positions of B, taking care that the different positions of A are equal distances apart. In Fig. 227 motion commences with the links inclined 30° to the vertical, the links being each 4 inches long. The positions of A are taken such that the inclination of the links decreases 5° each time, being numbered 1, 2, 3, etc., the corresponding positions of B having the same number. Fig. 228 shows the time displacement curve of A which is a straight line, the velocity being uniform. The other line shows the time-displacement of B, the equal intervals of time between successive positions being taken as $\frac{1}{10}$

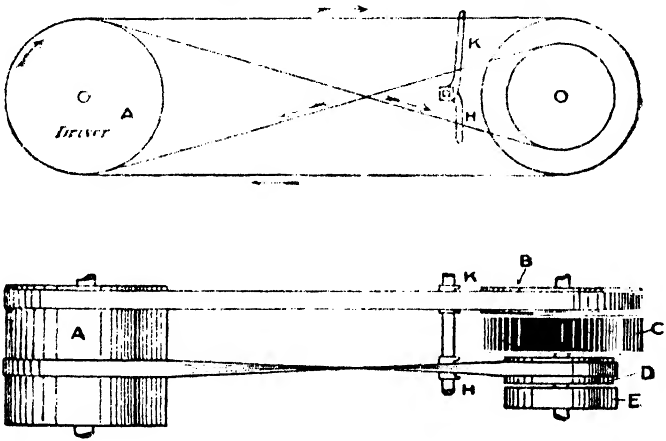


FIG. 226.—Reversing and quick return by belting.

second. It should be noticed that the movement of B during each interval of $\frac{1}{10}$ second becomes less and less as the inclination of the links to the vertical decreases, or, in other words, if the velocity of A is uniform, the velocity of B keeps decreasing as the inclination of the links to the vertical decreases, and the force exerted at B increases (see Fig. 49).

Watt's Straight Line Motion.—In its simplest form this motion consists of two links, AB and DC (Fig. 229), of equal length, connected at B and C by another link in the middle of which a point P will move over an approximately straight line for a limited movement of AB and DC. The levers AB and DC turn about fixed centres at A and D respectively, and the reader should draw in the mechanism for all possible positions of the links, and plot the locus of the point P. It will be found to be a figure of *eight*, as in

Fig. 229. When used in practice the motion has only a small movement of the levers AB and DC, so that the point P traces out an approximate straight line.

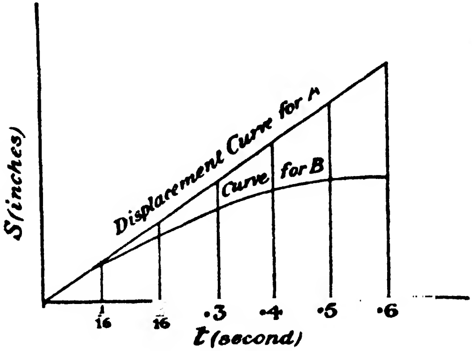


FIG. 228.

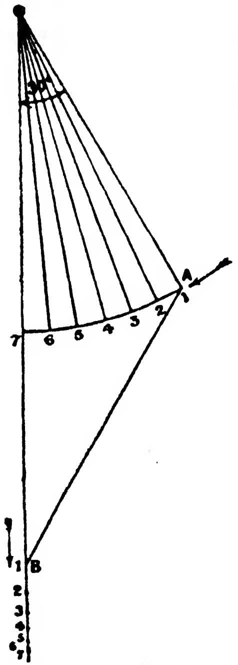


FIG. 227.—Position diagram for toggle joint.

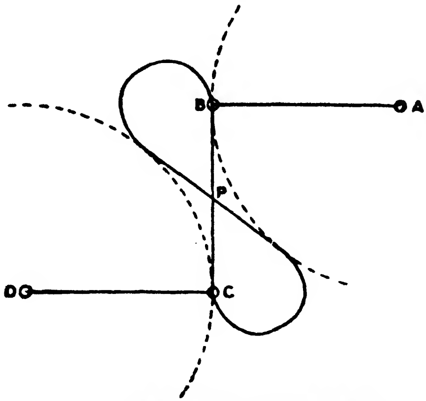


FIG. 229.—Watt's straight-line motion.

If the links AB and DC are unequal, the point P is so chosen that—

$$\frac{PC}{PB} = \frac{AB}{DC}$$

in which case P moves in the best straight line for a limited movement of AB and DC.

Ratchets.—A continuous motion is transmitted from one shaft to another by gear wheels as shown in Chap. X. If an intermittent motion is required, ratchet wheels may be used. The ratchet wheel A shown in Fig. 230 is keyed to a spindle B, and is provided with saw-shaped teeth; the pawl C is pivoted at the end of a lever D, which rides loosely on the spindle B or on another which is parallel to the spindle B. The lever D has a vibrating motion imparted to it, and when it vibrates from, say, right to left, the pawl pushes the ratchet wheel through a certain angle in a contra-clockwise direction. The pawl is kept in contact with the ratchet wheel either by its own weight or by springs. On the return of the lever D from left to right, the pawl slides over the teeth of the wheel, the wheel

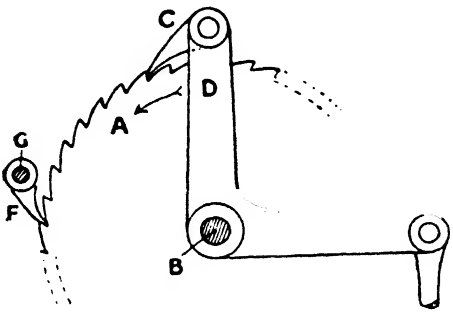


FIG. 230.—Ratchet.

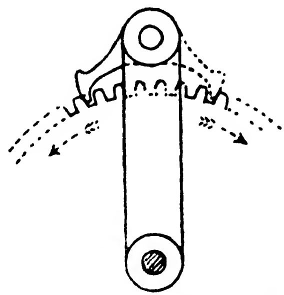


FIG. 231.—Reversible ratchet motion.

remaining at rest. To prevent the wheel reversing during the return stroke of the lever D a catch F turning on a fixed axis G is sometimes provided. During the forward stroke of the lever D from right to left, the ratchet wheel slides under the catch F.

To obtain a reversal of the motion of the ratchet wheel, using a single pawl, the teeth of the wheel are made of ordinary shape (Fig. 231), and the pawl is thrown over to occupy the dotted position, in which position the direction of movement of the wheel would be clockwise.

Example.—A ratchet wheel has 30 teeth, and the stroke of the lever carrying the pawl is such that the pawl moves over two teeth on its return stroke. The ratchet wheel is keyed to a screw of $\frac{1}{4}$ inch pitch, which gives the feed motion to the tool in a lathe. How many strokes must the lever make to give a feed of $\frac{1}{3}$ inch to the tool?

One stroke of the lever moves the wheel forward 2 teeth = $\frac{2}{30}$ revolution.

To give a feed of $\frac{1}{2}$ inch, the screw, and therefore the ratchet wheel, must make $\frac{1}{2} \div \frac{1}{4} = 2$ revolutions.

Hence, to turn the ratchet wheel through 2 revolutions the lever must make

$$2 \div \tfrac{2}{30} = 30 \text{ forward strokes.}$$

Cams.—Cams are used in machines for converting a rotary motion into any desired reciprocating motion. The plate cam

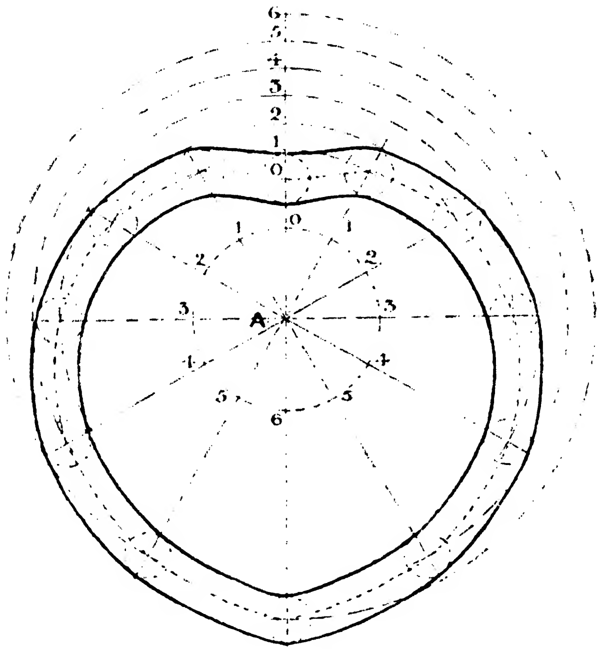


FIG. 232.—Plate cam.

(Fig. 232) has a groove of the required shape cut in one face of a plate, and as the plate rotates, a reciprocating motion is given to a hardened steel roller which works in the groove. In the edge cam the roller works on the curved edge of the cam (Fig. 233). We proceed to work out the correct shape of the groove or edge for one or two cases.

Example 1.—Design a plate cam to give a uniform reciprocating

motion of 1·8 inches stroke, the diameter of the roller being ½ inch, and the least distance between the centres of roller and camshaft 1½ inches.

We first draw the centre of the groove (shown dotted in Fig. 232) as follows :—

From A the centre of the camshaft mark off Ao equal to 1½ inches, make o6 equal to the stroke required, namely 1·8 inches ; divide o6 into, say, six parts at 1, 2, 2, 4, 5. With centre A and *any* convenient radius describe the dotted circle dividing it into 12 equal parts, 6 on each side of the line A6. Draw the radii A1, A2, A3, etc., and produce them as shown. With centre A and radius A1 draw an arc to cut the radial line

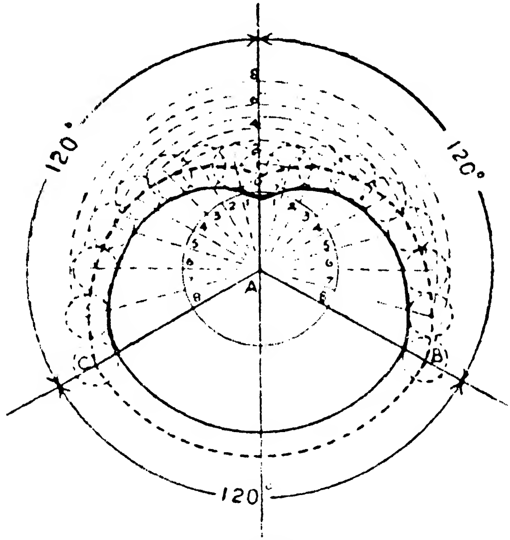


FIG. 233.—Edge cam.

A1 ; with the same centre and radius A2 draw an arc to cut the line A2, and so on. Join the points so obtained by a smooth curve ; this curve (shown dotted) will be the centre line of the groove required. The width of the groove must be the same as the diameter of the roller which has to work in it. Draw, therefore, a series of circles of diameter equal to that of the roller, in this case ½ inch, with their centres on this curve ; parallel curves inside and outside the centre line and touching these circles will give the groove required.

As the cam rotates at uniform speed about the centre A, the roller will reciprocate backwards and forwards along the line A6 with uniform speed. While the cam turns through 30°, *i.e.* the angle oA1, the roller moves a distance o1, for the next 30° (angle 1A2) the roller moves

through 12, and so on, for equal angles, and therefore equal times, the roller moves equal distances, in this example $\frac{1 \cdot 8}{6}$ or $0 \cdot 3$ inch for every 30° rotation of the cam. In one-half of a revolution the roller moves one stroke from o to 6, and in the other half of the same revolution it returns to o again.

Example 2.—Design an edge cam to give a uniform rise of 1 inch during $\frac{1}{3}$ of revolution, then a rest for $\frac{1}{3}$ of a revolution followed by a uniform fall of 1 inch during the remainder of the revolution. Take the diameter of the roller as $\frac{1}{2}$ inch.

With centre A and any convenient radius, draw a circle and divide it into 3 equal parts of 120° (Fig. 233). Set out the curve for the centre of the roller shown dotted, the construction lines for which are shown in the figure, o8 being the lift of 1 inch. Then draw the shape of the cam as shown. While the cam turns through the angle oAB ($\frac{1}{3}$ of a revolution) a uniform rise o8 (1 inch) is given to the roller; then follows a period of rest through the angle BAC ($\frac{1}{3}$ of a revolution), and a uniform fall 8o (1 inch) for the remaining $\frac{1}{3}$ of the revolution.

Examples XXIV.

1. In the crank and connecting rod mechanism shown in Fig. 218, the crank is 9 inches long and the connecting rod 3 feet long. How far is the piston from the middle of its stroke when the crank angle is (1) 30°; (2) 90°; (3) 135°?

2. If the crank in Question 1 is rotating at 120 revolutions per minute, what is the approximate velocity of the piston for each of the crank angles given?

3. In the quick return motion shown in Fig. 223, the crank OC is 4 inches long and the distance OA between the centre of the crank shaft and the fulcrum of the oscillating lever is 10 inches. The crank makes 120 revolutions per minute. How many times greater is the average speed of the ram during the return stroke than during the cutting stroke?

4. Find the angular velocity of the lever of Question 3 when in its mid position.

5. A ratchet wheel has 36 teeth, and the stroke of the lever carrying the pawl is such that the pawl moves over 3 teeth on its return stroke. The ratchet wheel is keyed to a screw of $\frac{1}{4}$ inch pitch which gives the feed motion to the tool in the lathe. How many strokes must the lever make to give a feed of 1 inch to the tool?

6. Draw a cam groove to work with a roller $\frac{3}{4}$ inch diameter, the rise and fall of the cam roller to be uniform. Take the minimum distance of the centre of the roller from the centre of rotation as 2 inches, and the maximum distance $4\frac{1}{2}$ inches.

7. Draw an edge cam to give a uniform rise of $\frac{3}{4}$ in for $\frac{1}{3}$ of a revolution, then a period of rest for $\frac{1}{4}$ revolution, followed by a uniform fall for the remainder of the revolution; roller $\frac{1}{2}$ inch diameter.

8. Draw an edge cam to give a uniform rise of $1\frac{1}{2}$ inches for $\frac{3}{4}$ revolution followed by a uniform fall during the remainder of the revolution.

CHAPTER XXV

HYDRAULICS

Fluids.—Matter may be divided into three kinds: (1) Solid; (2) Liquid; and (3) Gaseous (including vapours). Matter in the 2nd or 3rd state is called fluid. The great difference between a fluid and a solid is, that the particles of a solid body adhere together, and are capable of offering considerable resistance to any change of shape of the body; particles of a fluid do not, but move freely over one another. Consequently, a quantity of fluid matter cannot maintain its shape, except by the help of a solid vessel to hold it. Liquids differ from gaseous matter in that the particles are in contact one with another, while the particles of a gas may be very much further apart. A gas allowed to enter an empty vessel expands throughout the vessel, filling it completely; while a liquid lies in the lower portion occupying the same volume which it did before entry. Our consideration at present is confined to liquids, and mainly to water.

Although liquids change shape easily, they do possess some resistance to rapid change of shape, and this property is called viscosity. This resistance is a tangential or shearing resistance, and is proportional to the speed of sliding motion of one particle over another. At zero speed the resistance to sliding is zero; that is, the force between two particles of a liquid at rest is entirely perpendicular to their surfaces in contact or wholly normal. Further, fluids cannot withstand tension, so the only kind of force which can be transmitted by the particles of a fluid at rest is a pressure perpendicular to the particles. Similarly, the only force which can exist between a liquid at rest and the walls of the vessel containing it, is perpendicular to the walls.

Fluid Pressure.—The intensity of pressure within a fluid or against the walls of a containing vessel is measured in pounds per square inch or per square foot.

At a given depth in a liquid at rest, the intensity of pressure is the same in all directions. This is not very easily shown experimentally in a simple way; it is a most important property of liquids

(and gases), and may be proved to follow from the fact that the pressure is everywhere normal. We shall see shortly that there is a different pressure at *different* depths in a liquid, but in water under high pressure in hydraulic machinery this is negligible, and the pressure is practically the same in all directions and in all connected parts. The principle may be further explained diagrammatically as follows:—

If two frictionless water-tight pistons or plungers both communicate with the same vessel of water (Fig. 234), the force on each will be proportional to the area of section of the plungers; that is, the pressure *per square inch* is the same on each. Thus, if one plunger is 40 square inches in area, and the total outward pressure or force on it is 20 tons, the pressure of the water is $\frac{20}{40} = \frac{1}{2}$ ton per

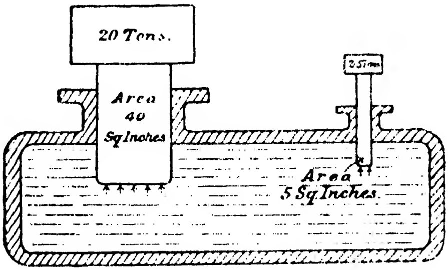


FIG. 234.

square inch. If the sectional area of the other plunger is 5 square inches, the total outward force on it will be $5 \times \frac{1}{2} = 2 \cdot 5$ tons. These forces are represented in Fig. 234 by the dead loads of 20 tons and 5 tons, balanced by the water pressure. The difficulty of showing this point experimentally, as represented in Fig. 234, is that it is not possible to produce frictionless plungers.

Hydraulic Pressure Machines.—Hydraulic power is transmitted by forcing water at high pressures along pipes from a central pumping station. It is particularly useful where the demand for power is intermittent (there being no loss of heat energy when the machinery is not working), and where great forces have to be exerted slowly, and through short distances.

Hydraulic Press.—The construction of a hydraulic press is shown in Fig. 235, and the method of working is as follows:—

On the upward stroke of the lever L the plunger A is raised. The valve B is raised, and water flows past it to follow the plunger. On the downward stroke of L, B closes, and water is forced out

along the pipe C into the ram cylinder D, pushing up the ram E. The top of the ram forms a platform, F, on which the material to be pressed is placed. Columns K carry another fixed platform G,

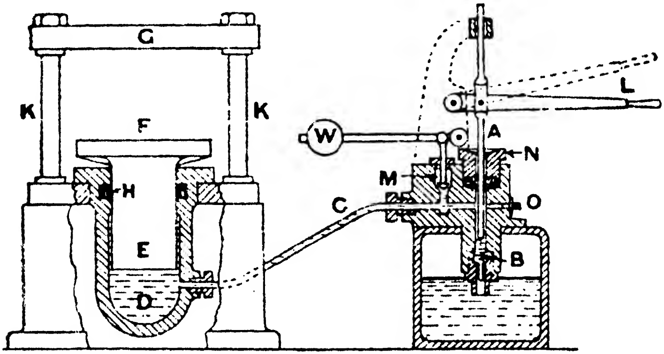


FIG. 235.—Hydraulic press.

and as F is forced upwards, the material is compressed between F and G. To prevent water leaking out of the ram cylinder D, the leather ring H is inserted in an annular recess, as shown. The greater the pressure of the water (and therefore the upward force

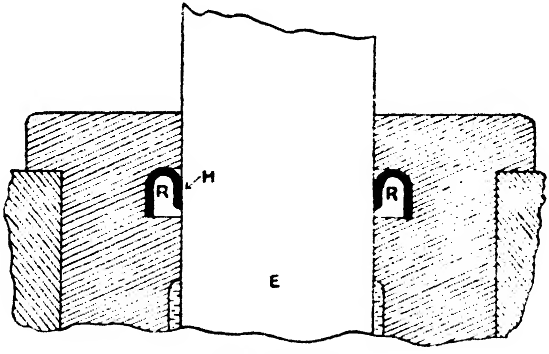


FIG. 236.—Packing for hydraulic pressure.

on F), the tighter is the joint formed by H, as will be seen by referring to Fig. 236, which shows an enlarged view of this water-tight joint. The cup leather H is placed in a recess, R (Fig. 236), which is turned in the ram cylinder D in such a way that the water

under pressure can pass into the annular space inside the leather. The result is, that the greater the water pressure, the tighter the leather is pressed against the ram E, and the better the joint.

As the material is compressed more and more between F and G (Fig. 235), the pressure of the water rises, and to prevent damage to the machine, a safety-valve, M, is fitted. This valve (which is similar to the lever safety-valve already described in Chap. III.) opens when the pressure rises to a dangerous height, and allows water to escape, and so relieves the pressure. When the material is compressed as much as is desired, the ram E is allowed to return again by its own weight, by opening the valve at O, and so allowing water to escape.

The base of the ram cylinder D is made hemispherical because that shape is the strongest to resist great internal pressures.

Example 1.—In a hydraulic press an effort of 40 lbs. is applied to the lever 28 inches from its fulcrum. The plunger is 2 inches from the fulcrum, and its diameter is 1 inch. The ram is 10 inches diameter. Find the pressure exerted on the ram if the efficiency of the machine at this load is 85 per cent.

Force exerted on the plunger by the lever $= 40 \times \frac{28}{2} = 560$ lbs.

$$\text{Area of plunger} = \frac{\pi d^2}{4} = 0{\cdot}7854 \text{ square inch.}$$

$$\text{Area of ram} = \frac{\pi}{4} \times 10^2 = 78{\cdot}54 \text{ square inches.}$$

$$\left.\begin{array}{r}\text{Neglecting friction,}\\ \text{pressure of water}\end{array}\right\} = \frac{560}{0{\cdot}7854} \text{ lbs. per square inch.}$$

Total pressure on ram = this water pressure (pounds per square inch) × area of ram (square inches) × efficiency, or

$$\text{Total pressure} = \frac{560}{0{\cdot}7854} \times 78{\cdot}54 \times 0{\cdot}85 = 47{,}600 \text{ lbs., or } 21{\cdot}25 \text{ tons.}$$

Example 2.—In a hydraulic press the leverage of the handle is 12 to 1, and the diameter of the plunger is 1 inch. What must be the diameter of the ram if a force of 30 lbs. on the end of the lever will raise a load of 15 tons on the ram? Take the efficiency of the press as 90 per cent.

Let d = diameter of ram required.

$$\text{Neglecting friction, water pressure} = \frac{\text{force on plunger}}{\text{area of plunger}} = \frac{30 \times 12}{0{\cdot}7854}$$

$$= \frac{360}{0{\cdot}7854} \text{ lbs. per sq. in.}$$

Load lifted = this water pressure × area of ram × efficiency
$= 15 \times 2240$ lbs.

Hence $$\frac{360}{0{\cdot}7854} \times 0{\cdot}7854 \times d^2 \times \frac{90}{100} = 15 \times 2240$$

$$324d^2 = 15 \times 2240$$

$$d = \sqrt{\frac{15 \times 2240}{324}} = 10{\cdot}18 \text{ inches.}$$

Example 3.—A load of 8 tons is lifted by a hydraulic press when an effort of 15 lbs. is applied on the end of the handle whose leverage is 14 to 1. Assuming an efficiency of 90 per cent., find the proportion between the diameter of the ram and the plunger.

$$\text{Force on plunger} = 15 \times 14 = 210 \text{ lbs.}$$

Let d = diameter of plunger in inches, and D the diameter of the ram in inches.

Water pressure neglecting friction $= \dfrac{210}{0{\cdot}7854d^2}$ lbs. per square inch.

Load lifted = this water pressure × area of ram × efficiency.

$$8 \times 2240 = \frac{210}{0{\cdot}7854d^2} \times 0{\cdot}7854\mathrm{D}^2 \times 0{\cdot}9$$

$$\frac{\mathrm{D}^2}{d^2} \times 210 \times 0{\cdot}9 = 8 \times 2240$$

$$\frac{\mathrm{D}^2}{d^2} = \frac{8 \times 2240}{189}$$

$$\frac{\mathrm{D}}{d} = \sqrt{\frac{8 \times 2240}{189}} = 9{\cdot}7$$

i.e., the diameter of the ram must be 9·7 times the diameter of the plunger.

Hydraulic Jack.—Fig. 237 shows a sectional view of a hydraulic jack made by Messrs. Tangye Ltd., of Birmingham. The ram O fitted with the ram leather N is stationary, whilst the cylinder M together with the remainder of the machine is raised with the load. The action of the machine is as follows: On lifting the pump plunger D by means of the hand lever R, after tightening the lowering screw L, water is drawn from the cistern G through the suction valve I, and on the return of D this water is forced out through the delivery valve J into the cylinder M above the ram and so raises the load which usually will be supported on the top of the cistern A. To lower the cylinder the lowering screw is slackened; the cylinder then descends by its own weight, the water above the ram leather N escaping back to the cistern G. The screw C is for charging the cistern with water, and B for admitting air into the space above the water in the cistern G. The load may also be lifted from the claw on the bottom of the cylinder M, but if this is

the case, the maximum load which can be lifted from A cannot be lifted from the claw owing to the tendency to bend the ram of the jack caused by the load being on one side of the centre.

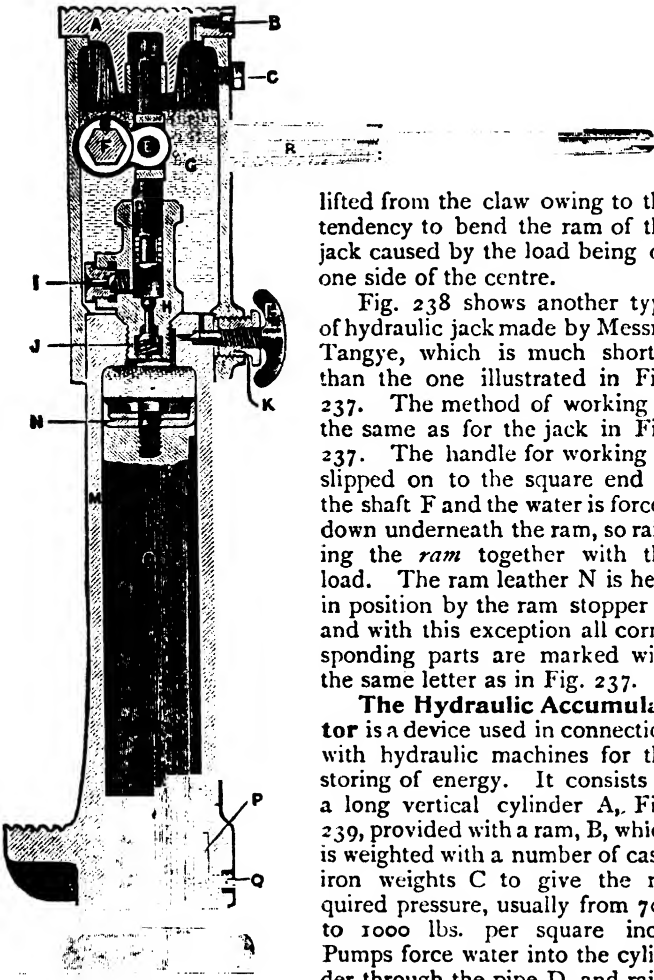


FIG 237.—Tangye's hydraulic lifting jack.

Fig. 238 shows another type of hydraulic jack made by Messrs. Tangye, which is much shorter than the one illustrated in Fig. 237. The method of working is the same as for the jack in Fig. 237. The handle for working is slipped on to the square end of the shaft F and the water is forced down underneath the ram, so raising the *ram* together with the load. The ram leather N is held in position by the ram stopper S, and with this exception all corresponding parts are marked with the same letter as in Fig. 237.

The Hydraulic Accumulator is a device used in connection with hydraulic machines for the storing of energy. It consists of a long vertical cylinder A, Fig. 239, provided with a ram, B, which is weighted with a number of cast-iron weights C to give the required pressure, usually from 700 to 1000 lbs. per square inch. Pumps force water into the cylinder through the pipe D, and raise the ram against the resistance offered by the weights C. When the ram is at the top of its stroke the crosshead E, to which the weights C are attached by the bolts

shown, comes in contact with the lever F and pushing it over stops the pumps working. If now some of the hydraulic machines (such as a lift or crane) start working they draw water at first from the cylinder and, consequently, the weighted ram descends. When the ram has descended a certain distance the lever F is released and the pumps start working again.

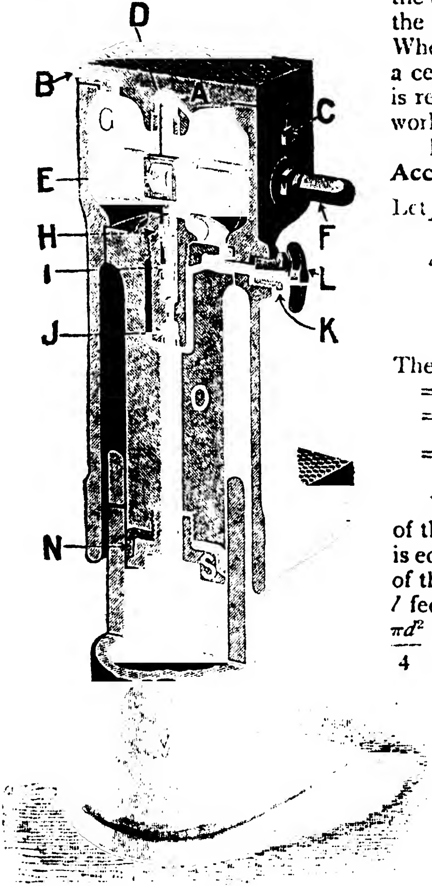


FIG. 238.—Tangye's hydraulic jack.

Energy stored in the Accumulator.

Let p = water pressure in pounds per square inch

d = diameter of ram in inches.

l = length of stroke of ram in feet.

Then total load on ram

$$= \text{area of ram} \times \text{water pressure}$$
$$= \text{weight C (Fig. 239)}$$
$$= \frac{\pi d^2}{4} \times p.$$

When the ram is at the top of the stroke the energy stored is equal to the potential energy of this weight when at a height l feet, hence energy stored $= \frac{\pi d^2}{4} \times p \times l$ foot-pounds.

Example.—A hydraulic accumulator has a ram 15 inches diameter, and is loaded with 60 tons. The stroke of the ram is 8 feet. How much energy can be stored, and what is the water pressure in pounds per square inch?

energy = load on ram × stroke

$$= 60 \times 2240 \times 8$$
$$= 1{,}075{,}200 \text{ foot-pounds.}$$

$$\text{Neglecting friction, water pressure} = \frac{\text{load on ram}}{\text{area of ram}}$$

$$= \frac{60 \times 2240}{0{\cdot}7854 \times 15 \times 15} = 760 \text{ lbs. per square inch.}$$

Hydraulic Lift.—Fig. 240 shows diagrammatically a common form of hydraulic lift. The cage C of the lift is fixed directly to the top of the ram B which works in the hydraulic cylinder A. Water entering the cylinder A raises the ram and therefore the cage. In order that the water may only have to lift a load approximately equal to the contents of the cage, the weight of the cage is balanced by the weight E, a wire rope, D, being attached to the roof of the cage, as shown.

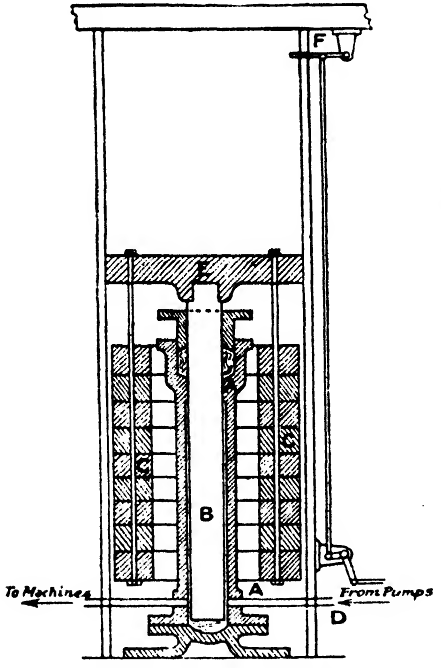


FIG. 239.—Hydraulic accumulator.

When the ram is at the bottom of its stroke it is almost completely surrounded with water; but as it rises, less of it is immersed in the water and therefore the upward force exerted on the ram by the water is less (pp. 337 and 340). For instance, when the ram rises 34 feet, the lifting force on it will diminish by 14·7 lbs. per square inch. Hence in order to keep the lifting force on the ram constant the pressure of the water admitted to the cylinder should be increased at the rate of 14·7 lbs. per square inch for each 34 feet of lift. In actual practice the water is supplied at constant pressure, and to keep the lifting force as uniform as possible the balance weight E is fitted. The lower the water pressure the greater is the importance of the balance weight, and the higher the pressure the less is its importance, and in many cases it is omitted altogether.

Example.—Neglecting friction, what is the greatest load (including the weight of ram and cage) that can be lifted by a hydraulic lift whose ram is 5 inches diameter and greatest lift 68 feet, the pressure of the water supplied being 700 lbs. per square inch?

$$\left.\begin{array}{r}\text{Total lifting force on the}\\ \text{ram when at the bottom}\end{array}\right\} = \frac{\pi}{4} \times 5^2 \times 700$$
$$= 13{,}744 \text{ lbs.}$$

Loss of lifting force per square inch when the ram is raised 68 feet $= 14{\cdot}7 \times \frac{68}{34}$
$= 29{\cdot}4$ lbs. per square inch.

$$\text{Total loss} = 29{\cdot}4 \times \frac{\pi}{4} \times 5^2 = 577 \text{ lbs.}$$

$$\text{Greatest load lifted} = 13{,}744 - 577 = 13{,}167 \text{ lbs.}$$

N.B.—If the pressure of the water supply were only 100 lbs. per square inch, the lifting force on the ram when at the bottom of its stroke would be—

$$\frac{\pi}{4} \times 5^2 \times 100 = 1963 \text{ lbs.}$$

This would be reduced by the same amount, viz. 577 lbs., when the ram is raised 68 feet, the proportionate reduction being very much greater than in the previous case.

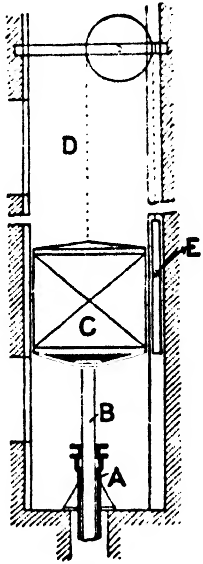


Fig. 240.—Hydraulic lift.

Hydraulic Crane.—Fig. 241 shows one form of hydraulic crane, a section of the hydraulic cylinder being shown on the left. To the bottom of the fixed cylinder A a pulley block B is mounted. Another pulley block C is mounted on the top of the ram D. The lifting rope or chain is fixed at one end to a lug E on the cylinder; it then passes round the pulleys in the blocks B and C to the load which is to be lifted as shown. In the crane shown there are three pulleys in each of the blocks B and C, therefore when the ram D is raised, say, 1 foot, the *six* lengths of rope each lengthen one foot, and the load W is lifted 6 feet, or, in other words, the load W is lifted with 6 times the speed of the ram.

Example.—In the crane shown in Fig. 241, the diameter of the ram is 5 inches, water pressure 700 lbs. per square inch, and the efficiency is 90 per cent. What is the greatest load that can be lifted?

$$\text{Force on ram} = \frac{\pi}{4} \times 5^2 \times 700$$

Hence

$$\left(\frac{\pi}{4} \times 25 \times 700\right) \times 1 \times \frac{90}{100} = W \times 6$$

$$W = \frac{25 \times \pi \times 700}{4} \times \frac{90}{100} \times \frac{1}{6} = 2060 \text{ lbs.}$$

Pressure or Head at different depths in Liquids.—If we imagine a vertical cylindrical-shaped portion of a liquid at rest in a tank (Fig. 242), we can easily estimate the difference of pressure at the two ends of such a cylinder. For if A square feet is the area of cross-section of the cylinder and h its height in feet, p_1 being

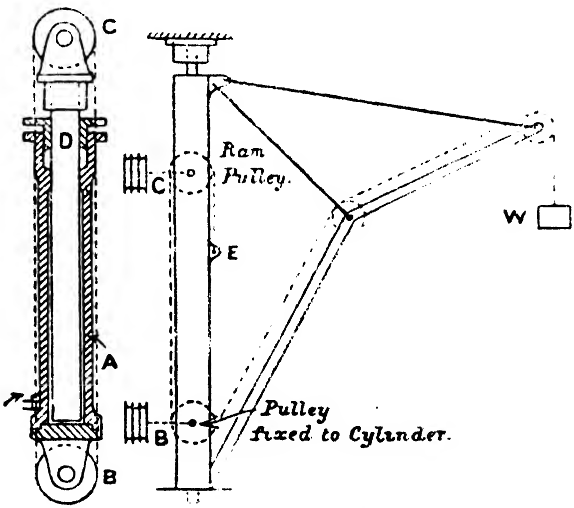


FIG. 242.—Hydraulic crane.

the pressure per square foot at the top end and p_2 that at the bottom end, the upward force on the lower end is—

$$p_2 \times A$$

and the downward force on the top end is—

$$p_1 \times A$$

The *horizontal* pressures which the surrounding liquid exerts on the *vertical* curved surface of the cylinder are in equilibrium among themselves, and being wholly horizontal, exert no vertical force The only other vertical force on the cylinder of liquid is its own

weight downwards; the volume of the cylinder of liquid is $A \times h$ cubic feet, and if the liquid weighs w pounds per cubic foot, the total weight of the cylinder of water is—

$$w \times A \times h \text{ pounds}$$

which must balance the excess of pressure on the base of the cylinder over that on the upper end, or—

$$p_2A - p_1A = wAh$$

or $$p_2 - p_1 = wh$$

That is, the increase of pressure for a depth h feet is wh pounds per square foot, or w pounds per square foot for every foot increase of depth.

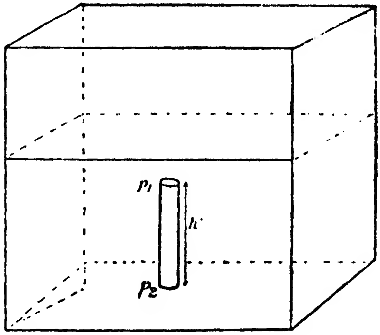


FIG. 242.

If one end of the cylinder is in the surface we see that the *upward* pressure of the surrounding liquid is just equal to the weight of the cylinder of water.

Reckoning from the surface of a liquid, the pressure at a depth h feet will exceed the atmospheric pressure at the surface by

$$wh \text{ pounds per square foot}$$

and for pressures at considerable depths the atmospheric pressure may generally be neglected.

In hydraulics the pressure of water is very frequently expressed in feet of water, not pounds per square foot. For instance, we should speak of pressure as a *head* of h feet, which would mean the pressure due to a head of h feet, *i.e.*, wh pounds per square foot.

Example.—If water at 40° F. weighs 62·4 lbs. per cubic foot, find the pressure at a depth of 100 feet.

The pressure increases at the rate of 62·4 lbs. per *square foot* for each foot of depth, hence at 100 feet depth it is—

$$62{\cdot}4 \times 100 = 6240 \text{ lbs. per square } \textit{foot}$$

or $$\frac{6240}{144} = 43 \text{ lbs. per square } \textit{inch}$$

that is, 0·43 lb. per square *inch* per foot of depth.

Pressure on Horizontal Surfaces in Liquids.—Any horizontal surface at a depth h feet below the surface of a liquid weighing w pounds per cubic foot has upon it a pressure equal to—

$$\text{Area in square feet} \times w \times h \text{ pounds.}$$

Example.—Find the pressure on the base of a rectangular tank 7 feet deep, 5 feet long, and 3 feet wide when the tank is full of water; water weighs 62·4 lbs. per cubic foot.

Area of base $= 5 \times 3 = 15$ square feet
Total pressure or force on base $= 15 \times 62{\cdot}4 \times 7 = 6552$ lbs.

NOTE.—The total pressure on the base would be the same whether the sides of the tank were vertical, or whether they sloped inwards or outwards, it depending only on the area of the base and the depth of water.

Pressure on Oblique Surfaces in Liquids.—It is not quite so simple a matter to estimate the total force exerted by hydraulic pressure on an inclined surface immersed in a liquid, as in the case of a horizontal surface, because the pressure per square foot is not the same all over it. The pressure varies proportionally to the depth below the surface of the liquid, so that unless all the surface is at the same depth (that is, horizontal) the pressure is different at different parts of the surface. In such cases the rule is that *the average pressure per square foot on the surface is equal to that at the centroid, or centre of gravity of the area.*[1] Or the total pressure in pounds is equal to the area in square feet multiplied by the pressure in pounds per square foot at the centroid, or to wh where h is the depth of the c.g. in feet, and w is the weight of the liquid in pounds per cubic foot.

Example 1.—Find the total pressure on a dock gate whose width is 25 feet, when the depth of water is 10 feet.

Depth of c.g. of wetted area = 5 feet
Pressure at c.g. $= 62{\cdot}4 \times 5$ lbs. per square foot
Area of wetted surface $= 25 \times 10 = 250$ square feet
Total pressure on gate $= 62{\cdot}4 \times 5 \times 250$
$= 78{,}000$ lbs.

Example 2.—A submerged vertical sluice gate is 3 feet long, and 2 feet deep, the 3-feet sides being horizontal. The top side is 20 feet below the surface of the water. Find the total water pressure on the gate.

Depth of c.g. of gate $= 20 + 1 = 21$ feet.
Pressure at c.g. $= 62{\cdot}4 \times 21$ lbs. per square foot.
Total pressure $= 62{\cdot}4 \times 21 \times 6 = 7862$ lbs.

Centre of Pressure.—Vertical or other inclined plane surfaces immersed in a liquid are acted on by parallel forces, consisting of the pressures on its various parts. These may be combined by the rules for finding the amount and position of the resultant of a number of parallel forces (Chap. II.) if the area be divided into

[1] Centroid or c.g. Refer to Chap. XIV. for position in various shaped areas.

very small portions. The rule for finding the *amount* of the resultant is given in the previous paragraph. That for finding the position of the resultant is beyond the scope of this book. The point at which the resultant pressure acts on the surface is called the *centre of pressure*. For a rectangular area having one side in the surface of the liquid, the vertical depth of the centre of pressure is $\frac{2}{3}$ of the vertical depth of the rectangle.

Example 1.—The depth of water in a dock is 30 feet. Find (*a*) the pressure on the dock wall in tons per foot of its length; (*b*) the position of the centre of pressure; (*c*) the overturning moment about the bottom of the wall in tons-feet per foot length, due to the water pressure. (Water weighs 62·4 lbs. per cubic foot.)

(*a*) Depth of c.g. of wetted area $= \frac{30}{2} = 15$ feet

Area wetted per foot length of wall $= 30$ square feet

$$\text{Total pressure on wall per foot length} = \frac{30 \times 15 \times 62{\cdot}4}{2240} = 12{\cdot}53 \text{ tons.}$$

(*b*) Depth of centre of pressure $= \frac{2}{3} \times 30 = 20$ feet below the water surface

(*c*) Moment about bottom of wall $= 12{\cdot}53 \times 10 = 125{\cdot}3$ tons-feet.

Example 2.—A vertical rectangular sluice gate is 4 feet by 3 feet, and is submerged with the longer sides horizontal, the top side being 2 feet below the surface of the water. Find the position of the centre of pressure.

$$
\begin{aligned}
\text{Total pressure on sluice gate BC (Fig. 243)} & \\
&= \text{pressure at c.g.} \times \text{area} \\
&= 62{\cdot}4 \times (2 + 1{\cdot}5) \times (4 \times 3) \\
&= 62{\cdot}4 \times 3{\cdot}5 \times 12 = 2621 \text{ lbs.} \\
\text{Total pressure on area AB} &= \text{pressure at c.g.} \times \text{area} \\
&= 62{\cdot}4 \times 1 \times 4 \times 2 = 500 \text{ lbs.} \\
\text{The pressure on AB will act } \tfrac{2}{3} \times 2 &= \tfrac{4}{3} \text{ feet from the surface of the water} \\
\text{Total pressure on the area AC} &= \text{pressure on AB} + \text{pressure on BC} \\
&= 500 + 2621 \\
&= 3121 \text{ lbs.} \\
\text{The pressure on AC will act } \tfrac{2}{3} \times 5 &= \tfrac{10}{3} \text{ feet from the surface of the water}
\end{aligned}
$$

Let x = depth of centre of pressure on the sluice gate BC.
Taking moments about the surface of the water, we get—

$$
\begin{aligned}
x \times 2621 + 500 \times \tfrac{4}{3} &= 3121 \times \tfrac{10}{3} \\
7863x + 2000 &= 31210 \\
x &= \frac{31210 - 2000}{7863} = 3{\cdot}71 \text{ feet.}
\end{aligned}
$$

Buoyancy. Floating Bodies.—A solid body floating in a liquid is supported by the vertical upward pressures of the liquid upon it. This pressure which acts on the submerged portion of the body is just as it would be on the same space if occupied by

the liquid, instead of by *part* of the floating body. But we have seen that in the case of a cylinder of liquid the net vertical force is just equal to the *weight* of the cylinder of liquid. The same is true of any other shaped body, for we could imagine it as consisting of a large number of small cylinders. Hence, we have the principle that the vertical upward liquid pressure which balances the weight of a floating body is just equal to the weight of a volume of liquid equal to that of the submerged part of the body. In other words, the supporting pressure is equal to the weight of the floating body, and also to the weight of liquid which it displaces in floating.

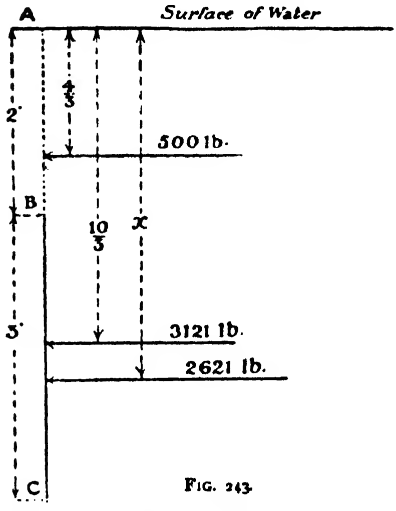


FIG. 243.

Density and Specific Gravity.—The density of a substance is its weight per unit volume. Thus the density of water is 62·4 lbs. per cubic foot at 40° F., and about 62 lbs. per cubic foot at 100° F. The density of *cast* iron is about 450 lbs. per cubic foot or 0·26 lb. per cubic inch, and of *wrought* iron about 480 lbs. per cubic foot.

The specific gravity of a substance is the ratio of the weight of any volume of the substance to the weight of an equal volume of water. For example—

$$\text{Specific gravity of cast iron} = \frac{\text{weight of 1 cubic foot of cast iron}}{\text{weight of 1 cubic foot of water}}$$

$$= \frac{450}{6 \cdot 24} = 7 \cdot 2$$

The specific gravities of substances are to one another in the same ratios as their densities, and the density of any substance in pounds per cubic foot is about

$$62 \cdot 4 \times \text{specific gravity}.$$

Example 1.—A cubical block of wood 2 feet edge of specific gravity 0·45 floats in water. Find the volume of water it displaces, and the proportion of the volume displaced.

Weight of 1 cubic foot of wood $= 62\cdot4 \times 0\cdot45$ lbs.

$$\text{Weight of water displaced} = \text{weight of block} = 2^3 \times 62\cdot4 \times 0\cdot45 \text{ lbs.}$$

$$\text{Volume of water displaced} = \frac{8 \times 62\cdot4 \times 0\cdot45}{62\cdot4} = 3\cdot6 \text{ cubic feet}$$

$$\text{which is } \frac{3\cdot6}{8} = 0\cdot45 \text{ of the volume of the block,}$$

i.e. volume displaced = specific gravity × volume of block.

Example 2.—A cast-iron body of weight 56 lbs. is suspended by a string and wholly immersed in water. Find the upward force on the weight and the tension of the supporting string. (Specific gravity of cast iron 7·2.)

$$\text{Weight of 1 cubic foot of cast iron} = 7\cdot2 \times 62\cdot4 = 450 \text{ lbs.}$$

$$\text{Volume occupied by 56 lbs. of cast iron} = \tfrac{56}{450} \text{ cubic foot.}$$

$$\text{Weight of water displaced} = \tfrac{56}{450} \times 62\cdot4 \text{ lbs.}$$

$$\text{Hence upward pressure on weight} = \text{weight of water displaced}$$

$$= \frac{56 \times 62\cdot4}{450} = 7\cdot77 \text{ lbs.}$$

$$\text{Tension of the string} = 56 - 7\cdot77 = 48\cdot23 \text{ lbs.}$$

NOTE.— $\dfrac{\text{weight of body}}{\text{upward pressure}} = \dfrac{56}{7\cdot77} = 7\cdot2$, the specific gravity of the iron.

EXAMPLES XXV.

1. In a hydraulic press an effort of 35 lbs. is exerted on the end of the lever whose leverage is 14. The diameter of the plunger is $1\frac{1}{4}$ inch, and that of the ram 10 inches. If the efficiency of the press is 90 per cent., what pressure will be exerted on the ram?

2. In a hydraulic press the diameter of the plunger is 1 inch, and of the ram 10 inches. The leverage of the handle is 12 to 1. If the efficiency of the press is 90 per cent., what effect must be exerted on the handle in order to give a total pressure on the ram of 20 tons?

3. The diameter of the plunger of a hydraulic jack is 1 inch, and of the ram is 2 inches. The leverage of the handle is 15 to 1. It is found that an effort of 53 lbs. on the handle just raises a load of 1 ton. What is the efficiency of the machine at the load?

4. If the stroke of the plunger of the jack in Question 3 is $\frac{1}{2}$ inch, how many working strokes must be made by the handle in order to lift the weight 3 inches?

5. A hydraulic accumulator has a ram 12 inches diameter, and is loaded with 50 tons. The stroke of the ram is 6 feet. How much energy can be stored, and what is the water pressure in pounds per square inch?

6. Neglecting friction, what is the greatest load (including the weight of ram and cage) that can be raised by a hydraulic lift whose ram is 8 inches diameter if the water pressure is 700 lbs. per square inch, and the greatest lift is 50 feet?

7. The block on the ram of a hydraulic crane is fitted with four pulleys, and that on the cylinder has four pulleys; the diameter of the ram is 6 inches, and the water pressure is 700 lbs. per square inch. What is the greatest load that can be lifted if the efficiency is 85 per cent.?

8. At a pressure of 700 lbs. per square inch, what is the charge for 1000 gallons of water at 2*d*. per horse-power per hour?

9. A hydraulic crane is supplied with water at a pressure of 700 lbs. per square inch, and uses 3 cubic feet of water in order to lift 8 tons through a height

of 9 feet. How much energy has been supplied to the crane, and what proportion of it has been converted into useful work?

10. The bottom of a rectangular water tank measures 8 feet by 3 feet 6 inches. When the tank contains 800 gallons of water, what will be the depth of the water, and what would be the total pressure on the bottom, on each side and on each end respectively? One gallon of water weighs 10 lbs., and 1 cubic foot weighs 62·4 lbs.

11. A submerged vertical rectangular sluice gate 2 feet by 1 foot 6 inches has the 2-feet sides horizontal, the top side being 15 feet below the surface of the water. Find the total water pressure on the gate.

12. The depth of sea water in a dock is 35 feet. Find (*a*) the pressure on the dock wall in tons per foot of its length; (*b*) the position of the centre of pressure; (*c*) the overturning moment about the bottom of the wall in tons-feet per foot length (35 cubic feet of sea water weigh 1 ton).

13. Find the position of the centre of pressure on the sluice gate in Example 11.

14. A spherical block of wood of specific gravity 0·42 and 3 feet diameter floats in water. Find the volume of water it displaces.

15. A cast-iron body weighing 112 lbs. is suspended by a rope, and is wholly immersed in water. Find the upward force on the body, and the tension of the rope. (Specific gravity of cast iron is 7·2.)

CHAPTER XXVI

RECIPROCATING PUMPS

Suction or Lift Pump.—This pump consists of a cylindrical pump barrel B (Fig. 244) in which works an air-tight plunger or bucket A. This bucket has one or more valves C which can only open upwards to allow water to pass through it to the top. A pipe, F, is attached to the bottom of the pump barrel and leads down to the water in the well or other source from which the water is to be pumped. At the top of this pipe is a valve E which opens upwards to allow water to pass from the pipe F into the pump barrel B.

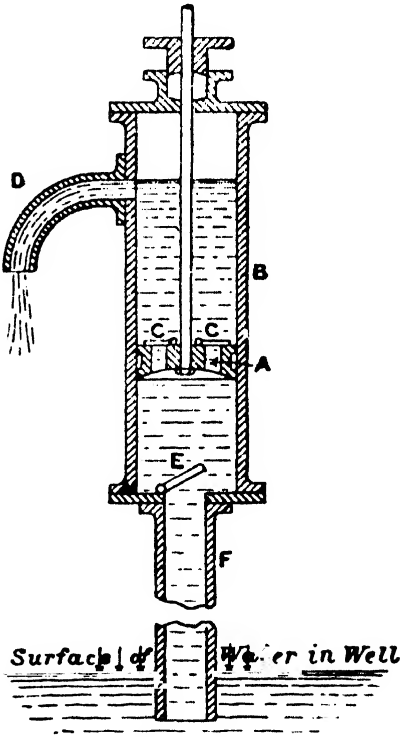


FIG. 244.—Suction pump

When starting the pump for the first time the barrel and suction pipe F will be full of air at atmospheric pressure. Suppose the bucket A is at the top of its stroke. As the bucket moves downwards E is closed and the valves C open and so allow the air which was previously in the barrel beneath the bucket to pass through into the barrel above it. On the next upward stroke of the bucket a partial vacuum is formed

underneath it, and the atmospheric pressure on the top of the bucket being greater than the reduced pressure of the air under it, the valves C are kept closed. Now, the pressure at the bottom of the pipe F is that due to the pressure of the atmosphere on the surface of the water; the result is that the valve E opens and some of the air in pipe F is forced through into the pump barrel beneath the bucket and water enters the pipe F to take its place until the pressure of the air inside together with that due to the water equals the atmospheric pressure. This process is repeated on successive strokes of the bucket until all the space beneath it is full of water. The next upward stroke of the bucket forms, as before, a partial vacuum under it, and the atmospheric pressure acting on the surface of the water in the well forces open the valve E and allows water to follow the bucket. When the bucket descends again, E closes, and the water under it flows through the valves C into the barrel above, from which it is lifted by the bucket on its next upward stroke and delivered through the spout D.

It will be seen, then, that the water is lifted from the well by the atmospheric pressure itself, and that the greatest height through which it can be lifted depends upon the atmospheric pressure. At standard atmospheric pressure 14·7 lbs. per square inch, or $14 \cdot 7 \times 144 = 2116$ lbs. per square foot, the maximum height h through which the water could be lifted will be—

$$h = \frac{p}{w} \text{ (see Chap. XXV., p. 337)}$$

$$= \frac{2116}{62 \cdot 4} = 34 \text{ feet, nearly.}$$

Owing to imperfections in the pump (the bucket not being perfectly air-tight) and frictional resistances, the maximum height of the bottom of the bucket, when at the top of its stroke, above the surface of the water in the well seldom in practice exceeds about 26 feet. The spout D, however, may be any height above the surface because the water above the bucket is simply lifted up by it and is independent of the atmospheric pressure.

Example 1.—Find the greatest vertical height at which the pump barrel could be fixed above the surface of the water in the well when the height of the mercury barometer is 28 inches.

When the barometer stands at 30 inches the atmospheric pressure is that due to a head of 34 feet of water as shown above: hence at 28 inches it is—

$$34 \times \tfrac{28}{30} = 31 \cdot 7 \text{ feet.}$$

Owing to the reasons mentioned above the height would be made much less than this. The pipe F (Fig. 244) may not be vertical, and may

be of considerable length; the longer it is the greater is the loss due to frictional resistance between the water and the pipe, and consequently the less would be the height at which the pump could work.

Example 2.—The barrel of a lift pump is 5 inches diameter, and when the bucket is at the bottom of the stroke the height of the entrance of the spout to the pump barrel above the top of the bucket is 20 feet. What will be the greatest tension in the rod lifting the bucket, and if the stroke of the bucket is 4 inches, how many upward strokes must it make in order to deliver 100 gallons of water?

The tension in the rod will be equal to the weight of water above the bucket. The volume of the water will be in cubic feet—

$$\text{area of barrel (in square feet)} \times 20 \text{ feet}$$

$$= \frac{0{\cdot}7854 \times 5^2}{144} \times 20 \text{ cubic feet}$$

$$\text{Hence tension in rod} = \frac{0{\cdot}7854 \times 25}{144} \times 20 \times 62{\cdot}4 = 171 \text{ lbs.}$$

$$\text{Volume of water lifted per stroke} = \frac{0{\cdot}7854 \times 25}{144} \times \frac{4}{12}$$

$$= 0{\cdot}0453 \text{ cubic foot.}$$

$$\text{Now } 100 \text{ gallons} = 100 \div 6{\cdot}24 \text{ cubic feet; hence,}$$

$$\text{Strokes required} = \frac{100}{0{\cdot}0453 \times 6{\cdot}24} = 354$$

Force Pump.—Fig. 245 shows a force pump in diagrammatic form. The plunger A is solid without any valves, and on its upward stroke water enters the pump barrel B through the valve C just as in the case of the suction pump previously considered. During this stroke the valve D is kept closed by the head of water in the delivery pipe E. On the return stroke of the plunger the valve C is closed, and water under pressure is forced out of the pump barrel through the valve D into the delivery pipe E. This type of pump is intermittent in its action, water only being delivered through the delivery pipe during the downward stroke of the plunger. In order to maintain a more continuous delivery of water with this type of pump an air vessel is attached to the delivery pipe.

Fig. 246 shows a sectional view of a single acting force pump fitted with an air vessel on the delivery side. On the upward stroke of the plunger A, water enters through the suction valve C; on the downward stroke the suction valve is closed and the water is forced through the delivery valve D against the pressure in the delivery pipe. The air in the air vessel E is compressed to a greater pressure than that corresponding to the head of water at the bottom of the delivery pipe, so that during the next upward stroke of the plunger the pressure of the air maintains the flow of water through the delivery pipe, thereby ensuring a more continuous flow of water.

Example 1.—A single acting force pump has to deliver water against a head of 240 feet. If the diameter of the plunger is 5 inches, what must be the average force exerted on the pump plunger during the delivery stroke if the efficiency of the pump is 85 per cent.?

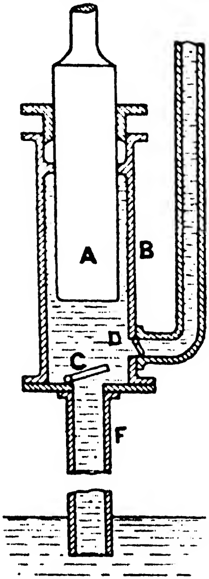


FIG. 245.—Single acting force pump.

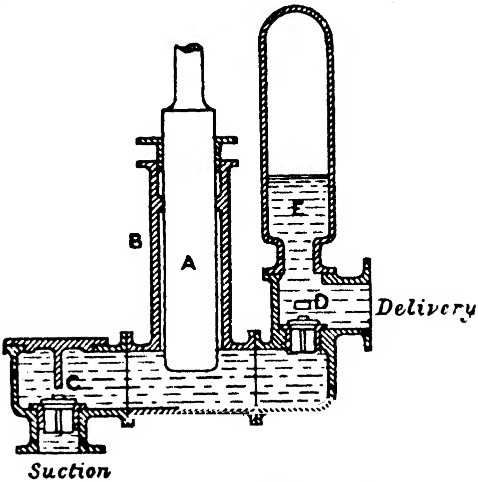


FIG. 246.—Force pump with air vessel.

The pressure due to a head of 240 feet

$$hw = 240 \times 62{\cdot}4 \text{ lbs. per square foot}$$

$$\text{or } \frac{240 \times 62{\cdot}4}{144}$$

$$= 104 \text{ lbs. per square inch}$$

Area of plunger $= 0{\cdot}7854 \times 5^2 = 19{\cdot}63$ square inches.

Total force on plunger neglecting frictional losses $= 104 \times 19{\cdot}63$ lbs.

Hence, allowing for the efficiency of 85 per cent.

Force required on plunger $= 104 \times 19{\cdot}63 \times \frac{100}{85} = 2400$ lbs.

Example 2.—If the stroke above the pump is 6 inches and it makes 120 double strokes per minute, what horse-power will be required to drive it?

$$\text{Work done per stroke} = \text{Force on plunger} \times \text{stroke}$$

$$= 2400 \times \tfrac{6}{12} \text{ foot-pounds}$$

$$\text{Work done per minute} = 2400 \times \tfrac{6}{12} \times 120 \text{ foot-pounds}$$

$$\text{Horse-power} = \frac{\text{work done per minute in foot-pounds}}{33{,}000}$$

$$= \frac{2400 \times 6 \times 120}{12 \times 33{,}000} = 4{\cdot}36 \text{ H.P.}$$

Example 3.—If the slip of the pump in Example 1 is 5 per cent., how many gallons will it deliver per minute?

$$\text{Volume swept out by pump plunger} = \text{area of plunger} \times \text{stroke}$$
$$= 19{\cdot}63 \times 6 \text{ cubic inches}$$
$$= \frac{19{\cdot}63 \times 6}{1728} \text{ cubic feet}$$

Owing to slip the pump does not deliver this volume of water per stroke, but only $\frac{95}{100}$ of it.

$$\text{Hence, water delivered per stroke} = \frac{19{\cdot}63 \times 6}{1728} \times \frac{95}{100} \text{ cubic feet.}$$

$$\text{Water delivered per minute} = \frac{19{\cdot}63 \times 6}{1728} \times \frac{95}{100} \times 120$$
$$= 7{\cdot}77 \text{ cubic feet}$$
$$= 7{\cdot}77 \times \frac{62{\cdot}4}{10} = 48{\cdot}48 \text{ gallons.}$$

Double Acting Force Pump.—In a double acting pump water is delivered every stroke, not every other stroke as is the case with the single acting pump. During the upward stroke of the plunger A (Fig. 247) water enters the pump barrel through the suction valve B, and at the same time water is forced out from above the plunger through the delivery valve C into the delivery pipe. On the downward stroke of the plunger the valves B and C are closed; water enters the pump barrel above the plunger through the suction valve E, and at the same time the water which was drawn in underneath the plunger during the previous upward stroke is forced out from below the plunger through the delivery valve D into the delivery pipe. By this means a continuous supply of water is delivered through the delivery pipe, and an air vessel is not required for this purpose.

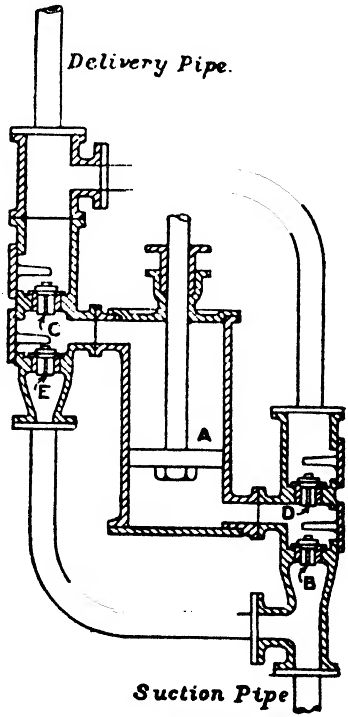


FIG. 247.—Double acting force pump.

In quick-running pumps or in cases where the delivery pipe is very long the accelerating force required on the pump plunger, and therefore on the water, causes a large increase in the stresses in the different parts of the pump, and gives under certain conditions separation of water in the pipe, and violent hammer actions may be set up. To reduce the effect of these inertia forces air vessels are usually put on the delivery pipe of double acting pumps. For the same reasons, if the pump runs at a high speed and the suction pipe is long, an air vessel will be also required on the suction side.

Example.—A double acting force pump of stroke 6 inches has to deliver water against a head of 125 feet. If the diameter of the plunger is 5 inches, and it makes 180 double strokes per minute, what horse-power will be required to drive the pump if its efficiency is 85 per cent.? If it is direct driven by a continuous-current electric motor which works at 200 volts, what current must be supplied to the motor if its efficiency is 90 per cent.?

$$\text{Pressure in pump barrel} = \frac{125 \times 62{\cdot}4}{144}$$

$$= 54 \text{ lbs. per square inch}$$

$$\text{Work done on the water per stroke} = \frac{\pi}{4} \times 5^2 \times 54 \times \frac{6}{12} \text{ ft.-lbs.}$$

$$\text{Useful horse-power expended in pumping} = \frac{\pi \times 25 \times 54 \times 6}{4 \times 12} \times \frac{180 \times 2}{33{,}000}$$

$$= 5{\cdot}78 \text{ H.P., and}$$

$$\text{Horse-power required to drive the pump} = \frac{5{\cdot}78 \times 100}{85} = 6{\cdot}8 \text{ H.P.}$$

$$\text{Electrical horse-power supplied to motor} = 6{\cdot}8 \times \tfrac{100}{90} = 7{\cdot}55 \text{ E.H.P.}$$

$$\frac{\text{Volts} \times \text{ampères}}{746} = 7{\cdot}55$$

$$\text{Current in ampères} = \frac{7{\cdot}55 \times 746}{200} = 28{\cdot}16.$$

Three-Throw Pump.—In cases where water is required under a constant *high* pressure the three-throw pump is frequently used. In this pump the crank shaft has three cranks arranged at 120°, each crank driving a pump plunger in its own cylinder. By this means large cylinders are avoided, and if each pump cylinder is double acting there are two working strokes per revolution of the crank in each cylinder, making in all six working strokes per revolution. If the pump is single acting there will be three working strokes per revolution; by this means a continuous supply of water at fairly constant pressure is obtained without, using an air vessel.

EXAMPLES XXVI.

1. The barrel of a lift pump is 8 inches diameter, and when the bucket is at the bottom of its stroke the height of the entrance of the spout to the pump

barrel is 80 feet. What will be the greatest tension in the rod lifting the bucket; and, if the stroke of the bucket is 1 foot, how many gallons will be lifted per stroke?

2. A single acting force pump has to deliver water against a head of 250 feet. If the diameter of the plunger is 4 inches, and the efficiency of the pump 85 per cent., what must be the average force exerted on the pump plunger during the delivery stroke?

3. If the stroke of the pump in Question 2 is 4 inches, and it makes 120 delivery strokes per minute, what horse-power will be required to drive it? If the slip is 5 per cent., how many gallons will it deliver per minute?

4. Find the horse-power required to raise 200 cubic feet of water per minute to a height of 100 feet by a pump whose efficiency is 70 per cent.

5. A double acting force pump has a plunger 6 inches diameter and the length of the stroke is one foot. The total head is 300 feet, and the pump makes 40 double strokes per minute. Assuming no slip, find the discharge of the pump in gallons per minute. A steam engine drives this pump direct. Find the necessary diameter of the steam cylinder, assuming the mean effective steam pressure to be 80 lbs. per square inch, and the mechanical efficiency of engine and pump together 80 per cent.

6. A pump is directly driven by an electric motor whose efficiency is 85 per cent., and delivers 100 gallons of water per minute under a head of 125 feet. The efficiency of the pump is 80 per cent., and the motor works at 200 volts. What current must be supplied to the motor? If the price of a Board of Trade Unit is one penny, what will be the cost of running the pump for one hour?

CHAPTER XXVII

WATER IN MOTION

WHEN water flows from a tank through a sharp-edged orifice, the amount flowing in a given time depends upon the depth or head of water in the tank and the size of the orifice, and if these are known the flow is fairly definite, and an orifice can be used to measure

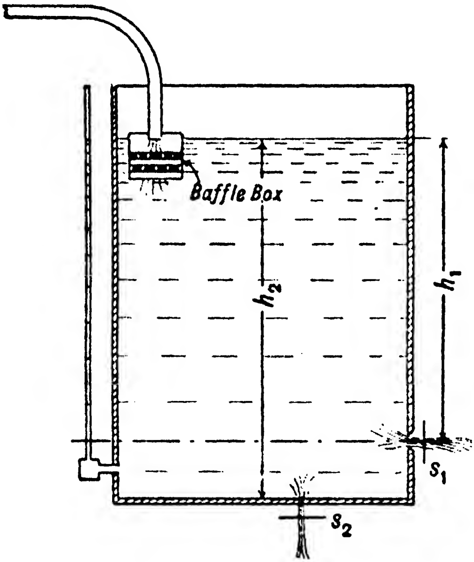


FIG. 248.

considerable quantities of water consumed at a steady rate. Orifices are often circular, but other shapes may also be employed. The stream on issuing contracts to a smaller cross-section than the area of the orifice as indicated in Fig. 248. The fraction which the sectional area at S_1 or S_2 bears to the full area of the orifice is

called the coefficient of contraction of the orifice. If the level of still water is h_1 or h_2 feet above the centre of an orifice (Fig. 248), the velocity of outflow is nearly that of a body falling freely through the height h_1 or h_2 as the case may be, namely $\sqrt{2gh_1}$ or $\sqrt{2gh_2}$ (see p. 256, Chap. XX.). The ratio which the actual velocity v at, say, S_1 bears to the velocity $\sqrt{2gh_1}$ due to falling h_1 feet is called the coefficient of velocity, and is generally about 0·97.

The amount of water passing, say, per second, would be represented by a cylinder having a sectional area equal to the stream at S_1 and length v, or—

$$Q = a_1 \times v = a_1 \times 0{\cdot}97 \times \sqrt{2gh_1}$$

where Q = discharge in cubic feet per second, a_1 = area of section of contracted stream in square feet = area of orifice in square feet multiplied by the coefficient of contraction.

If a = area of orifice in square feet, this may also be written

$$\begin{aligned} Q &= a \times \text{coefficient of contraction} \times 0{\cdot}97 \times \sqrt{2gh_1} \\ &= a\sqrt{2gh_1} \times \text{coefficient of discharge} \\ &= ka\sqrt{2gh_1} \end{aligned}$$

where k the coefficient of discharge is equal to the product of the coefficients of velocity and contraction. The value of k for circular orifices is about 0·62 and does not greatly differ for other shapes. The coefficient of contraction is about 0·64, so that—

$$0{\cdot}97 \times 0{\cdot}64 = 0{\cdot}62$$

Experiment 1. To verify that Q is proportional to $\sqrt{h}$.—The simple apparatus shown in Fig. 248 may be conveniently used for this purpose. A wrought-iron tank is fitted with a gauge consisting of a glass tube to which is attached a graduated scale. The level of the water in this tube will be the same as in the tank. An orifice is fitted to the side of the tank near the bottom as at S_1. The water-level is kept constant by allowing water to flow into the tank from the top at the same rate as water runs out through the orifice. The pipe which admits water into the tank has a baffle box on the end to still down whirling and eddies as far as possible, the idea being to keep the water in the tank at rest as far as possible.

The following results were obtained in a particular experiment using a sharp-edged circular orifice. Keeping the head constant, water was allowed to flow through the orifice for 1 minute, the quantity W discharged in this time being collected and weighed; the discharge for different heads is tabulated below.

Plotting W^2 and h the straight line through the origin O shown in Fig. 249 is obtained, showing that W^2 is directly proportional to h or that W is proportional to $\sqrt{h}$. It should be noticed that there is no need to measure the area of the orifice for this experiment, because the

discharge Q cubic feet is represented by W pounds to another scale, and to show the proportionality between Q^2 and h the discharge Q may be expressed in any units.

Discharge W (lbs.).	Head h (feet).	W^2.
10·97	3·0	120·34
10·03	2·5	100·90
8·95	2·0	80·10
7·75	1·5	60·06
6·30	1·0	39·69

Experiment 2. To find the Coefficient of Discharge of an Orifice.—Proceeding as in the above experiment the discharge for various heads is measured in pounds per second. Knowing the area of

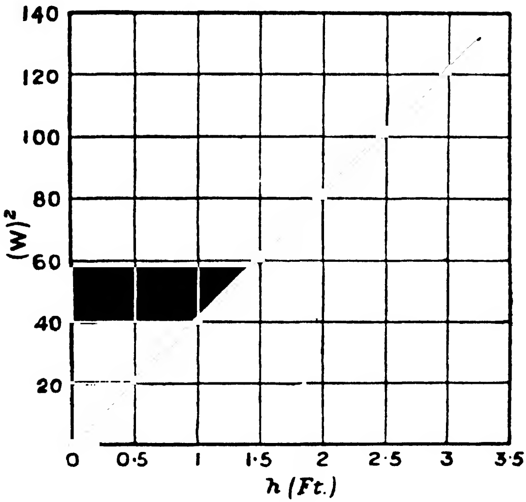


FIG. 249.

the orifice, the coefficient of discharge may be calculated. The results obtained with different-shaped orifices are shown in the table on p. 353.

Example 1.—How many cubic feet will be discharged per hour through a sharp-edged circular orifice ½ inch diameter under a constant head of 4 feet, the coefficient of discharge being 0·62?

Area of orifice $= \frac{\pi}{4} \times \left(\frac{1}{2}\right)^2 = 0{\cdot}1963$ square inch $= \frac{0{\cdot}1963}{144}$ square feet

$$Q = ka\sqrt{2gh}$$

$$= 0{\cdot}62 \times \frac{0{\cdot}1963}{144} \times \sqrt{64{\cdot}4 \times 4} \text{ cubic feet per second}$$

$$= 0{\cdot}62 \times \frac{0{\cdot}1963}{144} \times \sqrt{64{\cdot}4 \times 4} \times 3600 \text{ cubic feet per hour}$$

$$= 48{\cdot}73 \text{ cubic feet per hour.}$$

Discharge per second		Orifice.	Head. h feet.	Coefficient of discharge. $k = \frac{Q}{a\sqrt{2gh}}$
W lbs.	Q cubic feet $= \frac{W}{62{\cdot}4}$			
0·183	0·00293	Circular orifice ¼ inch diameter = 0·000340 square foot.	3·0	0·619
0·167	0·00268		2·5	0·628
0·149	0·00239		2·0	0·620
0·129	0·00207		1·5	0·619
0·105	0·00168		1·0	0·614
0·233	0·00374	Square orifice ¼ inch side = 0·000434 square foot.	3·0	0·618
0·213	0·00341		2·5	0·618
0·190	0·00305		2·0	0·621
0·165	0·00264		1·5	0·619
0·134	0·00215		1·0	0·619

Example 2.—Find the diameter of a sharp edged circular orifice to give a discharge of 600 gallons per hour under a constant head of 10 feet.

$$\text{Discharge} = 600 \times \frac{10}{62{\cdot}4} \text{ cubic feet per hour}$$

$$= 600 \times \frac{10}{62{\cdot}4} \times \frac{1}{3600} = \text{cubic feet per second}$$

$$= \frac{1}{37{\cdot}5} \text{ cubic foot per second}$$

hence

$$\frac{1}{37{\cdot}5} = 0{\cdot}62 \times a \times \sqrt{64{\cdot}4 \times 10}$$

$$\frac{1}{37{\cdot}5} = 0{\cdot}62 \times a \times 25{\cdot}4$$

$$a = \frac{1}{37{\cdot}5 \times 0{\cdot}62 \times 25{\cdot}4}$$

$$= 0{\cdot}001693 \text{ square foot}$$

$$= 0{\cdot}001693 \times 144 = 0{\cdot}244 \text{ square inch}$$

and $$\frac{\pi d^2}{4} = 0{\cdot}244$$

$$d^2 = \frac{0{\cdot}244 \times 4}{\pi}$$

$$d = \sqrt{\frac{0{\cdot}244 \times 4}{\pi}} = 0{\cdot}557 \text{ inch diameter.}$$

Head and Energy of Water.—Water may possess energy of two important kinds. For instance, if it is at a considerable height it possesses potential energy which may be converted into kinetic energy by allowing the water to fall freely. In Fig. 250 *one pound* of water at A, the top of a tank being at a level h feet above the levels of B and C, possesses potential energy

$$h \times 1 = h \text{ foot-pounds.}$$

If the valve V is opened, neglecting any resistance to flow, the water issues at C with a velocity $v = \sqrt{2gh}$, and its kinetic energy (see p. 286, Chap. XXII.) per pound is—

$$\frac{1}{2g} \times v^2 = \frac{2gh}{2g} = h \text{ foot-pounds}$$

exactly the same as for the potential energy at A.

If the valve V is closed, water at the lower level B is under a pressure corresponding to the head h of water, and may be employed in doing work such as driving a piston or plunger. If p is the pressure in pounds per *square foot* we have seen that (p. 337)

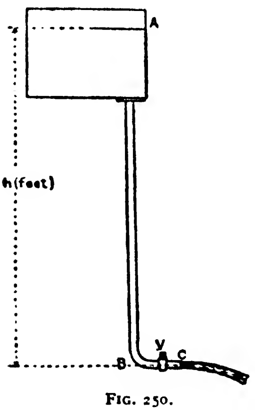


FIG. 250.

$$p = wh = 62{\cdot}4h$$

where w is the weight of one cubic foot of water.

The volume through which this one pound of water would displace a plunger is $1/w$ cubic foot, and the work it could do is—

$$p \times \frac{1}{w} = wh \times \frac{1}{w} = h \text{ foot-pounds}$$

exactly the same energy as one pound at A. The energy thus

transmitted per pound of water under a pressure of p pounds per *square foot* used is then—

$$\frac{p}{w} \text{ foot-pounds.}$$

This quantity is spoken of as the "pressure head" of water, *i.e.* the number of feet of water required to produce the pressure p pounds per square foot.

Similarly the quantity $\frac{v^2}{2g}$ is called the "kinetic head" of water moving with a velocity v.

When potential or other energy is converted into kinetic energy work is done, and in a pipe full of flowing water the work done between two sections in accelerating the water is that of a higher pressure at one section pushing forward against a lower pressure at another where the velocity is higher. The total head in a pipe, if resistances are negligible, remains constant. Thus, if in Fig. 251 water flows from a tank through a pipe of variable cross-section, so that the velocity v is different in different parts, namely, v_A at section A, v_B at section B, and v_C at section C, all in feet per second, and the pressures at A and B (pounds per square foot) are p_A and p_B respectively, the total head of water is the same at A, B, C and D, but is made up as follows :—

HEADS OF WATER (FIG. 251).

Section.	Potential or gravitational head.	Pressure head.	Kinetic head.	Total head.
D	h	0	0	h
A	h_A	$\frac{p_A}{w}$ or $\frac{p_A}{62 \cdot 4}$	$\frac{v_A^2}{2g}$ or $\frac{v_A^2}{64 \cdot 4}$	$h_A + \frac{p_A}{62 \cdot 4} + \frac{v_A^2}{64 \cdot 4} = h$
B	h_B	$\frac{p_B}{w}$ or $\frac{p_B}{62 \cdot 4}$	$\frac{v_B^2}{2g}$ or $\frac{v_B^2}{64 \cdot 4}$	$h_B + \frac{p_B}{62 \cdot 4} + \frac{v_B^2}{64 \cdot 4} = h$
C	0	0	$\frac{v_C^2}{2g}$ or $\frac{v_C^2}{64 \cdot 4}$	$\frac{v_C^2}{64 \cdot 4} = h$

$$\text{or } h = h_A + \frac{p_A}{62 \cdot 4} + \frac{v_A^2}{64 \cdot 4} = h_B + \frac{p_B}{62 \cdot 4} + \frac{v_B^2}{64 \cdot 4} = \frac{v_C^2}{64 \cdot 4}$$

Example 1.—Water flows in a pipe of varying section from a reservoir, the surface of which is 50 feet above the outlet of the pipe into the air. Neglecting friction, find the velocity of the water leaving the open

end of the pipe and the pressure 20 feet above this outlet, if the velocity of the water at this level is 18 feet per second.

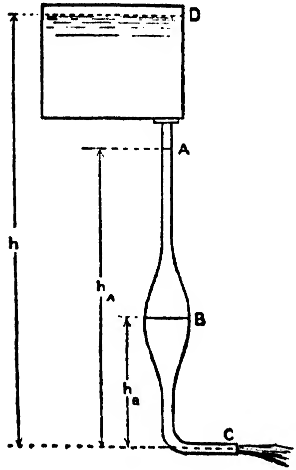


FIG. 251.

Let v be the velocity in feet per second, then—

$$\frac{v^2}{2g} = 50 \text{ or } v = \sqrt{64 \cdot 4 \times 50} = 56 \cdot 6 \text{ feet per second}$$

And at the level 20 feet above the outlet, since the total head made up of (1) potential or gravitational head, (2) kinetic head, and (3) pressure head is equal to 50 feet,

$$20 + \frac{18^2}{64 \cdot 4} + \frac{p}{62 \cdot 4} = 50$$

$$20 + 5 \cdot 03 + \frac{p}{62 \cdot 4} = 50$$

$p = 24 \cdot 97 \times 62 \cdot 4 = 1557$ lbs. per square foot

Example 2.—A pipe is running full of water and discharges 6 cubic feet per second. At a certain point A, the cross-section of the pipe is 2 square feet, and at B 20 feet vertically below A the cross-section is 0·5 square foot. Find the difference of pressure at A and B. The total head at A is equal to the total head at B. Equating these in feet,

$$h_A + \frac{v_A^2}{64 \cdot 4} + \frac{p_A}{62 \cdot 4} = h_B + \frac{v_B^2}{64 \cdot 4} = \frac{p_B}{62 \cdot 4}$$

$$v_A = \frac{6}{2} = 3 \text{ feet per second, and } v_B = \frac{6}{0 \cdot 5} = 12 \text{ feet per second.}$$

$$(h_A - h_B) + \frac{v_A^2}{64 \cdot 4} = \frac{v_B^2}{64 \cdot 4} + \frac{p_B - p_A}{62 \cdot 4}$$

$$20 + \frac{3 \times 3}{64 \cdot 4} = \frac{12 \times 12}{64 \cdot 4} + \frac{p_B - p_A}{62 \cdot 4}$$

$$\frac{p_B - p_A}{62 \cdot 4} = 20 \cdot 14 - 2 \cdot 23 = 17 \cdot 81$$

$$p_B - p_A = 17 \cdot 81 \times 62 \cdot 4$$
$$= 1111 \cdot 3 \text{ lbs. per square foot.}$$

Flow of Water in Pipes.—When water flows with a fairly high velocity through pipes there is a considerable resistance to motion and considerable loss of head by the water, so that the assumption that the total head remains constant would not be correct. The loss of "head" of water due to resistances is approximately proportional to the square of the velocity

(unless the velocity is very small), and also depends upon the size and condition of the pipe. For round pipes the head lost in l feet length of pipe is roughly—

$$\text{Loss of head (in feet)} = 0{\cdot}03\frac{l}{d} \times \frac{v^2}{2g}$$

where d = diameter of pipe in feet, v = velocity in feet per second $g = 32{\cdot}2$; but the coefficient varies considerably, and 0·03 is merely an average value.

Example.—Find how much water will flow per hour through a water main 18 inches diameter, and one mile long from a reservoir 20 feet above the outlet of the pipe.

$$h = \frac{0{\cdot}03lv^2}{2gd}$$

Here $h = 20$ feet, $l = 5280$ feet, $d = 1{\cdot}5$ feet

$$v^2 = \frac{h \times 2gd}{0{\cdot}03 \times l} = \frac{20 \times 64{\cdot}4 \times 1{\cdot}5}{0{\cdot}03 \times 5280} = 12{\cdot}2$$

$$v = \sqrt{12{\cdot}2} = 3{\cdot}5 \text{ feet per second}$$

$$\text{Area of main} = \frac{\pi}{4} \times (1{\cdot}5)^2 = 1{\cdot}767 \text{ square feet}$$

$$\begin{aligned}\text{Discharge per second} &= 1{\cdot}767 \times 3{\cdot}5 \text{ cubic feet}\\ &= 1{\cdot}767 \times 3{\cdot}5 \times 3600\\ &= 22{,}260 \text{ cubic feet per hour.}\end{aligned}$$

Force of a Jet.—When a steady stream of water strikes an object it exerts a pressure upon it. Suppose a horizontal jet of water impinges on a vertical surface (Fig. 252), and the water flows off at right angles to its original direction, then the force exerted by the jet on the surface is equal and opposite to that exerted by the surface on the water. This force serves to change the direction of the water, and, in fact, to completely destroy its velocity and its momentum in the direction of the jet. We have seen in Chapter XXI. that the force is equal to the change of momentum which it causes per second. Hence, if we know how many pounds of water strike the surface per second, and its velocity, we can calculate the total change of momentum per second, which is the force exerted by the jet on the surface. Thus, suppose the velocity of the jet is 10 feet per second, and it carries 80 lbs. of water to the surface per

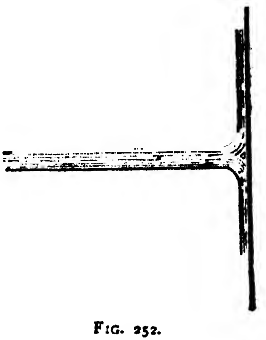


FIG. 252.

second, then the momentum of the water reaching the surface per second is—

$$\text{mass} \times \text{velocity} = \frac{80}{32 \cdot 2} \times 10 = 24 \cdot 8 \text{ units}$$

and the change per second, or the force exerted is 24·8 lbs.

In most forms of water motors, such as water wheels and water turbines, the flow of water against a blade or vane drives the machine, and the driving force exerted by the water is estimated by calculating the change of momentum of the water between entering and leaving the machine, but as the initial and final directions of the water are generally inclined to one another, the change of momentum is to be measured by vectors and not by an arithmetic difference.

Example 1.—A horizontal jet of water carrying 1000 lbs. of water per minute strikes a vertical wall with a velocity of 40 feet per second. What force will be exerted on the wall?

$$\text{Weight of water per second} = \tfrac{1000}{60} \text{ lbs.}$$
$$\text{Force} = \text{momentum change per second}$$
$$= \frac{1000}{60 \times 32 \cdot 2} \times 40 = 20 \cdot 7 \text{ lbs.}$$

Example 2.—A jet of water 2 inches diameter moving with a velocity of 80 feet per second impinges at right angles on a fixed plate. Calculate the momentum of the water striking the plate per second, and the force exerted on the plate.

Cross-sectional area of jet $= \frac{\pi}{4} \times 2^2 = \pi$ square inches $= \frac{\pi}{144}$ square feet.

$$\text{Volume of water reaching the plate per second} = \frac{\pi}{144} \times 80 \text{ cubic feet.}$$

$$\text{Weight of water reaching the plate per second} = \frac{\pi \times 80}{144} \times 62 \cdot 4 = 108 \cdot 9 \text{ lbs.}$$

$$\text{Momentum per second} = \frac{108 \cdot 9}{32 \cdot 2} \times 80 = 270 \text{ units.}$$

$$\text{Force exerted on the plate} = \text{momentum change per second}$$
$$= 270 \text{ lbs.}$$

Water Wheels.—Large wheels were formerly in considerable use to develop the power in streams for driving mills, and are still used to some extent, being of simple construction and easily kept in good working order. There are two principal types.

(1) **The Overshot Wheel** (Fig. 253) on which the water flows at the highest level and from which it leaves near the lowest

level. The total fall is nearly equal to the diameter of the wheel, and the energy available for each pound of water flowing is nearly d foot-pounds where d is the diameter of the wheel in feet.

Example. — The cross-section of the stream supplying an overshot wheel is 4 square feet, and its velocity of flow is 2·5 feet per second. The total height of fall is 20 feet. If the efficiency of the wheel is 70 per cent., what will be its brake horse-power?

Volume of water supplied to wheel per second = 4 × 2·5 = 10 cubic feet.

Weight of water per second 10 × 62·4 = 624 lbs.

Energy given out by this weight in falling 20 feet = 20 × 624 = 12,480 foot-pounds.

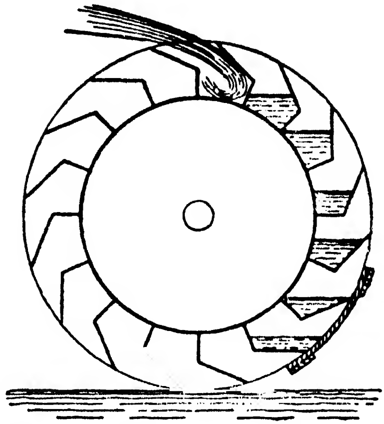


FIG. 253.—Overshot water wheel.

Horse-power given by water to the wheel = $\frac{12480}{550}$, and the B.H.P. is 70 per cent. of this, hence—

$$\text{Brake horse-power of wheel} = \frac{12480}{550} \times \frac{70}{100} = 15{\cdot}8 \text{ B.H.P.}$$

(2) **Undershot Wheel** (Fig. 254).—In this type the water is allowed to flow down and attain such a velocity that, impinging on

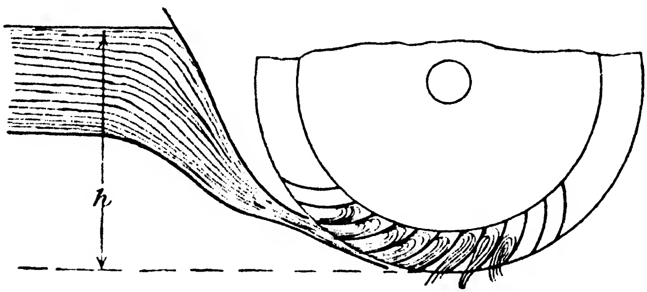


FIG. 254.—Undershot water wheel.

the blades of the wheel near the bottom of the wheel, it exerts the driving force, converting its kinetic energy into available work on the driving-shaft. This type of wheel is called an *impulse* wheel.

Example.—The cross-section of the stream supplying an undershot wheel is 3 square feet, and its velocity of flow is 2·5 feet per second. The level of the stream is 5 feet above the bottom of the wheel, and the efficiency of the wheel is 60 per cent. What will be the B.H.P. of the wheel?

Volume of water striking wheel per second $= 3 \times 2{\cdot}5 = 7{\cdot}5$ cubic feet.
Weight of water „ „ „ „ $= 7{\cdot}5 \times 62{\cdot}4 = 468$ lbs.
Work done on the wheel per second $= 468 \times 5$ foot-pounds.

The work available on the driving shaft per second is 60 per cent. of this, namely, $\frac{60}{100} \times 468 \times 5$ foot-pounds.

$$\text{Brake horse-power of wheel} = \frac{60 \times 468 \times 5}{100 \times 550} = 2{\cdot}55 \text{ B.H.P.}$$

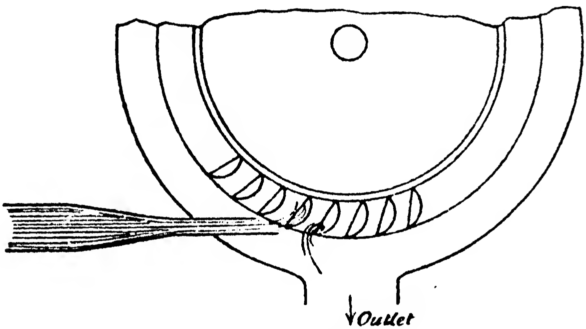


FIG. 255.—The Pelton wheel.

The Pelton Wheel.—A modern form of impulse wheel, suitable for high falls, is the Pelton wheel (Fig. 255) in which water, under the full available head, is led to a nozzle from which it issues in a jet of high velocity, and plays on the blades or buckets of the wheel which are shaped as shown in Fig. 256. The bucket (Fig. 256) is so shaped that if at rest it would as nearly as possible reverse the velocity of flow v. But, as the buckets are moving forward with a velocity which is generally about half that of the water (say, $\frac{1}{2}v$), the velocity of the water relative to the wheel on striking the bucket is $v - \frac{1}{2}v = \frac{1}{2}v$, and, on leaving, its *relative* velocity is still nearly $\frac{1}{2}v$ reversed in direction, and its actual

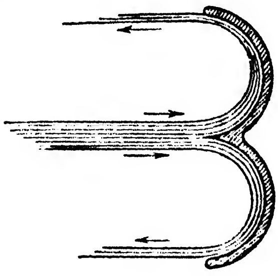


FIG. 256.

velocity ($\frac{1}{2}v - \frac{1}{2}v = 0$) is almost zero, the whole of its kinetic energy, except for losses, being given to the wheel. The principal losses are friction in the pipe, jet, and buckets, and those due to the fact that all the water cannot strike the successive buckets in the ideal position and direction. The efficiency of the wheel is often as high as 80 per cent.

Example.—5 cubic feet of water are supplied per second to a Pelton wheel under a head of 125 feet. The area of the nozzle is 0·1 square foot, and the B.H.P. of the wheel is 56; find (1) Velocity of the jet; (2) work available per second; (3) efficiency of the wheel.

(1) Velocity of water × area = 5 cubic feet.

$$v = \frac{5}{0{\cdot}1} = 50 \text{ feet per second.}$$

(2) Weight of water striking wheel per second = 5 × 62·4 = 312 lbs.

$$\begin{aligned}\text{Available energy} &= 312 \times \text{head}\\ &= 312 \times 125 \text{ foot-pounds}\\ &= 39{,}000 \text{ foot-pounds.}\end{aligned}$$

(3) 56 H.P. = 56 × 550 = 30,800 foot-pounds per second.

$$\text{Efficiency of wheel} = \frac{\text{useful work}}{\text{work supplied}} = \frac{30{,}800}{39{,}000} = 0{\cdot}79 \text{ or } 79 \text{ per cent.}$$

Turbines.—Turbines are water wheels dealing with water at comparatively high velocity or pressure, or both. There are many varieties, but they are roughly classed as impulse or pressure (or reaction) machines, according as the available energy is, or is not, transformed wholly into kinetic energy before entering the wheel. The Pelton wheel may be classed as a water wheel or as an impulse water turbine. In pressure or reaction turbines, the wheel is enclosed in a case containing guides which direct the water on to the wheel blades, the pressure in this case being above atmospheric pressure. Thus the water entering the wheel has both pressure energy and kinetic energy; on leaving, it has no energy except the small amount of kinetic energy corresponding to the velocity with which it flows to waste. Fig. 257 shows the arrangement of guides and blades in a radial inward-flow pressure turbine. The water fills the casing C, and is directed by the guides G, obliquely on to the blades B, on which it exerts a pressure, turning the wheel in a contra-clockwise direction, in the figure. The water flows through the wheel, pressing it forward and being deflected itself in a backward direction so that on leaving the inside of the wheel it has a velocity relative to the wheel opposed to the direction of rotation, and an *actual* velocity which is radially inward; that is, it has no velocity in the direction of motion of the blade. The greater part of the energy of the water is thus

spent in driving the wheel. The blades, or vanes, are so shaped at A as to be in the direction of the velocity of the entering water

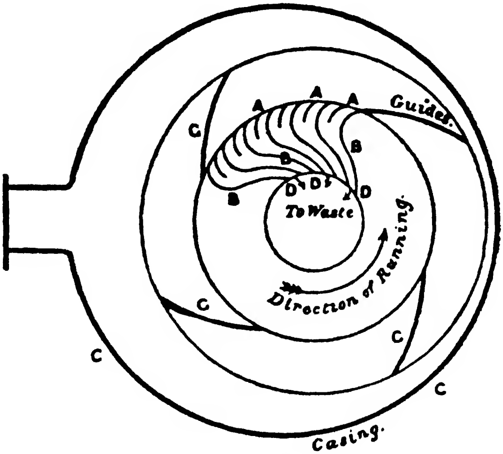


FIG. 257.—Inward flow pressure turbine.

relative to the wheel, so that the water may enter smoothly without shock. At D, similarly, they are shaped so that they slope in the

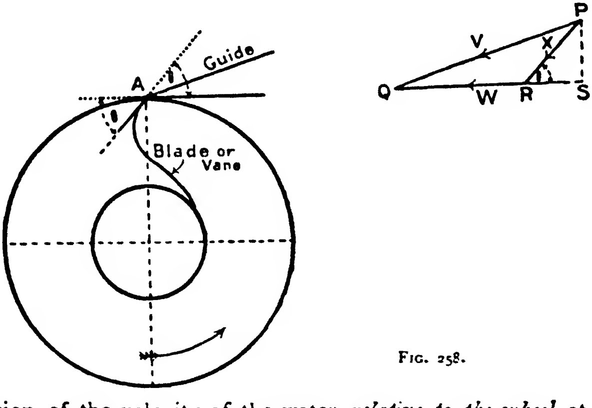


FIG. 258.

direction of the velocity of the water *relative to the wheel* at exit. The angles of the blade tips may accordingly be determined by vectors giving the velocity of the water relative to the wheel.

For example, if in Fig. 258 PQ represents the velocity, V, of the water at A (parallel to the guide vane) in magnitude and direction, and RQ represents the velocity, W, of the circumference of the wheel (*i.e.* the tips of the moving blades) at A, the velocity of the water relative to the wheel is PR or X, for in vectors—

$$X + W = V$$

or $X = V - W$ the velocity of the water relative to the wheel (see Chap. XX., p. 263).

Hence the angle, PRS, or θ, gives the proper angle for the vane at A, and this is shown on the blade to the left of Fig. 258.

The radial inward velocity of the water is represented by PS (*i.e.* the component $X \sin \theta$) and the tangential velocity relative to the blade is represented by SR (*i.e.* the component $X \cos \theta$). Sometimes the vanes are radial at A, that is, $\theta = 90°$.

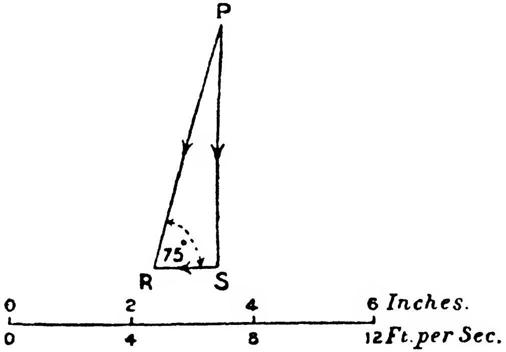


FIG. 259.

Example 1.—If water enters a turbine wheel at a radial velocity of 8 feet per second and the blades are inclined 75° to the circumference, find the velocity of the water relative to the blades and the tangential component of this relative velocity.

Choosing a convenient scale, say, 1 inch to 2 feet per second, draw PS (Fig. 259) 4 inches long representing the radial velocity of 8 feet per second. Draw SR at right angles to PS, and from P draw PR inclined $90 - 75 = 15°$ to PS, or 75° to SR. Then PR represents in magnitude and direction the velocity of the water relative to the blade; it will be found to scale 4·14 inches, or $4{\cdot}14 \times 2 = 8{\cdot}28$ feet per second. The vector SR represents the tangential component of this relative velocity; it scales 1·07 inches or $1{\cdot}07 \times 2 = 2{\cdot}14$ feet per second. These results may also be found by calculation, thus—

$$\text{Relative velocity PR} = \frac{\text{radial velocity (PS)}}{\sin 75^\circ}$$

$$= \frac{8}{0{\cdot}9659} = 8{\cdot}28 \text{ feet per second as before.}$$

$$\text{Tangential component SR} = \text{radial velocity} \times \tan 15^\circ$$

$$= 8 \times 0{\cdot}2678$$

$$= 2{\cdot}14 \text{ feet per second as before.}$$

Example 2.—Water enters a turbine wheel at an angle of 30° to the circumference with a velocity of 65 feet per second. If the speed of the circumference of the wheel is 45 feet per second, find the velocity of the water relative to the wheel and the proper angle for the blades.

Using a scale of 10 feet per second to 1 inch, draw RQ (Fig. 260)

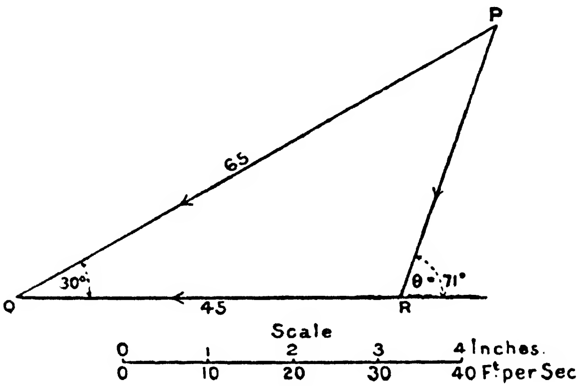


FIG. 260.

4·5 inches long to represent the velocity of the circumference, namely 45 feet per second. Draw QP 30° to RQ and 6·5 inches long to represent the velocity of the water, namely 65 feet per second. Then PR which scales 3·44 or 34·4 feet per second is the velocity of the water relative to the wheel. The angle θ measures 71°, and this will be the angle which the tips of the blades must make with the circumference of the wheel.

Centrifugal Pumps.—If water were led into the exhaust end of a turbine such as Fig. 257, and the turbine were driven in the reverse direction (*i.e.* clockwise) water would be pumped from the centre to the outside of the wheel, and would be forced from the casing C to a certain height. Such an arrangement is called a centrifugal pump, and with modifications of the blades to suit the various conditions of speed, such pumps are largely used for lifting water.

Example.—The radial velocity of water in a centrifugal pump is 5 feet per second; the vanes make an angle of 35° with the outer circumference. What is the velocity of the water relative to the wheel, and what is the tangential component of this velocity?

Using a scale of 2 feet per second to 1 inch, draw SP $2\frac{1}{2}$ inches long (Fig. 261) to represent the radial velocity of 5 feet per second.

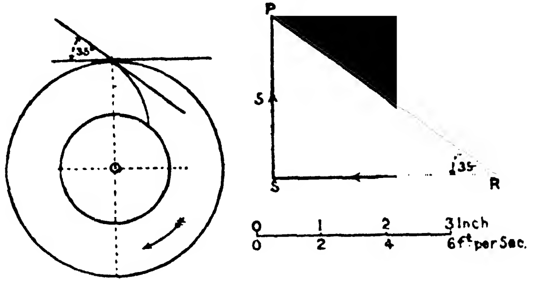


FIG. 261

Draw SR at right angles to PS and RP inclined 35° to SR to meet in R. Then RP which scales 4·35 inches, or 8·7 feet per second represents the required relative velocity, and RS, which scales 3·6 inches or 7·12 feet per second represents the tangential component of this velocity.

This result may also be found by calculation as follows:—

$$\text{Relative velocity RP} = \frac{5}{\sin 35} = \frac{5}{0{\cdot}5736}$$

$$= 8{\cdot}7 \text{ feet per second as above.}$$

$$\text{Tangential component RS} = \frac{5}{\tan 35^\circ} = \frac{5}{0{\cdot}700}$$

$$= 7{\cdot}14 \text{ feet per second.}$$

EXAMPLES XXVII.

1. How many cubic feet will be discharged per hour through a circular sharp-edged orifice 1 inch diameter under a constant head of 12 feet, the coefficient of discharge being 0·62?

2. Find the diameter of a sharp-edged circular orifice to give a discharge of 65·9 gallons per hour, under a head of 3 feet.

3. A sharp-edged circular orifice, ¼-inch diameter, discharges 10·03 lbs. per minute of water under a constant head of 2·5 feet. Calculate the coefficient of discharge.

4. Water flows in a pipe of varying section from a reservoir, the surface of which is 80 feet above the outlet of the pipe into the air. Neglecting friction, find the velocity of the water leaving the open end of the pipe, and the pressure 50 feet above the open end, if the velocity of the water at this level is 20 feet per second.

5. A pipe is running full of water, and discharges 8 cubic feet per second. At a certain point, A, the cross-section is 4 square feet, and at B, 10 feet vertically below A, the cross-section is 2 square feet. Find the difference of pressure between A and B.

6. A horizontal pipe, 1 foot diameter, is 1 mile long. The pipe discharges 4 cubic feet of water per second when running full. Find the loss of head in feet between the two ends of the pipe $\left(h = \dfrac{0{\cdot}03lv^2}{2gd}\right)$.

7. A horizontal jet of water, 4 inches diameter, moving with a velocity of 50 feet per second, strikes a vertical wall. Calculate the force exerted by the jet on the wall.

8. Find, from the following data, the horse-power available in a given water fall :—

Available height of fall, 120 feet; cross-section of the stream, 10 square feet; velocity of the stream, 100 feet per minute.

9. The cross-section of a stream supplying an overshot water wheel is 3 square feet, and its velocity of flow is 2·5 feet per second. The total height of fall is 12 feet. If the efficiency of the wheel is 70 per cent., what will be its brake horse-power?

10. The cross-section of a stream supplying an undershot water wheel is 4 square feet, and its velocity of flow is 2 feet per second. The surface of the stream is 6 feet above the bottom of the wheel, and the efficiency of the wheel is 60 per cent. Calculate the B.H.P. of the wheel.

11. Three cubic feet of water are supplied per second to a Pelton wheel under a head of 250 feet. The area of the nozzle is 0·1 square foot, and the B.H.P. of the wheel is 68. Find (1) velocity of the jet; (2) work available per second; (3) efficiency of the wheel.

12. Water enters a turbine wheel with a radial velocity of 7 feet per second, and the blades are inclined 60° to the circumference; find the velocity of the water relative to the blades, and the tangential component of the velocity.

13. Water enters a turbine wheel at an angle of 35° to the circumference, with a velocity of 80 feet per second. If the speed of the circumference of the wheel is 60 feet per second, find the velocity of the water relative to the wheel, and the proper angle for the blades.

14. The radial velocity of water in a centrifugal pump is 6 feet per second; the vanes make an angle of 33° with the circumference. What is the velocity of the water relative to the wheel, and what is the tangential component of this velocity?

15. The vanes of a centrifugal pump make an angle of 35° with the circumference. The velocity of the circumference of the wheel is 65 feet per second, and the radial velocity of the water is 5 feet per second. Find (1) the velocity of the water relative to the vanes; (2) the actual velocity of the water leaving the wheel.

16. Neglecting frictional losses, to what height may the water be lifted by the pump in Question 15?

Angle.		Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
Degrees.	Radians								
0°	0	0	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	[illegible]
						0212	0253	0294	0334	0374	8	8	12	16	20	24	28	32	[illegible]
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	[illegible]
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	[illegible]
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	[illegible]
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	[illegible]
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	[illegible]
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	[illegible]
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	[illegible]
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	[illegible]
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	[illegible]
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	[illegible]
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	[illegible]
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	[illegible]
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	[illegible]
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	[illegible]
18	2553	2577	2601	2525	2648						2	5	7	9	12	14	16	19	[illegible]
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	[illegible]
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	[illegible]
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	1
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	1
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	[illegible]
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	1
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	[illegible]
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	1
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	[illegible]
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	1
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	1
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	[illegible]
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	1
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	[illegible]
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	1
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	1
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	1
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	1
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	1
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	1
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	1
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	1
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	1
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	1
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	

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51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	9
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7280	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7529	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	6	5
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	3	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	7915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9479	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	8647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9003	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9091	9996	0	1	1	2	2	3	3	3	4

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·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2513	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2814	1	1	2	3	3	4	4	5	[illegible]
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	9
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	4610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

ANSWERS TO EXAMPLES

Examples I. Page 23.

(1) 24·1 lbs. (2) 49·4 lbs. (3) 25·6 lbs.; 30° to the 35 lb. force.
(4) 18·1 lbs. and 22·5 lbs. (5) 2·19 tons; 2·68 tons.
(6) Jib 4·06 tons; tie-rod 2·19 tons. (7) 38·5 lbs.; 66° north of west.
(8) 23·4 lbs.: 33·2° north of west.
(9) 21·5 lbs. north-west; 101·5 lbs.; 60° east of north.
(10) 19·8 tons compression; 49·5 tons tension.

Examples II. Page 38.

(1) 3350 lb.-ft. (2) 45 lb.-ft. contra-clockwise.
(3) 14 lb.-inches; Q = 6·66 lbs.; 10·5 lb.-inches; P = 4·70 lbs.
(4) 105·5 lbs. (5) 140·9 lbs. (6) 849 lbs.
(7) $4\frac{3}{8}$ cwt. at R.H.; $4\frac{1}{8}$ cwt. at L.H. (8) 10·13 tons at R.H.; 6·87 tons at L.H.
(9) 648 lbs.; 1061 lbs. (10) 199·5 lbs. at E; 280·5 lbs. at D.
(11) R_F = 225 lbs.; W = 200 lbs.; R_F = 200 lbs.; W = 225 lbs.
(12) 165 lbs. (13) 9·18 inches from the weight of 17 lbs.
(14) 59·13 lbs. (15) 24·38 lbs. (16) 10,120 lbs. (17) 23·9 lbs.

Examples III. Page 50.

(1) 3·19 inches. (2) 10·25 lbs.; $3\frac{9}{11}$ inches; $\frac{10}{11}$ inch.
(3) 99·1 lbs. per square inch above atmospheric. (4) 123·4 lbs.
(5) 271 lbs.; 386 lbs. at $57\frac{1}{2}°$ to horizontal. (6) 252 lbs. at Q; 496 lbs. at P.

Examples IV. Page 60.

(1) Tie-rod = 3500 lbs.; jib = 9800 lbs.
(2) Tie-rod = 7000 lbs.; jib = 13,300 lbs.
(3) (1) Tie = 5250 lbs.; jib = 9800 lbs.
(2) Tie = 7000 lbs.; jib = 11,550 lbs.
(3) Tie = 6120 lbs.; jib = 10,680 lbs.
(4) AD = 22 cwt. comp.; BD = 38·1 cwt. comp.; DC = 19·05 cwt. tension.
(5) AD = BE = 5434 lbs. comp.; DE = 1672 lbs. tension; DC = EC = 4750 lbs. tension.
(6) AD = BF = 7·16 tons comp.; DC = FC = 6·26 tons tension; EC = 5·32 tons tension; DE = EF = 1·47 tons tension.
(7) BC = 1385 lbs. tension; CA = 693 lbs. comp.; DC = 1385 lbs. comp.; DB = 1385 lbs. tension.
(8) AD = BF = 2750 lbs. tension; DE = EF = 5500 lbs. tension; DC = EC = FC = 5500 lbs. comp.

(9) Members in tension:—DA = EA = 1·15 tons; EF = 2·31 tons; GH = 8 tons; GA = HB = 3·45 tons. Members in comp.:—DC = 2·31 tons; FG = CF = 2·31 tons; HC = 6·93 tons. Stress in DE = 0.
(10) DE = FG = HK = LM = 5000 lbs. tension; FA = KB = 3000 lbs. tension; CD = EF = KL = MC = 4000 lbs. comp.; CE = CL = 3000 lbs. comp.; CG = CH = 6000 lbs. comp.; DA = MB = GH = 0.

EXAMPLES V. PAGE 73.

(1) (*a*) 141·75; (*b*) 2′ − 1½″; (*c*) 1194⅜ lbs. (2) (*a*) 5·18 feet; (*b*) 67 inches. (3) 20,697,600 ft.-lbs. (4) 95,000,000 ft.-lbs. (5) 10,603 ft.-lbs. (6) 6963 ft.-lbs. (7) 9163 ft.-lbs. (8) 200,277 ft.-lbs. (9) 500 lbs. (10) 28·8 lb.-ft. (11) 31·25 inch-lbs.; 15·94 inch-lbs. (12) 67·5 inch-lbs.; 39·37 inch-lbs. (13) 2,208,937 ft.-lbs. (14) 194,400 ft.-lbs. (15) 384,200 ft.-lbs. (16) 220·26 ft.-lbs. (17) 5·21; 32·6 per cent. (18) 799·26 lbs. (19) 2·97 lbs.; 13·4 per cent. (20) 149⅓ lbs.; 0·613 ft. per second.

EXAMPLES VI. PAGE 83.

(1) 0·357. (2) 67·5 lbs. (3) 1674 lbs. (4) 192·8 lbs. (5) 0·225; 20·5 lbs. inclined 102° 42′ to the 4·5 lb. force. (6) 564·48 ft.-lbs. (7) 103·4 lbs. (8) 791·68. (9) 0·268. (10) 17·13 lbs. (11) 38·24 lbs. (12) 4·04 lbs. (13) 60·3. (14) 60 lbs.; 79,200 ft.-lbs. (15) 420 tons; 7920 ft.-tons. (16) 9,504,000 ft.-lbs.

EXAMPLES VII. PAGE 97.

(1) 41·5 lbs. (2) 226·2; 55 per cent. (3) 2·78; 25,846 ft.-lbs. (4) (*a*) 48 per cent.; (*b*) 52 per cent.; (*c*) 53 per cent. (5) (*a*) 12 lbs.; (*b*) 33 lbs. (6) 147·4 E.H.P.

EXAMPLES VIII. PAGE 105.

(1) Steel on oak, F = 0·16W; brass on oak, F = 0·18W; oak on oak, F = 0·50W; brass on steel, F = 0·15W; coefficients of friction, 0·16, 0·18, 0·50, 0·15.
(2) $t^2 = 0·1022l$. (3) $w = 0·00549p$. (5) P = 1·5 + 0·61W; F = 2·7 + 0·15W. (6) P = 0·09 + 0·072W; F = 2·633 + 0·789W. (7) $H = 1082 + 0·305t$. (8) W = − 430 + 21·19 I.H.P. (9) W = 0·372 + 0·828 B.H.P.

EXAMPLES IX. PAGE 121.

(1) 13·57 H.P. (2) 0·297 H.P. (3) 254,130. (4) 6·10. (5) 6·06. (6) 1858·5. (7) 3·98. (8) 110·4. (9) 60·6. (10) 72 per cent. (11) 68·2. (12) 2*s*. 0¾*d*. (13) 4·36. (14) 102·1. (15) 10·27. (16) 3798. (17) 13,428,000. (18) 5·5 ampères; 0·183 pence. (19) 89·02 I.H.P (20) 276·34.

EXAMPLES X. PAGE 141.

(1) (*a*) 205·7 revs. per min.; (*b*) 199·5 revs. per min. (2) 25 inches. (3) 280 revs. per min.; 672 revs. per min. (4) 10·68. (5) 21·81. (6) 1650 lbs.; 2970 lbs. and 1320 lbs.; 37½ inches. (7) 21·85 inches. (8) 25·4 (9) 24; 2·35 inches. (10) 100. (11) (*a*) 187·5; (*b*) 250; (*c*) 125 revs. per min. (12) 216. (13) 275 lbs. (14) 235·7 revs. per min. (15) 23⅓ feet per min. (16) 3535 lbs.

EXAMPLES XI. PAGE 157.

(1) (*a*) 10 lbs.; (*b*) 10·06 lbs. (2) 79·3 lbs. (3) (*a*) 46·5; (*b*) 5·4.
(4) 1 in 100. (5) 288. (6) 39·6 lbs. (7) 125·7.
(8) 9·8 tons. (9) 4·6 tons. (10) 18,095 lbs. (11) 6750 revs. per min.
(12) 3375 revs. per min. (13) 7·87 lbs. (14) 50·3 per cent.
(15) 2·13 per cent. (16) 33·9 tons.

EXAMPLES XII. PAGE 175.

(1) 30. (2) 480 lbs.; 46·6 per cent. (3) 330 lbs. (4) 24.
(5) (*a*) 302 lbs.; (*b*) 241·6 lbs. (6) 90·7 per cent. (7) 367 lbs.; 1430 lbs
(8) Drivers, 80 and 90 teeth; followers, 20 and 30 teeth.
(9) Driver, 20; follower, 80. (10) 26·6 revs. per min.
(11) With back gear out:—360, 216, 150, 90 revs. per min.
With back gear in:—40, 24, 16⅔, 10 revs. per min.

EXAMPLES XIII. PAGE 183.

(1) 13 lbs. (2) 56·94 lbs.; 38·7° to the 80 lb. force. (3) 114·9 lbs.; 315·7 lbs.
(4) Rafters, 2500 lbs. comp.; tie, 2165 lbs. tension.
(5) Tie, 9·66 tons tension; jib, 13·65 tons comp.
(6) 38·5 lbs.; 66° north of west. (7) 52·5 lbs.; 20·5° south of east.

EXAMPLES XIV. PAGE 189.

(1) 6·27 inches from apex of cone. (2) 5·44 inches from the 7-inch ball.
(3) 3·46 inches. (4) 3·46 inches. (5) 3·38 inches.
(6) 4·85 inches from the plane end of the cylinder.

EXAMPLES XVI. PAGE 214.

(1) 5·29 tons per sq. inch. (2) 0·0096 inch; 0·00096.
(3) 20,000 lbs. per sq. inch; 30,000,000 lbs. per sq. inch.
(4) 15,300,000 lbs. per sq. inch. (5) 158 lbs. (6) 0·0318 inch.
(7) 3·96 tons per sq. inch.; 13,700 tons per sq. inch.
(8) 8640 lbs. per sq. inch. (9) 0·64 inch. (10) 23·56 tons.
(11) $d = 1$ inch; $p = 2$ inches. (12) 50 per cent. (13) 0·8 inch.

EXAMPLES XVII. PAGE 235.

(1) Bending moments:—142, 64, 16, 0 tons-feet. Shearing forces:—13, 8, 2 tons.
(2) 54·96, 37·68, 54·62, 57·48 tons-feet. (3) 75 tons-feet.
(4) 1·5 tons per sq. inch. (5) 1·18 tons per sq. inch. (6) 1·42 tons per sq. inch.
(7) 4·76 tons per sq. inch. (8) 13·4 feet. (9) 1·36 tons. (10) 0·26 inch.
(11) 1·01 inch. (12) 4·05 tons per sq. inch. (13) 990 lbs.; 3960 lbs.
(14) 15,000 lbs.

EXAMPLES XVIII. PAGE 241.

(1) 47,750 lb.-inches. (2) 7928 lbs. per sq. inch. (3) 216. (4) 2·07 inches.
(5) 430·5. (6) 14,137 lb.-inches; 0·47, say 0·5 inch. (7) 666⅔ lb.-inches.

Examples XX. Page 264.

(1) $733\frac{1}{3}$ yards ; 22 minutes 48 seconds.
(2) 49·2 miles per hour, *or* 72·16 feet per second. (3) 330 feet.
(4) 26·18 radians per second. (5) 25·13 radians per second ; 720 feet per min
(6) 23·57 radians per second ; 712·5 feet per min. (7) 76·6 feet per second.
(8) $1\frac{1}{3}$ feet per second per second. (9) 0·733 foot per second per second.
(10) (*a*) $\frac{11}{15}$ foot per second per second ; (*b*) 1760 feet ; (*c*) 3960 feet ; (*d*) 75 secs.
(11) 970·5 feet. (12) 3·52 seconds.
(13) 3 feet per second per second due east. (14) (*a*) 3 ; (*b*) 0·314.
(15) 186·5 feet per second per second in a direction 30·4° north of west.
(16) 13·24 feet per second ; 7·88 feet per second ; 15·4 feet per second.
(17) 25 feet per second in a direction 37° north of west. (18) 19·54 knots.

Examples XXI. Page 280.

(1) 612,170 units. (2) 30·6 lbs ; 38·01 units. (3) 201 units ; 67 units.
(4) 102 lbs. (5) 45·54 tons. (6) 33·7 feet per second. (7) 310·5 lbs.
(8) 3597 lbs. ; 143,850 units ; 6·9 feet per second.
(9) 1·15 feet per second per second ; 517·5 feet ; 4·64 H.P.
(10) 9200 lbs. ; 981 H.P. (11) 72·1 feet per second per second ; 1082 lbs.
(12) 27·4. (13) 337 feet ; 250 feet. (14) 168·8 lbs. ; (1) 180 lbs. ; (2) 191·2 lbs.
(15) 3730 lbs. (16) 20,700.
(17) (*a*) 57·09 feet per second per second ; (*b*) 18·50 feet per sec. ; (*c*) 0·0045 sec.
(18) 1·95 feet per second. (19) 0·175 radian per second per second.
(20) 37·5. (21) 133 seconds. (22) 262·3 lb.-feet ; 98,885 foot-lbs.

Examples XXII. Page 299.

(1) 7·83 per cent. (2) 35·3 lbs. (3) 12·19 E.H.P. ; 9·094 units ; 18·18 amps.
(4) 23·53 lbs. (5) 970·4 foot-lbs. (6) 50,504 foot-lbs. ; 397 feet.
(7) 7768 foot-lbs. (8) 769 feet per second. (9) 420 lbs. ; 0·97 inch.
(10) 5040 lbs. ; 2240 foot-lbs. (11) 13·52. (12) 26,832 lbs.
(13) 35·9 feet per second ; 26·2 feet per second. (14) 790,700 foot-lbs.
(15) 26·87 feet per second ; 85·2 revs. per min. (16) 600 foot-lbs.
(17) 109,670 foot-lbs. ; 205·3 foot-lbs. ; 534 revs. (18) 7263 lbs.
(19) 55,000 foot-lbs. ; 211,200 lbs. (20) 500,000 foot-lbs.
(21) 81·8 foot-lbs. ; 0·476 foot. (22) 110·9 foot-lbs. ; 0·57 foot.
(23) 172 revolutions per minute.

Examples XXIII. Page 313.

(1) 197·4 feet per second per second. (2) 7·35 lbs. (3) 2298 lbs.
(4) 6080 lbs. (5) 732 lbs. ; 0·083 H.P. (6) 44·8 lbs. (7) $18\frac{2}{3}$ lbs.
(8) 9·79 inches ; 0·69 inch. (9) 109·6 feet per second per second.
(10) 5884 lbs. (11) 153·2. (12) 31·23. (13) 1 to 1·0073.

Examples XXIV. Page 326.

(1) (1) 7·51 inches ; (2) 1·08 inches ; (3) 6·9 inches.
(2) 5·71 feet per second ; 9·42 feet per second ; 5·47 feet per second.
(3) 1·7. (5) 48.

EXAMPLES XXV. PAGE 341.

(1) 15·46 tons. (2) 41·4 lbs. (3) 70·4 per cent. (4) 24.
(5) 300 foot-tons; 990 lbs. per square inch. (6) 15·22 tons. (7) 2100 lbs.
(8) 16·3 pence. (9) 302,400 foot-lbs.; 0·533.
(10) 4·57 feet; 8000 lbs.; 5213 lbs.; 2280 lbs. (11) 2948 lbs.
(12) (*a*) 17·5 tons; (*b*) $23\frac{1}{3}$ feet from surface of water; (*c*) 204·1 tons-feet.
(13) 15·762 feet from the surface. (14) 5·937 cubic feet.
(15) 15·55 lbs.; 96·45 lbs.

EXAMPLES XXVI. PAGE 348.

(1) 1742 lbs.; 2·18 gallons. (2) 1600 lbs. (3) 1·94 H.P.; 20·72 gallons.
(4) 54 H.P. (5) 98·01; 8·54 inches. (6) 20·78 amps.; 4·156 pence.

EXAMPLES XXVII. PAGE 365.

(1) 338·4. (2) 0·25 inch. (3) 0·619.
(4) 71·7 feet per second; 1484·5 lbs. per sq. foot. (5) 612·3 lbs. per sq. feet.
(6) 63·7 feet. (7) 422·8 lbs. (8) 226·9 H.P. (9) 7·15. (10) 3·26.
(11) (1) 30 feet per second; (2) 46,800 foot-lbs.; (3) 80 per cent.
(12) 8·08 feet per second; 4·04 feet per second.
(13) 46·2 feet per second; $83\frac{1}{4}°$ to the circumference.
(14) 11·01 feet per second; 9·23 feet per second.
(15) (1) 8·7 feet per second; (2) 58 feet per second. (16) 52 feet.

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